

Van Hiele's Theory and Think Pair Share Cooperative Learning Model and Their Effect on Madrasah Tsanawiyah Student's Level of Mathematical Thinking

Vivi Fitri Falentina¹, Arif Muchyidin², Toto Syatori Nasehudin²

¹SMP Innovative School Cianjur, Cianjur, Indonesia

² Department of Mathematics Education, IAIN Syekh Nurjati, Cirebon, Indonesia

Article Info

ABSTRACT

This study aimed to determine the effect of applying van Hiele's theory using a Think Pair Share (TPS) cooperative learning model on students' mathematical thinking levels. Following the research objectives, the researchers used experimental and quantitative research methods. In this study, two sampling techniques were used, namely using, purposive and random samples. Based on the sampling technique, class VIII F is the experimental class, and class VIII G is the control class with 32 students each. Based on the results of the analysis, it was found that the level of students' mathematical thinking after the implementation of van Hiele's theory used the TPS type cooperative learning model with an average post-test score of 63.38, which was considered sufficient, and based on the results of hypothesis testing using t-test calculations, $t\text{-count} = 3.825 > t\text{-table} = 2.04$. This means that applying van Hiele's theory utilizing a Think Pair Share (TPS) cooperative learning model affects the level of students' mathematical thinking.

This is an open-access article under the [CC BY-SA](#) license.



Corresponding Author:

Vivi Fitri Falentina
SMP Innovative school Cianjur, Indonesia
Email: vfitri14@gmail.com

1. INTRODUCTION

The quality of students' thinking is different, and the quality of students' thinking in mathematics can be seen in the way students think and solve a problem. The thinking process carried out by students in solving problems uses inductive and deductive methods depending on the thinking skills they have. Deductive way of thinking that distinguishes mathematical thinking from other ways of thinking. According to Mason [1], [2], mathematical thinking is a dynamic process that expands the scope and depth of mathematical understanding. In this process, a specialization process is carried out (specialization, paying attention to some exceptional cases or examples), a generalization

process (generalizing, focusing on a larger group of examples, looking for patterns and relationships), guessing (making guesses about the problem at hand, predicting relationships and outcomes), and belief (building beliefs about the understanding that has been built, looking for and communicating reasons why something is true).

A preliminary study of students at SMPN 1 Tengah Tani using interviews, questionnaires and tests found that the level of mathematical thinking possessed by students was still lacking, and geometry lessons were the subjects that students disliked the most because of monotonous learning using the methods that what had used so far. To develop thinking skills in mathematics, what can support it needs to be considered. Mathematical thinking skills can be supported by creating an atmosphere where self-confidence can grow, which is necessary but not enough[3].

In several studies, it is stated that the evidence in the field shows that the learning outcomes of geometry are still low and need improvement [4], [5]. In fact, among the various branches of mathematics, geometry occupies a position of the most concern [6]. It was found that there are still many junior high school students who do not understand the concepts of geometry. In addition, geometry is a material that is difficult to master after fractions and math problems in the form of stories [7], [8], and there are still many junior high school students who do not understand geometric concepts [9]–[12].

Mathematics lessons have several theories educators can apply to convey their material, one of which is van Hiele's theory. Van Hiele's theory, developed by Pierre Marie van Hiele and Dina van Hiele-Geldof around the 1950s, has been internationally recognized and has strongly influenced geometry in schools. Practical application of van Hiele's theory to improve the quality of students' thinking [13]–[15]. Van Hiele explained the stages of students' thinking (cognitive) level in geometry, namely: introduction (level 0), analysis (level 1), sorting (level 2), deduction (level 3), and rigour/accuracy (level 4) [10]. In each stage in van Hiele's theory, students are expected to be able to develop their way of thinking from essential to complex stages [14]–[17].

Mathematical thinking levels include knowledge, use of concepts, problem-solving, and reasoning or analysis. In this case, the development in understanding mathematical concepts is contained in van Hiele's learning theory as a theory to find out the mathematical thinking possessed by students to create an effective learning process in the realm of geometry. A meaningful mathematics learning process is a way to develop students' mathematical thinking [18]. Therefore, it is hoped that van Hiele's learning theory

is expected to be able to apply the principles and stages. Several studies that have been conducted have proven that the application of van Hiele's theory has a positive impact on learning geometry and provides an increase in students' mathematical thinking patterns[19]–[21].

A meaningful mathematics learning process is a way to develop students' mathematical thinking. Therefore, it is hoped that van Hiele's learning theory is expected to be able to apply the principles and stages. In addition, a learning process requires a suitable learning model to convey the learning objectives. One of the learning models that can be used is the cooperative learning model. The cooperative learning model is a learning model that consciously and systematically combines interactions between students as a life practice in actual society [22], [23]. Think Pair Share (TPS) is a suitable method that gives students time to think, respond, and help each other [24], [25].

Several studies that have been conducted have proven that the application of van Hiele's theory has a positive impact on learning geometry and provides an increase in students' mathematical thinking patterns. Looking at the problems that arise, namely the lack of understanding of students' geometric concepts and van Hiele's theory is believed to impact geometry learning positively. So it is necessary to conduct empirical studies to prove its means of research.

2. METHOD

For the research objective, namely, to determine the effect of the application of van Hiele's theory using the TPS type cooperative learning model on students' mathematical thinking levels, the researchers used a quantitative experimental approach. The design used is a static group comparison. In this design, there are already other groups as external standards. The research design was carried out by testing all respondents who were used as samples or experimental classes in this study. The population in this study were all students of class VIII MTs KHAS Kempek totalling 256 students. Researchers used two sampling techniques, the first using a purposive sample and the second using a random sample.

The purposive sampling technique was carried out by looking at several considerations based on school regulations, namely the separation of male and female classes. For male classes, teachers are only for men, while for female classes, there is no regulation. Because of these considerations, the researcher took a sample of the female

class. After agreeing with the school, the researcher used a second sampling technique, namely random sampling or lottery. This sampling is done by writing the name of the class, which consists of four groups, namely E, F, G, and H; then the paper is rolled up, and one of the papers and papers that already contains the written class name, in this case, the class that will be sampled in the study. So that the obtained class VIII F as the experimental class, class VIII E as the experimental class, and class VIII G as the control class.

The research instrument used to collect data is a test instrument (Post-test), which is a test used to measure students' mathematical thinking levels after using van Hiele's theory with the Think Pair Share (TPS) cooperative learning model and a questionnaire used to determine student responses to the application Van Hiele's approach uses the Think Pair Share (TPS) cooperative learning model..

3. RESULTS AND DISCUSSION

The test was carried out in the experimental class with 32 students participating. The calculated test results obtained the average test score was 63.38 with a standard deviation of 12.68. In addition, who calculated the post-test results in the control class and the average test score was 48.75 with a standard deviation of 12.73. The maximum score obtained is 73 achieved by a student, and the minimum score obtained is 27, owned by four students.

After knowing the average value of each class, it is necessary to compare it with the ideal standard curve to determine the average trend of the data. Based on the known data, a comparison table of the average value with the standard curve of the ideal criteria [26] can be made as follows:

Table 1. Comparison of Average Value with Normal Curve

Class Type	Test type	Score	Comparison Criteria
Experiment	<i>Post test</i>	63,38	High
Control	<i>Post test</i>	48,75	Moderate

After the comparison, based on table 1 it can be seen that the post-test scores in the experimental class are more significant than those in the control class. The experimental class is included in the high criteria, while the control class is included in the moderate. It can be concluded that learning using van Hiele's theory combined with a Think Pair Share (TPS) cooperative learning model produces more optimal learning.

After knowing the average value in the experimental class is greater than the control class. Students are grouped based on the level of mathematical thinking according to van Hiele's theory. Data on the level of mathematical thinking of experimental class students as a whole are presented in the following table:

Table 2. Student's Mathematical Thinking Level Data

Van Hiele's Level of Thinking	The number of students
Level 0 (recognition)	5
Level 1 (analysis)	10
Level 2 (sequence)	17
Total students	32

Based on table 2, it can be seen that most of the students have reached level 2. This means that students are at an advanced stage in the sequence process.

A good learning process will produce good results as well. At least two critical aspects are needed to structure mathematics learning: the nature of mathematics and the level of development of students' mathematical thinking. The nature of mathematics is abstract, axiomatic, symbolic and deductive, which are generally difficult for students to understand. In this case, the teacher's role is very significant in the success of student learning by adjusting the concept of learning mathematics based on the level of development of students' mathematical thinking.

A series of lessons is needed for a good understanding of the concept, namely by studying the initial concept first to understand the second or subsequent concepts. Van Hiele's theory is a series of mathematics learning in geometry that has been applied and is believed to help the learning process according to the level of students' thinking. The characteristics of van Hiele's theory are applied when the research reaches the third stage [17] because the VIII MTs students, on average, have the ability in the third stage.

In the first stage, the researcher and students ask questions about the circle sub-material that will be studied by asking and giving students time to answer basic questions such as understanding, differences or similarities, and the reasons for answering them. The questions are intended to determine the student's initial abilities. In the second stage, students are given tasks that involve different relationships according to the material that has been arranged in sequence by the teacher. Until the third stage, the teacher introduces the terminology of the material and requires students to try to express it in their language. The teacher's role is to bring the object being studied to the level of understanding through discussion between students in their language. The teacher introduces relevant

mathematical terminology when students demonstrate the studied object and discuss it in their language.

Students who have received good learning will never forget the lesson. From the memories that students have, it can be drawn the ability of a student; If students can remember the learning events that the teacher has given, it will be known how their abilities are. Students' abilities can be seen from their learning outcomes; In the previous discussion, the research results on student responses to the application of van Hiele's theory using the Think Pair Share (TPS) cooperative learning model and learning outcomes regarding students' mathematical thinking levels have been stated.

Based on the results of data analysis, it is known that the learning process using van Hiele's theory combined with the Think Pair Share (TPS) cooperative learning model is strong or good, with an average interpretation of 73.56%. This condition is influenced by how the material is delivered based on the level of thinking or the sequence of material that students can accept. The learning process is not monotonous. This is indicated by the differences in students' enthusiasm and learning methods in the experimental class and control class according to the response of each indicator.

Student responses to the first indicator, namely students' interest in learning mathematics, amounted to 74.42%, which was classified as solid or reasonable; Based on research during the learning process, most students have studied the material to be studied in class; it can be seen by preparing to bring the tools to be used in learning, bringing examples of objects in everyday life, reading and working on questions on the subject. Worksheets, as well as paying attention to and following each stage of learning starting from the think, pair, and share stages properly.

The second indicator, namely changes in student learning patterns, lies at 79.95%, which is classified as solid or reasonable. Seen at the stage where students are required to understand the language, they try to analyze carefully what they should do to students who are used to accepting axioms and applying them to problems without knowing the reason. After applying van Hiele's theory, students can make meaning in their language to understand clearly and know the similarities or differences. When students are asked to explain their work regarding the calculation of the circumference of a circle with a known radius, while the formula they know to calculate the circumference of a circle is to use diameter, with the understanding and properties understood, students change the value of the radius to the value of the diameter so that they can solve the problem. In addition,

students become more active when learning in class. The increase in student responses at the pair and share stage was seen in the first, second, and subsequent meetings.

The third indicator, presentation of information, has an average of 74.22%, classified as solid or fair; researchers who act as teachers during the learning process act as facilitators and mediators. During the learning process, the teacher explains questions that students have not understood and mediate the statements that students debate. In naming an angle whose magnitude is 180° , some students say that the diameter forms an angle, and some say that the angle is a straight line.

In the fourth indicator, namely strengthening students' abilities and understanding, an average of 74.61% is classified as solid or reasonable. When the sharing process has been carried out, the teacher straightens the students' opinions according to the axioms. In the fifth indicator, observing group activity, an average of 72.66% is classified as solid or reasonable. The teacher, in this case, observes group discussion activities by observing each group and helps if there are groups that are having difficulties.

The sixth indicator helps students evaluate the problem-solving results, which is 69.53% classified as solid or reasonable. The problem evaluation activity encountered problems. Namely, most students have difficulty when given questions or the application of circular material in daily activities. In applying van Hiele's theory at level three, namely deduction, students are challenged to accept lessons so that when asked questions about the application of thinking levels, students who are still at level two, namely informal deduction, have not been able to complete them correctly. Even some students cannot finish it at all.

In the seventh indicator, namely the advantage of learning in pairs, an average of 73.05% is classified as solid or reasonable. Students look excited during the pairing process by uniting their respective opinions to be presented. In the eighth indicator, think has an average of 72.27%, classified as solid or reasonable. This can be seen when the teacher asks questions and gives students time to think and use them well. However, the habit carried out during learning is memorizing an axiom or formula that makes it difficult for students to think and takes a very long time to answer the questions given by the teacher.

Student responses are sufficient to support the level of mathematical thinking that students have, which is the way students have after receiving mathematics learning. The

results of the research that has been carried out show that students' average level of thinking is quite adequate, with an average value of 63.38.

The average post-test value represents an indicator which is the level of students' mathematical thinking, with the number of students at the first level (level-0) being five students, at the second level (level-1) are ten students, and at the third level (level-2) is 17 students. Based on the values obtained, the researchers stated that thinking about the material being studied was very helpful in the basic skills acquired. This means that if the initial indicators have been completed, students will more easily understand the material in the following indicator, and vice versa; if the initial indicators have not been mastered, students will have more difficulty understanding the next indicator. This can be seen from the difference in the average post-test value obtained by the experimental and control groups.

Based on the average value obtained from the experimental and control classes to determine whether there is an effect of applying van Hiele theory using the Think Pair Share (TPS) cooperative learning model, conclusions can be drawn by calculating the hypothesis test. After calculating the value of $t_{count} = 3.83 > t_{table} = 2.04$, it means that there is an effect. Based on the calculation of hypotheses and student responses that are classified as good when learning, this study can say that the application of learning materials using van Hiele's theory using a Think Pair Share (TPS) type of learning model influences students' thinking levels.

Because this research can be said that there is an influence, this research can be said to be relevant to research that has been done previously. Van Hiele's theory considered one of the pieces of evidence in overcoming the problem of learning mathematics in geometry material, can be proven in this study where the situation of students who have difficulty thinking is helped by the application of van Hiele's theory [27]–[29]. In addition, the contextual approach was more significant in improving critical thinking skills than conventional learning [30]–[33]. This conclusion follows what the researchers did. Thus, it can be said that the results of research conducted on applying van Hiele's theory using a Think Pair Share (TPS) cooperative learning model on students' mathematical thinking levels have supported previous studies.

4. CONCLUSION

Based on the results of the discussion on the effect of applying van Hiele's theory using the Think Pair Share (TPS) type cooperative learning model on students' mathematical thinking levels, it can be concluded that: The level of students' mathematical thinking after applying van Hiele's theory using the Think Pair Share (TPS) cooperative learning model has an average post-test score of 63.38. This shows a value in the range of 55 - 69. So it belongs to the excellent category. Based on the value obtained, it can be seen that the level of mathematical thinking of students, namely, five students are at the first level (level-0), ten students are at the second level (level-1), and 17 students are at the third level (level-2), thus most of the students are at level 2. And based on the results of hypothesis testing using t-test calculations, it was obtained that $t_{\text{count}} = 3.83 > t_{\text{table}} = 2.04$. This means that applying van Hiele's theory using the Think Pair Share (TPS) cooperative learning model affects students' mathematical thinking levels.

REFERENCES

- [1] J. Mason, L. Burton, and K. Stacey, *Thinking mathematically*, 2nd ed. London: Prentice – Hall, 2010.
- [2] B. J. Mason, *Thinking Mathematically*. London: Pearson Education, 1982.
- [3] N. A. P. Yaman and Jailani, “Learning to Think Mathematically Through Reasoning and Problem Solving in Secondary School Mathematics : A Literature Review,” in *5th ICRIEMS Preceedings*, 2018, pp. 77–82.
- [4] A. Muljo, “Teaching Materials Based Development Of Art Traditional Geometri Kasab Aceh,” *5th ICRIEMS Proc.*, pp. 295–302, 2017.
- [5] R. Istiyani, A. Muchyidin, and H. Raharjo, “Analysis of Student Misconception on Geometry Concepts Using Three-Tier Diagnostic Test,” *J. Cakrawala Pendidik.*, vol. 37, no. 2, pp. 223–236, Jul. 2018, doi: 10.21831/cp.v37i2.14493.
- [6] W. N. Yanuarto, “Encouraging Students Creativity By Secret Inside Balloon Game In Geometry Class,” in *Proceeding of International Conference On Research, Implementation And Education Of Mathematics And Sciences*, 2015, no. May, pp. 17–19.
- [7] U. I. Kusuma and H. Retnawati, “Analysis of Sixth Graders’ Difficulties in Solving Mathematics Word Problems on Whole Numbers, Fractions, and Decimals,” *J. Phys. Conf. Ser.*, vol. 1320, no. 1, 2019, doi: 10.1088/1742-6596/1320/1/012008.
- [8] L. Sugiarti and H. Retnawati, “Analysis of student difficulties on algebra problem solving in junior high school,” *J. Phys. Conf. Ser.*, vol. 1320, no. 1, 2019, doi: 10.1088/1742-6596/1320/1/012103.
- [9] H. Suyitno, E. N. Eka, and R. B. Veronica, “The Development of Geometry Concepts Understanding Based on NCTM Reference in Learning Using Discovery Learning Model,” *Unnes J. Math. Educ.*, vol. 8, no. 1, pp. 42–48, 2019, doi: 10.15294/ujme.v8i1.28683.
- [10] A. K. D. Utami, Mardiyana, and I. Pramudya, “Analysis of junior high school students’ difficulty in

resolving rectangular conceptual problems," *AIP Conf. Proc.*, vol. 1868, no. August, 2017, doi: 10.1063/1.4995135.

[11] D. L. Sulistiowati, T. Herman, and A. Jupri, "Students' Geometry Skills Viewed from van Hiele Thinking Level," *5th ICRIEMS Proc.*, pp. 55-, 2018, [Online]. Available: http://seminar.uny.ac.id/icriems/sites/seminar.uny.ac.id.icriems/files/proceeding2018/ME07_Dwi_Laila_Sulistiowati.pdf.

[12] O. Sudarsono, A. Awalya, and I. Junaedi, "Jurnal The Ability to Complete Story Problems Reviewed from Van Hiele's Theory in Problem Based Learning," ... *Math. Educ.* ..., vol. 10, no. 1, pp. 77–83, 2021, [Online]. Available: <https://journal.unnes.ac.id/sju/index.php/ujmer/article/download/45247/18438>.

[13] Misnasanti and A. Mahmudi, "Van Hiele Thinking Level and Geometry Visual Skill towards Field Dependent-Independent Students in Junior High School," *J. Phys. Conf. Ser.*, vol. 1097, no. 1, 2018, doi: 10.1088/1742-6596/1097/1/012133.

[14] J. K. Alex and K. J. Mammen, "Geometrical sense making: Findings of analysis based on the characteristics of the van hiele theory among a sample of South African grade 10 learners," *Eurasia J. Math. Sci. Technol. Educ.*, vol. 12, no. 2, pp. 173–188, 2016, doi: 10.12973/eurasia.2016.1211a.

[15] A. H. Abdullah and E. Zakaria, "The Effects of Van Hiele's Phases of Learning Geometry on Students' Degree of Acquisition of Van Hiele Levels," *Procedia - Soc. Behav. Sci.*, vol. 102, no. Ifee 2012, pp. 251–266, 2013, doi: 10.1016/j.sbspro.2013.10.740.

[16] Baiduri, A. D. Ismail, and R. Sulfiyah, "Understanding the concept of visualization phase student in geometry learning," *Int. J. Sci. Technol. Res.*, vol. 9, no. 2, pp. 2353–2359, 2020.

[17] I. U. Machromah, C. K. Sari, and M. E. Rosaria, "Implementing Van Hiele Theory on Circle Module," *5 th ICRIEMS Proc.*, no. October, pp. 45–54, 2018, [Online]. Available: https://www.researchgate.net/profile/Christina_Sari/publication/336532810_Implementing_Van_Hiele_Theory_on_Circle_Module/links/5da4c58ea6fdcc8fc3527a9a/Implementing-Van-Hiele-Theory-on-Circle-Module.pdf.

[18] E. D. Minarti, R. Purwasih, and R. Sariningsih, "Mathematical Thinking Ability in Solving Mathematics Problems Consider Cognitive Styles of Field Independent and Field Dependent," *Proc. 5th Int. Conf. Res. Implement. Educ. Math. Sci. (5th ICRIEMS)*, pp. 597–602, 2017.

[19] H. T. N. Rizki, D. Frentika, and A. Wijaya, "Exploring students' adaptive reasoning skills and van Hiele levels of geometric thinking: A case study in geometry," *J. Phys. Conf. Ser.*, vol. 983, no. 1, 2018, doi: 10.1088/1742-6596/983/1/012148.

[20] R. B. Armah and P. S. Kissi, "Use of the van hiele theory in investigating teaching strategies used by college of education geometry tutors," *Eurasia J. Math. Sci. Technol. Educ.*, vol. 15, no. 4, 2019, doi: 10.29333/ejmste/103562.

[21] A. M. Ramlan, "The Effect of Van Hiele Learning Model Toward Geometric Reasoning Ability Based on Self-Efficacy of Senior High School Students," *J. Math. Educ.*, vol. 1, no. 2, pp. 63–72, 2016.

[22] L. Darling-Hammond, L. Flook, C. Cook-Harvey, B. Barron, and D. Osher, "Implications for

educational practice of the science of learning and development," *Appl. Dev. Sci.*, vol. 24, no. 2, pp. 97–140, 2020, doi: 10.1080/10888691.2018.1537791.

[23] L. Baloche and C. M. Brody, "Cooperative learning: exploring challenges, crafting innovations," *J. Educ. Teach.*, vol. 43, no. 3, pp. 274–283, 2017, doi: 10.1080/02607476.2017.1319513.

[24] P. Minh Huyen, D. Thi My Lan, and C. Author, "Journal of English Language Teaching and Applied Linguistics Using Think-Pair-Share Strategy to Support Students in Speaking Lessons," no. c, pp. 1–8, 2020, doi: 10.32996/jeltal.

[25] I. G. A. M. Sukelasmini, "The Implementation of Think Pair Share (TPS) Type of Cooperative Learning Model To Improve Student's Motivation And Nutrition Science Learning Achievement," *J. Educ. Action Res.*, vol. 3, no. 1, p. 9, 2019, doi: 10.23887/jear.v3i1.17081.

[26] A. Sudjono, *Pengantar Statistik Pendidikan*. Jakarta: Rajawali Pers, 2010.

[27] K. Kairuddin, ... B. S.-J. of M., and U. 2020, "Improvement of Students' High Order Thinking Skills (HOTS) Ability through the Application of Van Hiele Theory Assisted by Video Animation," *Journal.Unesa.Ac.Id*, vol. 2, no. 1, pp. 32–39, 2020, [Online]. Available: <https://journal.unesa.ac.id/index.php/JOMP/article/view/11908>.

[28] K. Wijayanti, A. Nikmah, and E. Pujiastuti, "Problem solving ability of seventh grade students viewed from geometric thinking levels in search solve create share learning model A R T I C L E I N F O," *Unnes J. Math. Educ.*, vol. 7, no. 1, pp. 8–16, 2018, doi: 10.15294/ujme.v7i1.21251.

[29] Y. D. Andila and E. Musdi, "Practicality of geometry learning set based on van hiele theory to increase students' mathematical communication ability," *J. Phys. Conf. Ser.*, vol. 1554, no. 1, 2020, doi: 10.1088/1742-6596/1554/1/012007.

[30] R. S. Wahyuningtyas and W. Wuryadi, "The influence of contextual teaching and learning (CTL) on critical thinking ability and conceptual understanding of skeletal system materials," *AIP Conf. Proc.*, vol. 2021, no. 2018, 2018, doi: 10.1063/1.5062828.

[31] M. Yusuf Nasution and S. Rezeqi, "Application of Contextual Learning to Improve Critical Thinking Ability of Students in Biology Teaching and Learning Strategies Class," *Int. J. Learn. Teach. Educ. Res.*, vol. 11, no. 3, pp. 109–116, 2015.

[32] R. S. Wahyuningtyas and W. Wuryadi, "The Influence of Contextual Teaching And Learning (CTL) towards Chritical Thinking and Problem Solving Ability on Skeletal System Materials," *5thICRIEMS Proc.*, pp. 117–123, 2018.

[33] S. Osa Kosassy, N. Gistituati, and M. Montesori, "The Effect of Contextual Learning Approach (CTL) to Improve Students' Critical Thinking Ability in Organization and Management Subject," vol. 178, no. ICoIE 2018, pp. 141–145, 2019, doi: 10.2991/icoie-18.2019.33.