The Impact of Releasing Domestic Dogs on the Spread of Rabies Disease and Its Prevention

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Abstract
The government's program to achieve rabies-free status by 2030 is an initiative to accelerate the eradication of rabies cases in Indonesia. Rabies is a highly contagious disease with a 100% fatality rate, and dogs are the main carriers of the virus. The government's efforts to minimize rabies cases include vaccination of susceptible animals, eliminating rabid dogs, and implementing dog population management (MPA). Field observations indicate that the practice of releasing domestic dogs allowing them to roam freely, has led to an increase in rabies cases. Using mathematical modeling, this husbandry system's impact can be simulated. Based on the model analysis, it is determined that a maximum of 15% of domestic dogs should be allowed to roam freely. If this threshold is exceeded, it becomes necessary to ensure a minimum of 20% vaccination coverage and eliminate at least 1% of rabid dogs.

Keywords: Dogs, Rabies Disease, Releasing Domestic Dogs.

INTRODUCTION

The national government's program is to make Indonesia rabies-free by 2030. However, in 2019, the number of provinces declared rabies-free decreased from the previous year, with only 8 out of 9 provinces declared rabies-free in 2018 [1]. This information highlights the government's seriousness in eradicating rabies to achieve the national program's goal. Government efforts to prevent rabies transmission include vaccination of susceptible animals, elimination of animals suspected of being exposed to rabies, and the implementation of dog population management (MPA).

Dogs are known as the main rabies transmitters, and the virus is fatal to all mammal species [2]. Out of 150 countries worldwide, in Asia and Africa, more than 95% of rabies deaths occur, with 40% being children under 15 years old. The total number of rabies-related deaths is approximately 59,000 people per year, with 99% of them caused by dog bites [3]. In Indonesia, the number of deaths due to rabies is still relatively high, ranging from 100 to 156 deaths per year, with an almost 100% fatality rate [4]. From 2015 to 2019, there were 404,306 recorded cases of rabies bites, resulting in 544 deaths.

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Regarding rabies incidence rates from 2018 to 2020, Bali has been identified as one of the regions with the highest incidence rates, with 26,130 cases in 2018, 37,372 cases in 2019, and almost 27,000 in 2020 [5]. A large number of stray dogs in Bali contributes to the increased spread of rabies, likely due to a lack of public education on responsible dog ownership [6]. As reported in 2021 [5], out of 96 people bitten by dogs in Kuta Utara Sub-district, 70% of them were bitten by stray dogs. Given these circumstances, further research is needed to analyze a mathematical model of the spread of rabies in the context of releasing domestic dogs.

The phenomena occurring in the development of rabies in dogs can be described through mathematical modeling. Analysis can be conducted analytically and through numerical simulation approaches. The analytical approach allows for obtaining exact solutions that closely approximate the actual solutions. It is crucial to understand the model's properties and the effects of the involved parameters, especially in the case of releasing dogs. By analyzing the differential equations analytically, we can identify the relationships between the involved subpopulations, determine equilibrium points, study the system's stability properties, and analyze the model's behavior. Once the analytical solutions are obtained, it is essential to complement them with numerical simulation approaches. We can obtain good approximations of the system's solutions

ISSN. 2087-2852
E-ISSN. 2338-1655

numerically with numerical methods, even when no analytical solutions are known. This approach allows us to model a more comprehensive system by considering a more significant number of parameters and conditions that occur in real-life situations.

This research utilizes a combination of quantitative data analysis and mathematical modeling. Epidemiological data on dog rabies cases, vaccination coverage, and dog demographics were collected from the affected area and used as comparative data. The data used in this study is the rabies cases data in dogs obtained from the Bali Provincial Department of Agriculture, 2008-2018 [7]. A mathematical model was developed to simulate the dynamics of rabies transmission in the context of releasing domestic dogs.

Self-modeling of rabies has been widely conducted, including [8], who introduced a simple model of rabies in dogs in Bali. The model consists of two compartments, namely S for healthy dogs and I for rabid dogs. The analysis results not only suggest the administration of vaccines to reduce the spread of rabies but also the need to limit the birth rate in the population of healthy dogs.

Based on the recommendations of a previous study [8], a model can be developed by dividing the dog population into two groups, stray dogs and domestic dogs, based on the level of interaction of each group. In 2018, a research [9] further developed the model proposed [8] by adding a vaccinated compartment (V), resulting in a model with three compartments. The analysis results indicated the need to reduce the birth rate and increase vaccination to reduce rabies transmission. Hailemichael *et al.* [10] discuss the population dynamics of rabies in dogs by dividing them into two categories: stray and domestic dogs. The basic model used is $SEIV$, resulting in a model consisting of eight compartments: four for stray dogs and four for domestic dogs. The model assumes that only stray dogs can transmit the disease and rabies is incurable. The designed model goes beyond a typical model by incorporating vaccination control and elimination measures. An effective method is to combine two measures: vaccination and elimination of rabies-infected dogs to reduce the spread of the rabies virus. The birth rate of the susceptible dog population, both domestic and stray dogs, is also an essential factor in the spread of rabies.

The mathematical model in this study is a development of the model proposed by Fitri *et al.* [8], dividing the dog population into two groups: domestic dogs and stray dogs, like the developed model by Nugraha *et al.* [6]. Then, the model is developed by adding a subpopulation of vaccinated/immune dogs, both domestic and stray, so that the model becomes similar to Roberta *et al.* [9]. The development of the model can help understand the impact of vaccination on the dog population. The designed model consists of four nonlinear differential equations. The impact of releasing domestic dogs and the influence of prevention measures such as vaccination and elimination are analyzed in the model. The study concludes with numerical simulations using the fourth-order Runge-Kutta method to strengthen the analytical results.

MATERIALS AND METHODS

This research utilizes a nonlinear differential equation approach to model a system involving the interaction (physical contact) between susceptible and infected subpopulations of dogs. This is because physical contact between susceptible and infected dogs can pose a risk for disease transmission. The interactions between subpopulations in the model are often nonlinear, meaning that changes in one subpopulation can affect the changes in other subpopulations. Therefore, this study employs a nonlinear differential equation approach to understand the development of the rabies subpopulations. Figure 1 shows cases of rabies in dogs in Bali from 2008 to 2018. The data is used as comparative data.

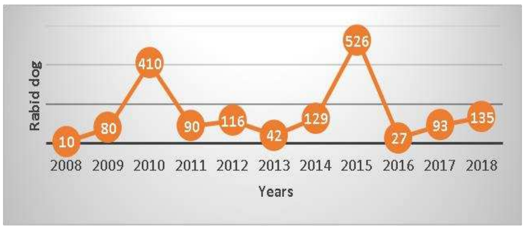

Figure 1. Rabid dogs in Bali, 2008-2018

Construction Model

In this study, the rabies transmission model in dogs consists of two population groups: stray dogs and domestic dogs. Each population group consists of susceptible and infected subpopulations. Both population groups assume there is interaction and only stray dogs can transmit the rabies disease. The model is expressed in the form of a nonlinear autonomous system.

The presence of released domestic dogs

ISSN. 2087-2852
E-ISSN. 2338-1655

contributes to the increasing population of stray dogs, especially those susceptible to rabies. A mathematical model was developed to simulate the dynamics of rabies transmission in the population of released domestic dogs. By analyzing this model, we can determine the threshold level of released domestic dogs to prevent increased rabies cases. If the level of released domestic dogs exceeds this threshold, it indicates an endemic tendency. Therefore, preventive measures such as vaccination and elimination of rabid dogs are implemented in this model.

Determination of the Equilibrium Points

The equilibrium points of a differential equation remain unchanged. Therefore, when the solution of a system is constant, equilibrium points are obtained. Generally, the equilibrium points of the model consist of two types: disease-free equilibrium point, which represents the extinction of rabies in dogs, and endemic equilibrium point, where rabies in dogs continues to persist.

Stability of the Equilibrium Points

The equilibrium point of a nonlinear model is asymptotically stable if the eigenvalues of the linearized Jacobian matrix have negative real parts. It is unstable if any of the eigenvalues of the linearized Jacobian matrix have positive real parts.

Numerical Simulation

Numerical simulations are performed to strengthen the analytical results and illustrate the model's behavior. The simulations are conducted using the fourth-order Runge-Kutta method. The parameter values and constraint level of released domestic dogs are determined to verify the results obtained analytically.

RESULT AND DISCUSSION

Model Formulation

Domestic dogs' susceptibility decreases when released, resulting in an increase in susceptible stray dogs. Both populations also increase due to births and decrease due to deaths and rabies transmission. Rabies transmission occurs through physical contact with infected stray dogs. This transmission process can be prevented through vaccination and elimination. If both susceptible domestic and stray dogs are late in prevention during transmission, the population of infected stray and pet dogs will increase. Rabid dogs, whether stray or domestic, decrease due to natural deaths.

Furthermore, we assume the existence of four subpopulations: susceptible dogs and dogs infected with rabies, each divided into stray and domestic dogs. Therefore, the model to be designed is similar to the interaction model between the two groups.

The susceptible (S_d) and infected (I_d) subpopulations of domestic dogs: Λ_d represents the recruitment rate of susceptible dogs, along with μ_d as the natural death rate. β_{ds} denotes the disease transmission rate within this group. The susceptible (S_s) and infected (I_s) subpopulations of stray dogs: Λ_s represents the recruitment rate of susceptible dogs (births) and μ_s as the natural death rate. The disease transmission rate within this group is denoted by β_{ss}. The rate of domestic dog release is donated by ϕ, resulting in a decrease in the susceptible subpopulation of domestic dogs and an increase in the susceptible subpopulation of stray dogs. The illustration of rabies transmission in stray and domestic dogs in the context of release is depicted in the diagram:

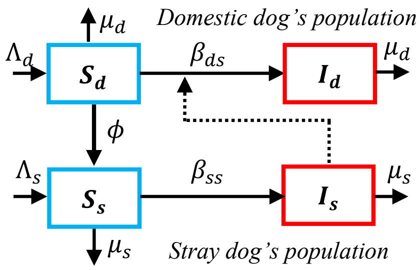

Figure 2. The compartmental model for rabies transmission

Model Equation

Based on Figure 2 above, the rabies transmission model with the presence of a system for releasing domestic dogs is formulated as the following system of nonlinear differential equations. The following Table 1 provides the model parameters along with their descriptions:

$$\left.\begin{aligned}\frac{dS_d}{dt} &= \Lambda_d - \phi S_d - \beta_{ds} S_d I_s - \mu_d S_d \\ \frac{dI_d}{dt} &= \beta_{ds} S_d I_s - \mu_d I_d \\ \frac{dS_s}{dt} &= \Lambda_s + \phi S_d - \beta_{ss} S_s I_s - \mu_s S_s \\ \frac{dI_s}{dt} &= \beta_{ss} S_s I_s - \mu_s I_s\end{aligned}\right\} \quad (1)$$

ISSN. 2087-2852
E-ISSN. 2338-1655

Table 1. Model Parameter

Symbol	Information	Value	Source
Λ_d	Number of domestic dog recruitment	120	[10]
Λ_s	Number of stray dog recruitment	32	[10]
β_{ds}	Rate of stray dog transmission to domestic dogs	0.00027	Assumed
β_{ss}	Rate of transmission from stray dog to stray dog	0.00054	[8]
μ_d	The natural death rate of domestic dogs	0.11	[10]
μ_s	The natural death rate of stray dogs	0.24	[10]
ϕ	The rate of releasing domestic dogs	[0,1]	Assumed

Disease-Free Equilibrium Points (DFE)

Equilibrium points when the left-hand of system (1) side equals zero are as follows:

$$\begin{aligned} \Lambda_d - \phi S_d - \beta_{ds} S_d I_s - \mu_d S_d &= 0, \\ \beta_{ds} S_d I_s - \mu_d I_d &= 0, \\ \Lambda_s + \phi S_d - \beta_{ss} S_s I_s - \mu_s S_s &= 0, \\ \beta_{ss} S_s I_s - \mu_s I_s &= 0. \end{aligned}$$

Based on the equation, two equilibrium points are obtained: disease-free equilibrium point and endemic equilibrium point. The disease-free equilibrium point

$$E_0 = \left(\frac{\Lambda_d}{\phi + \mu_d}, 0, \frac{\Lambda_s(\phi + \mu_d) + \phi\Lambda_d}{(\mu_s)(\phi + \mu_d)}, 0\right)$$

and the endemic equilibrium point are as follows.

$$S_s^* = \frac{\mu_s}{\beta_{ss}}, I_s^* = \frac{-B + \sqrt{B^2 - 4AC}}{2A},$$
$$S_d^* = \frac{\Lambda_d}{\beta_{ds} I_s^* + (\phi + \mu_d)}, I_d^* = \frac{\beta_{ds}}{\mu_d} S_d^* I_s^*.$$

$$A = 1,\ B = \frac{\beta_{ds}(\mu_s)^2 + (\phi+\mu_d)\mu_s - \beta_{ds}\beta_{ss}\Lambda_s}{\beta_{ds}\mu_s},$$
$$C = \frac{(\phi+\mu_d)((\mu_s)^2 - \beta_{ss}\Lambda_s) - \beta_{ss}\phi\Lambda_d}{\beta_{ds}\mu_s}$$

Stability Analysis

The stability of the equilibrium point of system (1), namely the equilibrium point E_0, will be analyzed. The linear approximation obtained in the section around the equilibrium points of system (1) can help determine the system's stability. The linear approximation of system (1) at the equilibrium point E_0 results in the Jacobian matrix.

$$J(E_0) = \begin{pmatrix} -(\phi + \mu_d) & 0 & 0 & -\beta_{ds} S_d^0 \\ 0 & -\mu_d & 0 & \beta_{ds} S_d^0 \\ \phi & 0 & -\mu_s & -\beta_{ss} S_s^0 \\ 0 & 0 & 0 & \beta_{ss} S_s^0 - \mu_s \end{pmatrix}$$

The characteristic values at this equilibrium point are given by:

$$\begin{vmatrix} \lambda + (\phi + \mu_d) & 0 & 0 & \beta_{ds} S_d^0 \\ 0 & \lambda + \mu_d & 0 & -\beta_{ds} S_d^0 \\ -\phi & 0 & \lambda + \mu_s & \beta_{ss} S_s^0 \\ 0 & 0 & 0 & \lambda - (\beta_{ss} S_s^0 - \mu_s) \end{vmatrix} = 0.$$

The resulting eigenvalues are:

$\lambda_1 = -(\phi + \mu_d),$
$\lambda_2 = -\mu_d,$
$\lambda_3 = -\mu_s,$
$\lambda_4 = \beta_{ss} S_s^0 - \mu_s.$

All four eigenvalues are real numbers, so for the equilibrium point E_0 to be stable, fourth characteristic values must be negative. Since $S_s^0 > 0$, we have $\lambda_4 = \beta_{ss} S_s^0 - \mu_s < 0$, which implies that $\frac{\beta_{ss}(\Lambda_s(\phi+\mu_d)+\phi\Lambda_d)}{(\phi+\mu_d)(\mu_s)^2} < 1$. Thus, based on stability theory, the equilibrium point E_0 is locally asymptotically stable if

$$\phi < \frac{\mu_d((\mu_s)^2 - \beta_{ss}\Lambda_s)}{\beta_{ss}(\Lambda_d + \Lambda_s) - (\mu_s)^2}.$$

If this is fulfilled, it can be said that rabies in dogs will experience extinction.

Model with Intervention

If interventions such as vaccination of susceptible dogs against rabies $(\nu_d = \nu_d = \nu)$ and elimination $(\xi_d = \xi_s = \xi)$ of rabid dogs are implemented, the two interventions are described as follows. ν_d represents vaccination in domestic dogs, and ν_s represents vaccination in stray dogs (assuming it remains constant). These interventions are periodically given according to the vaccination schedule, ensuring that the level of immunity against rabies in dogs is maintained. The elimination ξ_d is implemented in the infected subpopulation of domestic dogs, while in stray dogs, infected subpopulations receive the intervention ξ_s. The compartmental model with intervention in the diagram:

ISSN. 2087-2852
E-ISSN. 2338-1655

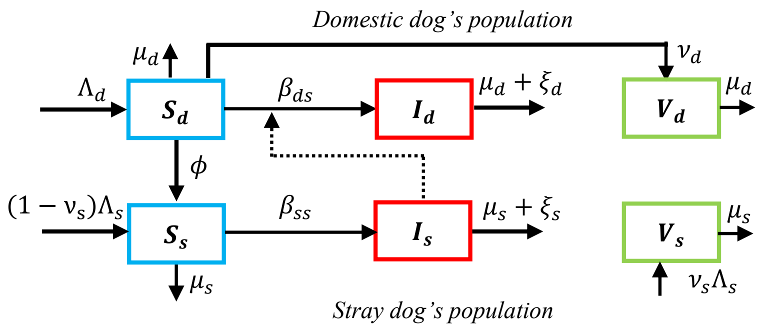

Figure 3. The compartmental model for rabies transmission with intervention

Model (1) becomes the following system.

$$\left.\begin{aligned}
\frac{dS_d}{dt} &= \Lambda_d - \phi S_d - \beta_{ds} S_d I_s - \mu_d S_d - \nu_d S_d \\
\frac{dI_d}{dt} &= \beta_{ds} S_d I_s - \mu_d I_d - \xi_d I_d \\
\frac{dV_d}{dt} &= \nu_d S_d - \mu_d V_d \\
\frac{dS_s}{dt} &= (1-\nu_s)\Lambda_s + \phi S_d - \beta_{ss} S_s I_s - \mu_s S_s \\
\frac{dI_s}{dt} &= \beta_{ss} S_s I_s - \mu_s I_s - \xi_s I_s \\
\frac{dV_s}{dt} &= \nu_s \Lambda_s - \mu_s V_s
\end{aligned}\right] \quad (2)$$

The first and second equations of system (2) do not depend on the variable V_d, and the fourth and fifth equations of system (2) do not depend on the variable V_s. it is sufficient to analyze the dynamics using only the first, second, fourth, and fifth equations of system (2). Similarly processed, system (2) produces two equilibrium points.

The equilibrium points are disease-free and endemic. The disease-free equilibrium point

$E_0 = (S_d^0, 0, S_s^0, 0)$, with

$$S_d^0 = \frac{\Lambda_d + \alpha}{\phi + \nu_d + \mu_d}$$

$$S_s^0 = \frac{(1-\nu_s)\Lambda_s(\phi + \nu_d + \mu_d) + \phi\Lambda_d}{\mu_s(\phi + \nu_d + \mu_d)}$$

The stability of the disease-free equilibrium point is determined by substituting S_s^0 into the equations $\beta_{ss}S_s^0 - (\mu_s + \xi_s)$. If $S_s^0 < \frac{(\mu_s+\xi_s)}{\beta_{ss}}$, then this equilibrium point is asymptotically stable, meaning rabies in dogs will be eradicated. To ensure this condition is met, the following must be fulfilled:

$$\frac{(1-\nu_s)\Lambda_s(\phi + \nu_d + \mu_d) + \phi(\Lambda_d + \alpha)}{\mu_s(\phi + \nu_d + \mu_d)} < \frac{\mu_s + \xi_s}{\beta_{ss}}$$

Therefore, if interventions are applied to the model, the constraint on the rate of releasing domestic dogs will change according to the magnitude of the interventions performed (vaccination and elimination). It indicates that if the releasing domestic dogs' rate exceeds the limit, actions must be taken on the model to prevent rabies from spreading in dogs, with the following magnitudes.

1. $\xi_s > \frac{\beta_{ss}\phi\Lambda_d}{\mu_s(\phi+\nu_d+\mu_d)} + \frac{\beta_{ss}(1-\nu_s)\Lambda_s - \mu_s^2}{\mu_s}$
2. $\nu_d > \frac{\beta_{ss}\phi\Lambda_d}{\mu_s(\mu_s+\xi_s) - \beta_{ss}(1-\nu_s)\Lambda_s} - (\phi + \mu_d)$
3. $\nu_s > 1 - \left(\frac{\mu_s(\mu_s+\xi_s)}{\beta_{ss}\Lambda_s} + \frac{\phi\Lambda_d}{\Lambda_s(\phi+\nu_d+\mu_d)}\right)$

Numerical Simulation

To strengthen the analysis results, numerical simulations are conducted using fourth-order Runge-Kutta method. The initial values used are presented in Table 2.

ISSN. 2087-2852
E-ISSN. 2338-1655

Table 2. The initial value

Symbol	Value	Source
S_d	3200	[10]
S_s	15	[10]
I_d	2800	[10]
I_s	20	[10]

The parameter values for the numerical simulation are provided in Table 2, resulting in the constraint $\phi = 15\%$. The numerical simulation is conducted based on various values of (ϕ) to observe the model trends without any intervention. If $\phi < 15\%$, the system exhibits local and global asymptotic stability towards the disease-free equilibrium point E_0, while if $\phi > 15\%$, the equilibrium point E_0 is unstable. The simulation results are presented ϕ in Figure 4, with variations in the value of $\{0\%, 10\%, 25\%, 50\%\}$.

The simulation results show differences in the solution curves when the value of $\phi < 15\%$ and $\phi > 15\%$. If $\phi < 15\%$, the curves converge to the disease-free equilibrium point, while if $\phi < 15\%$, the curves converge to the endemic equilibrium point. Thus, rabies in dogs will be eliminated when the percentage of free-roaming domestic dogs is less than 15%. If it exceeds this threshold, preventive measures such as vaccination or elimination of rabies-exposed dogs are necessary. Furthermore, if the percentage of free-roaming domestic dogs is set at $\phi = 50\%$, the role of vaccination $\nu_d = \nu_s = \nu$ or elimination $\xi_d = \xi_s = \xi$ is illustrated in Figure 5.

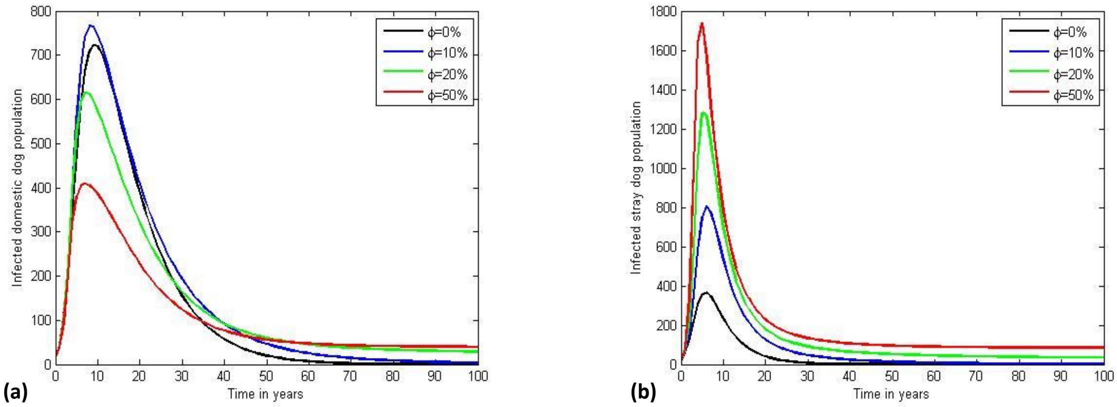

Figure 4. The numerical simulation results of system (1) with variations on the rate of releasing domestic dogs on the (a) Infected domestic dog population and (b) infected stray dog population

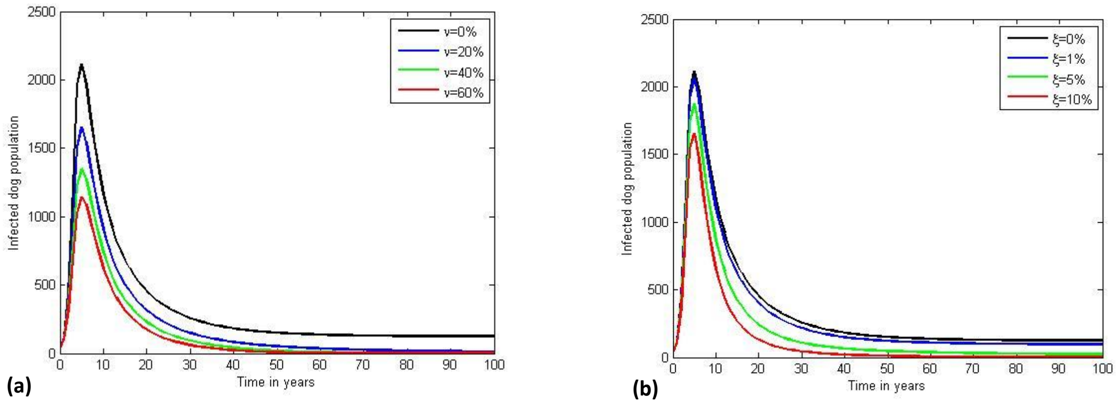

Figure 5. The impact of vaccination (a) and elimination (b) on rabies in dogs can be significant

Figure 5 (a) illustrates the impact of vaccination on the rabies dog subpopulation, while Figure 5 (b) depicts the effect of eliminating rabid dogs. When the value of $\phi > 15\%$, in order to reduce the spread of rabies, it is necessary to carry out a minimum vaccination rate of $\nu > 20\%$ or a minimum elimination of rabid dogs of $\xi > 1\%$.

CONCLUSION

Based on the model's simulation results, the system of releasing domestic dogs plays a crucial role in the spread of rabies. It is evident when the parameters in the designed model are given, as there is a limit to the rate of

ISSN. 2087-2852
E-ISSN. 2338-1655

domestic dog roaming. If the roaming rate exceeds this limit, it indicates the occurrence of rabies transmission. In such conditions, preventive measures such as vaccination or elimination of domestic and stray rabid dogs are necessary.

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ISSN. 2087-2852
E-ISSN. 2338-1655