

Student Thinking Structure Against Errors in Solving Mathematical Problems Based on Newman's Theory

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Abstrak: Jenis penelitian yang digunakan adalah kualitatif-deskriptif. Penelitian ini dilakukan pada 3 sekolah dengan siswa yang berjumlah 112 orang. Pemilihan subjek dilakukan dengan teknik *purposive sampling* dengan kriteria, yaitu siswa yang memiliki kesalahan dalam menyelesaikan masalah matematika. Data penelitian terdiri atas jawaban, rekaman hasil wawancara semi terstruktur dan hasil *think aloud* siswa. Melalui tiga data tersebut, kategori kesalahan penyelesaian masalah matematika akan dilihat berdasarkan teori Newman (1997) dan indikator tahapan berpikir siswa terhadap penyelesaian masalah matematika yang dikemukakan oleh Bloom (1956). Analisis data dilakukan melalui tahap mereduksi data, menyajikan data, dan menarik kesimpulan. Hasil penelitian menunjukkan bahwa struktur berpikir siswa yang mengalami kesalahan membaca, pada tahap mengingat mampu menguraikan informasi yang diketahui, namun siswa mengalami kesulitan dalam mengenali gambar. Sehingga struktur berpikir yang dimiliki belum mampu memenuhi tahap mengingat secara lengkap. Selanjutnya struktur berpikir siswa yang mengalami kesalahan memahami, hanya mampu memenuhi tahapan mengingat, namun pada tahap menafsirkan siswa belum dapat secara sempurna.

Keywords: Thinking Structure; Problem Solving; Newman's Theory

Abstract: This type of research is qualitative-descriptive. This research was conducted in three schools with 112 students. Subject selection was carried out using the *purposive sampling* technique with criteria, namely students who had errors in solving math problems. The research data consisted of answers, recordings of semi-structured interview results, and students' think-aloud results. Through these three pieces of data, categories of errors in solving math problems will be seen based on Newman's theory (1997) and indicators of students' thinking stages towards solving math problems proposed by Bloom (1956). Data analysis was carried out through the stages of reducing data, presenting data, and drawing conclusions. The findings revealed that students with reading errors had a thinking structure capable of deciphering known information during the remembering stage, but they had difficulty recognizing images. so that the structure of thinking that is owned has not been able to fulfill the complete recall stage. Furthermore, the thinking structure of students who have misunderstood is only able to fulfill the stages of remembering, but at the interpreting stage, students cannot do it perfectly.

INTRODUCTION

Problem solving is an integrated process of mathematics learning and an essential component of the learning program (Sukiyanto, 2020). Problem solving is also a cognitive activity that involves complex processes. In addition to solving problems, students must integrate cognitive, metacognitive, and self-regulation mechanisms from various strategies (Montague et al., 2011). The ability to solve

mathematical problems is very important because it involves higher-order thinking processes that require more modulation and control than basic skills (Amam, 2017).

When solving math problems, students are expected to be able to determine the method to be used and prepare alternative strategies if they experience difficulties or there is a change in the situation (Supiarmo et al., 2021; Sukiyanto, 2020). Difficulties in solving math problems are always experienced by students, which result in mistakes or failures (Wibawa et al., 2018). One of the factors that influences students' mathematical problem-solving errors is their thinking structure (Sukmaangara & Prabawati, 2019).

Through the initial observations that have been made, students who experience obstacles in the thought process cannot solve the given math problems. This also has an impact on the steps to solving the next math problem (Nazihah, 2018). overcome problems in students' thinking processes; this can be done by structuring these thinking structures into a more complete structure so that students can achieve a deep understanding of solving mathematical problems (Wulandari & Gusteti, 2021).

According to Newman's theory, student errors in solving math problems were grouped into several categories, including reading errors, understanding errors, transformation errors, analyzing errors, and errors in writing the final answer. Newman's Error Analysis (NEA) was developed to assist teachers when dealing with students who have difficulty solving math problems, especially problems that focus on math words (Garderen et al., 2013). NEA provides a framework for considering the reasons underlying student learning difficulties and helps teachers determine where students' misconceptions lie in determining effective teaching strategies to overcome them (Oktaviana et al., 2017).

METHOD

The type of research used in this study is qualitative-descriptive. This research was conducted on class VIII students at MTs Wali Songo Bululawang, MTs Al-Hidayah Batu, and MTs AN-Nashriyah Montong Goak, a total of 112 students. Subject selection was carried out using the purposive sampling technique with criteria, namely students who had errors in solving math problems. In addition, prospective subjects are given test sheets on which they can see and group students based on error categories. The research data consisted of students' answers to the story problem test, recordings of semi-structured interview results, and the results of students' thinking aloud. Through these three pieces of data, categories of errors in solving math problems will be seen based on Newman's theory (1997) and indicators of students' thinking stages toward solving math problems proposed by Bloom (1956). The results of the analysis of research data are used to determine the structure of students'

thinking. Then the data obtained were analyzed through various techniques, namely reducing data, presenting data, and drawing conclusions.

RESULT AND DISCUSSION

Subjects who had reading errors in this study were represented by S1 and S2, while subjects who had comprehension errors were represented by S3 and S4. The structure of students' thinking in solving mathematical problems in terms of Newman's mistakes is described as follows.

The Thinking Structure of Students Who Experience Reading Errors

S1 and S2 are subjects who have reading errors according to the category of reading errors described by Newman (1997). This is because S1 and S2 fail to recognize images and determine important information from a given problem. In the following, the answers, the results of think aloud recordings, and the results of semi-structured interviews are presented regarding the structure of the two subjects' thinking toward solving mathematical problems.

At the remembering stage, S1 and S2 can identify or describe information that is known to be simpler, even though initially it is not complete. S1 describes information related to the rows of seats in the building where the number of seats is in the first row to the fourth row but has difficulty deciphering the images presented by conveying the difference in ticket prices for two adjacent rows of seats. When interviewing, S1 can describe important information related to what is known about the problem, such as the number of seats in the first to fourth rows, the difference in ticket prices for two adjacent rows of seats, and the funds the committee wants to obtain, and S1 can also find out what is being asked, namely, determine the cheapest ticket price.

Slightly different from what S2 explained regarding the information in the problem, S2 can directly describe the number of seats in the first to fourth rows in the building and continue by conveying the difference in ticket prices for the two adjacent rows of seats where the committee wants to obtain funds of 22,500,000. During the interview, S2 was able to explain what was known in the problem, such as the number of seats in the first to fourth rows, but experienced a little confusion in deciphering the images presented because they did not match the known number of seats, the difference in ticket prices for two adjacent rows of seats, and the funds that the committee and Masters want to obtain. They can also find out what is being asked, namely determining the cheapest ticket prices based on.

Furthermore, at the stage of interpreting the problem S1 and S2 did not determine the number of seats in the fifth and sixth rows, but the two subjects directly determined the total number of seats in the building by adding up the seats in the first to fourth rows. This explains that S1 and S2 are not able

to connect the problem with the mathematical material that has been obtained previously, so S1 and S2 can be said to not fulfill the stage of interpreting the problem given based on the indicators of the stages of thinking in Table 1. Thus, of course, S1 and S2 also do not meet the indicator stages of implementing, analyzing, and evaluating.

S1 had difficulty understanding the information without knowing the fifth and sixth rows of seats in advance. Whereas S2 has difficulty deciphering the drawing, this is because the number of seats known in the building does not match the number of rows of seats shown in the drawing, and S2 also does not determine the fifth and sixth row of seats first. For more details, see the following part of the answer.

Handwritten student work for S1:

dik: baris pertama = 25 kursi
 baris kedua = 35 kursi
 ketiga = 50 kursi
 keempat = 70 kursi

180 kursi
 dit: harga tiket maksimum ?

Figure 1. S1 Answer Slices Determine Seat Rows in a Building

Handwritten student work for S2:

dik 1 2 3 4 = 10

= 25 , 35 , 50 , 70 ÷ 180

= 10 + 180

= 190

Figure 2. S2 Answer Slices Determine Seat Rows in a Building

Based on Figure 1, it is known that S1 made an error in determining the total number of seats. The total number of seats is obtained by adding up the first to the fourth row of seats, namely 25 seats, 35 seats, 50 seats, and 70 seats, so that the total number of seats in the building is 180 seats. This resulted in an S1 error at a later stage in implementing the settlement plan. Based on Figure 2, it is known that S2 made an error in determining the total number of seats. The total number of rows of seats is obtained by adding up the first, second, third, and fourth-row seats, namely 25, 35, 50, and 70 seats, so that the total number of seats is 180 seats, which is then summed again with the number of rows of seats in the building, namely row seats $1 + 2 + 3 + 4 = 10$, so we get $180 + 10 = 190$ seats. This results in an S2 error in the next stage, namely the stage of implementing a plan for solving mathematical problems. More details can be seen in Figures 3 and 4.

$$= 50x + 50(x+10) + 35(x+10) + 25(x+10) = 22.500$$

$$= 70x + 50x + 35x + 25x = 22.500$$

$$= 180x + 1.100 = 22.500$$

$$180x = 22.500 - 1.100$$

$$180x = 21.400$$

$$x = \frac{21.400}{180}$$

$$x = 118,88$$

Figure 3. S1 Answer Slices Analyzing the Number of Seats with a Difference in Ticket Prices

misalnya
 misalkan x harga tas murah

$$= 70(x+10) + 50(x+10) + 35(x+10) + 25(x+10)$$

$$= 700x + 500x + 350x + 250x$$

$$= 1200x + 600x$$

$$= 1.800x$$

Figure 4. S2 Answer Slices Analyzing the Number of Seats with a Difference in Ticket Prices

Based on Figure 3, it is known that S1 can make an example with (x) as the cheapest ticket price, but S1 makes a mistake in setting the difference in ticket prices for two adjacent rows of seats, namely $70x + 50(x+10) + 35(x+10) + 25(x+10) = 22,500$. As for the results of think aloud, it is known that S1 made a mistake in determining the difference in ticket prices for two adjacent rows, by setting the difference in ticket prices at IDR 10,000 for each row of seats. Furthermore, based on Figure 4, it is known that S2 can make an example with (x) as the cheapest ticket price, but S2 makes a mistake in setting the difference in ticket prices on two adjacent rows, namely $70(x+10) + 50(x+10) + 35(x+10) + 25(x+10)$. Therefore, S1 and S2 have not fulfilled the stages of applying, analyzing, and evaluating in solving problems, which can be seen more clearly in the following figure.

dik: 180 orang
 orang = 25 orang
 orang = 35 orang
 orang = 50 orang
 orang = 70 orang
 180 orang
 dit: harga tiket normal?
 misalkan x

$$= 50x + 50(x+10) + 35(x+10) + 25(x+10) = 22.500$$

$$= 70x + 50x + 35x + 25x = 22.500$$

$$= 180x + 1.100 = 22.500$$

$$180x = 22.500 - 1.100$$

$$180x = 21.400$$

$$x = \frac{21.400}{180}$$

$$x = 118,88$$

Figure 5. S1 answer fragment determines the cheapest ticket price Figure

dik: 1 2 3 4
 25 35 50 70
 180
 misalkan x harga tas murah

$$= 70(x+10) + 50(x+10) + 35(x+10) + 25(x+10)$$

$$= 700x + 500x + 350x + 250x$$

$$= 1200x + 600x$$

$$= 1.800x$$

Figure 6. S2 answer fragment determines the cheapest ticket price

Based on Figure 5, it is known that the cheapest ticket price found by S1 is $x = 8.30$. The results of an analysis performed on the interval $180x + 1.100 - 22.500$, where $x = 1500/180 = 8.30$, reveal the cheapest ticket price. Then based on figure 9, it is known that the cheapest ticket price found by S2 is 18.000. The cheapest ticket price is known from the results of the analysis, namely $700x + 500x + 350x + 250x = 1200x + 600x = 18000$.

The Thinking Structure of Students Who Have Misunderstood

Subjects who had misunderstandings in this study were represented by S3 and S4. Both subjects experienced a thinking structure intervention with the same tendency in solving mathematical

problems according to Newman's (1997) category. The structure of thinking of the two subjects is described as follows.

S3 and S4 can directly identify and describe important information about what is known and asked in the problem during the remembering stage. S3 can directly describe the information that is known regarding the rows of seats in the first to fourth rows and the difference in ticket prices in the two rows of seats with the funds that the committee wants to obtain, while the information asked is the cheapest ticket price. Meanwhile, S4 describes the information in several parts related to what is known and asked in the problem, although imperfectly. However, when conducting interviews, S3 and S4 were able to explain in detail the important data related to the problem that will be used to answer questions in the given problem.

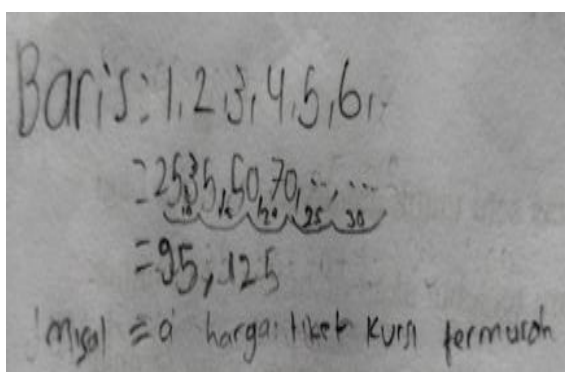


Figure 7. S3 answer fragment

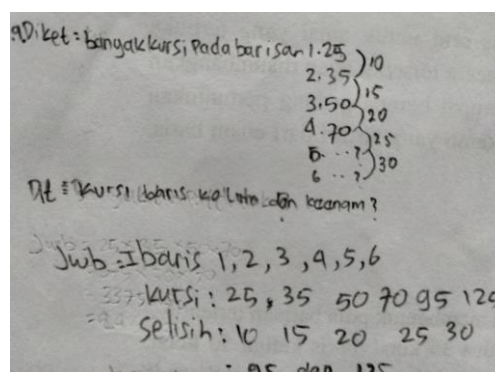


Figure 8. S4 answer fragment

The two subjects described the known information regarding the first to the fourth row of seats and the difference in ticket prices for the two adjacent rows of seats. In addition, S3 and S4 describe the information requested regarding the cheapest ticket prices for the fifth and sixth rows of seats. By decomposing the information carried out by S3 and S4 and explaining the information that is known and asked, it can be stated that the two subjects fulfill the remembering stage in solving problems.

Furthermore, at the interpreting and understanding stage, S3 and S4 were able to find out what was being asked, namely the cheapest ticket price, and then S3 was also able to identify the number of seats in the fifth and sixth rows in advance, namely 95 and 125 seats, but S3 did not add up the first to sixth-row seats as a whole in the building. S4 can also determine the number of seats in the fifth and sixth rows before determining the cheapest ticket price asked, but it experiences an error because it does not add up all the seats in the building. On the basis of the decomposition of the information carried out by S3 and S4, it can be said that the two subjects have not been able to fulfill the interpretation stage in solving the problem.

CONCLUSION

Based on the results of data analysis and discussion, it can be concluded that students' thinking structures towards reading errors, at the remembering stage can describe known information, but students have difficulty recognizing the images presented in the problem. So the thinking structure that students have towards reading errors has not been able to complete the remembering stage. Furthermore, the thinking structure that students have towards understanding errors is only able to fulfill the remembering stage, but at the interpreting and understanding stage students can only determine the number of seats in the fifth and sixth rows by determining the difference in seats in the first to fourth rows, but students do not determine the total number of seats in the building, so it can be said that they have not been able to fulfill the stage of interpreting perfectly.

REFERENCES

- Amam, A. (2017). *Penilaian Kemampuan Pemecahan Masalah Matematis Siswa SMP*. 2(1), 39–46.
- Biane, P. (1997). *Some properties of crossings and partitions*. 175, 41–53.
- Bloom, B. dan K. (1956). *Taksonomi Tujuan Pendidikan*. 53–126.
- M. Gunawan Supiarmo, Liny Mardhiyatirrahmah, T. (2021). Pemberian Scaffolding untuk Memperbaiki Proses Berpikir Komputasional Siswa dalam Memecahkan Masalah Matematika. *Jurnal Cendekia*, 05(01), 368–382.
- Nazihah, Z. (2018). Defragmenting Struktur Berpikir Mahasiswa Dalam Mengidentifikasi Homomorfisma Ring Pada Matakuliah Struktur Aljabar. *Jurnal Ilmiah Edukasi & Sosial*, 9(2015), 90–95.
- Newman, M. A. (1997). An analysis of sixth-grade pupils' error on written mathematical tasks. *Jurnal Victorian Institute for Education Research Bullatin*, 31–43.
- Oktaviana, D., Newman, K. K., & Diskrit, M. (2017). *Analisis tipe kesalahan berdasarkan teori newman dalam menyelesaikan soal cerita pada mata kuliah matematika diskrit*. 5(2), 22–32.
- Sukiyanto, S. (2020). Munculnya Kesadaran Metakognisi Dalam Menyelesaikan Masalah Matematika. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 9(1), 126. <https://doi.org/10.24127/ajpm.v9i1.2654>
- Sukmaangara, B., & Prabawati, M. N. (2019). Analisis Struktur Berpikir Peserta Didik Dalam Menyelesaikan Masalah Tes Kemampuan Berpikir Kritis Matematik Berdasarkan Dominasi Otak. *Prosiding Seminar Nasional & Call For Papers*, 3, 89–95.
- Van Garderen, D., Scheuermann, A., & Jackson, C. (2013). Examining how students with diverse abilities use diagrams to solve mathematics word problems. *Learning Disability Quarterly*, 36(3), 145–160. <https://doi.org/10.1177/0731948712438558>

- Wibawa, K. A., Nusantara, T., Subanji, & Nengah Parta, I. (2018). Defragmentation of Student's Thinking Structures in Solving Mathematical Problems based on CRA Framework. *Journal of Physics: Conference Series*, 1028(1). <https://doi.org/10.1088/1742-6596/1028/1/012150>
- Wulandari, S., & Gusteti, M. U. (2021). Defragmentation of Preservice Teacher's Thinking Structures in Solving Higher Order Mathematics Problem. *Journal of Physics: Conference Series*, 1940(1), 012099. <https://doi.org/10.1088/1742-6596/1940/1/012099>