PERFORMANCE OF THE BEST SOLUTION FOR THE PROHIBITED ROUTE TRANSPORTATION PROBLEM BY AN IMPROVED VOGEL'S APPROXIMATION METHOD

E.M.D.B. Ekanayake[1], E. M. U. S. B. Ekanayake[1]

[1]Department of Physical Sciences, Faculty of Applied Sciences, Rajarata University of Sri Lanka, Mihinthale, Sri Lanka.

ABSTRACT

The transportation problem (TP) is a significant factor in operational research. Numerous researchers have put forth various solutions to these problems. The goal is to reduce the overall cost of distributing resources from multiple sources to numerous destinations. If there are road risks (snow, flood, etc.), traffic limitations, etc., it might not be feasible to transport products from one place to another. In these circumstances, the appropriate route(s) can be given an extremely high unit cost, such as M (or ∞). Following that, a specific case of the prohibited transportation problem is introduced. Therefore, the focus of this study is to provide a novel algorithm that will reduce the cost of restricted transportation problems. With a few modifications, the traditional Vogel approach has been enhanced. The proposed method would perform better than the other approaches now in use. The numerical problem is resolved to demonstrate the effectiveness of the proposed approach and make comparisons with different approaches already in use.
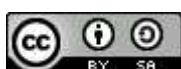
1. INTRODUCTION

One of the most colorful and demanding problems in the history of operations research is the transportation problem (TP). The fundamental goal of these problems is to get an initial basic feasible solution (IBFS) product from the source to the destination while keeping the supply limit and demand in check. Therefore, many researchers are researching the initial basic, feasible solution and the optimal solution to transportation problems. Hitchcock (1941) developed the first method for calculating the IBFS. Subsequently, various methods of calculating the IBFS were proposed. The most commonly used methods are the Northwest corner method by Charmes and Cooper (1954–1955) (Mhlanga et al., 2014), the Least cost method, and the Vogels approximation method, introduced by Reinfeld and Vogel (Mhlanga et al., 2014) in 1958. In addition, various novel methods for calculating the IBFS using various research methodologies have been established in recent years. Including "An effective alternative new approach to solving transportation problems" and a "Modified ant colony optimization algorithm for solving transportation problems" (Ekanayake et al., 2020). an improved algorithm to solve transportation problems for an optimal solution, a new approach to finding the initial solution to the unbalanced transportation problem (Shaikh et al., 2018). In all the optimization mentioned above, the focus is on cost minimization.

However, in a particular case of transportation problems, various algorithms have been created based on time. For example, Małachowski et al. (2019) published "Application of the transport problem from the criterion of time to optimize supply network with production fast running" 2019. "Algorithmic approach to calculate the minimum time shipment of a transportation problem" published in 2013 (Ullah et al., 2013) and "Problem of modeling road transport" (Ziółkowski & Lęgas, 2019) can be pointed out. Meanwhile, the multi-objective transportation problem can be mentioned as a problem that is solved by considering several things like time and cost. Related articles have been published by Kaur et al. (2018), Nomani et al. (2017), and Bharathi and Vijayalakshmi (2016).

All the articles mentioned above are exceptional cases of transportation problems. Here we will study transportation problems with restricted roads, a particular type of problem that could be more popular among researchers. it is called "prohibited route transportation problems." Some specific routes may not be accessible due to issues such as construction projects, poor road conditions, strikes, unforeseen disasters, and traffic laws. Such problems belong to this category, and in the case of the transportation problem, transportation on forbidden routes incurs a substantial cost (M or [infinity (∞)]). Therefore, this new approach tries to derive IBFS so that these paths are not included in the optimal solution to drive these problems. Examples of such reports are "problems with prohibited routes." published by Ekanayake et al. (2022), "an improved ant colony algorithm to solve prohibited transportation problems (Ekanayake et al., 2022)" published by Prah et al. (2022); and "A 2-phase method for solving transportation" published (Prah et al., 2022).

Also, Currin (1986) has made similar observations. In this way, this research paper aims to present a new algorithm to obtain a basic feasible solution for transportation problems with restricted roads. The study aims to provide a preliminary solution so that the most expensive route is not included in the calculation of the relevant answer, and whether that solution is optimal or close to it is the aim of the study. For this, an improved Vogel's approximation method is used, and satisfactory solutions are expected. It is not appropriate to use standard methods such as the "Northwest corner method," "least cost method," and

"Vogel's approximation method," used in solving general transportation problems, to solve transportation problems with restricted routes.

In some cases, the costs of prohibited roads may be included in the essential solutions obtained. Then the corresponding total transportation cost will be high. However, the primary objective of a transportation problem is to construct an algorithm that minimizes transportation costs. Where the value obtained as the initial solution is its minimum cost, it is known as the optimal solution. We presented a new algorithm to find the immediate solution to the prohibited road transport problem. Since the traditional Vogel method is unsuitable, it has been improved, and the related problems have been adjusted accordingly. Mathematical problems indicate the primary solutions obtained thereby. It has been shown by the transportation problems that the cost values obtained by this method do not include the cost of the prohibited route and provide the optimal solution or a solution close to it.

2. METHODS

Generally, express the transportation problem in a mathematical table as shown below since the problem is simple to solve (Jude et al., 2016).

Table 1 General representation of transportation tableau

To destination→ ↓From source	D_1	D_2	…	D_n	Supply a_i
S_1	C_{11} X_{11}	C_{12} X_{12}	…	C_{1n} X_{1n}	a_1
S_2	C_{21} X_{21}	C_{22} X_{22}	…	C_{2n} X_{2n}	a_2
⋮	⋮	⋮	⋮	⋮	⋮
S_m	C_{m1} X_{m1}	C_{m2} X_{m2}	…	C_{mn} X_{mn}	a_m
Demand b_j	b_1	b_2	…	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

- Where; "m" implies the quantity of sources $S_1, S_2,.. S_m$) with supply capabilities ($a_1, a_2,.. a_m$).
- Source - It is the location where commodities are placed.
- "n" implies the quantity of destinations ($D_1, D_2,.., D_n$) with the capabilities of demand ($b_1, b_2,.., b_n.$)
- Destinations - It is the location where commodities are transported.

Take into account that C_{ij} (here particular case in transportation problem, some C_{ij} values are infinite.) is the cost of transporting a unit from the ith source to the jth destination, and X_{ij} indicates the number of units transported from the source (i) to the destination (j). Given the following formula, the transportation problem can be represented mathematically as a linear programming problem.

Objective function;

$$\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} C_{ij}$$

$$\text{S.t; } \sum_{j=1}^{n} X_{ij} = a_i, i = 1,2,\dots,m$$

$\sum_{i=1}^{m} X_{ij} = b_j,\ j = 1,2,\dots,n$ and

$X_{ij} \geq 0$ for all $i = 1,2,\dots,m$, $j = 1,2,\dots,n$

One of the main requirements is to be a balanced transportation problem. i.e.,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

When a problem is unbalanced (total demand and total supply are not equal), the problem should be balanced by properly adding a dummy column or row before determining the solution. After solving IBFS and determining whether it is the optimum solution: 1,) IBFS are those that satisfy the transportation problem and also have a total number of allocations equal to (m + n-1); 2) When the total cost of transportation is the lowest, it is an "optimum solution." We use the modified Vogel's approximation approach in this study to find the most basic, feasible solutions to the problems. Before introducing the method, we need to study some assumptions and theoretical information; 3) Vogel's approximation method (VAM) (Balakrishnan, 1990).

One of the most popular methods is Vogel's approximation method, which can identify the best initial feasible solution to the transportation problem. The most important steps to solving problems by that method can be listed as follows (Priya & Maheswari, 2022): Step.1) First, the penalties for each row and column in the transportation table will be determined by subtracting the most negligible unit cost from the smallest unit cost; Step.2) Next, select the column or row with the greatest penalty value. Select the cell with the lowest unit cost that is a part of that column or row. After completing as much of the supply or demand for this cell as possible, remove the row or column. If both the row and the column are satisfied, step cross over the line; Step.3) Repeat the previous process until all supplies and demands are met. Complete the procedure, determine the shipping cost, and ensure that all supply and demand are satisfied.

Vogel's method mentioned above has been modified to provide solutions for these particular problems. The related numerical problems were obtained through various research papers and reference books. The aim is to test this improved Vogel method. Also, the steps of the improved Vogel's method can be shown as follows:

2.1. Proposed Algorithm

The steps consist of: Step.1) First, whether the transport problem is balanced or unbalanced should be checked. If the problem is unbalanced, it could be balanced by using a suitable dummy column or row; Step.2) Then calculate the reciprocal of the values in all the cells of the transportation problem table. (Do not calculate the reciprocal values in the dummy column or dummy row; Step.3) Then, for each column and row, subtract the highest value in that row or column from the next highest value to get the penalty value. Select the

row or column with the highest value among those penalty values. After assigning assignments to the highest common value in the respective column or row, cross the row or column that satisfies supply or demand; Step. 4) Follow the above steps to satisfy all demand and supply in the system. In an unbalanced transportation problem, after checking all suitable cells to give an assignment in the considered column or row, if the corresponding supply or demand values are not satisfied, finally select a dummy cell and give the assignment; Step 5) Finally, each assigned value is assigned to the original cost values of the corresponding transportation table, and the problem's initial solution is computed.

In this research, the relevant calculations are made subject to the following assumptions:

1) The maximum unit cost of the cell with the Prohibited path is said to be infinite (∞).

2) Dividing by one assumes that the value of infinity is equal to zero (Ufuoma, 2020).

How to obtain solutions using the above-mentioned improved Vogel's method is presented in detail in the following section. It is shown in each step. It is also essential to compare the solutions obtained by the proposed method with those obtained by the existing methods. Accordingly, the analytical comparison is mentioned in Tables 27, 28, and 47. Also, analytical graphs related to it are shown in this section.

3. RESULTS AND DISCUSSION

This section shows some of the transportation problems solved through the after-mentioned algorithms in detail. It is intended to show the effectiveness of this newly introduced method according to the result obtained.

Problem 1 (Prah et al., 2022)

Table 2 Balance prohibited route transportation problem table

	D_1	D_2	D_3	D_4	supply
S_1	∞	14	12	17	250
S_2	11	10	6	10	350
S_3	12	8	15	7	400
demand	180	320	120	380	

Table 3 Step 02 - Table of Calculating the reciprocal value of unit costs for each cell

	D_1	D_2	D_3	D_4	supply
S_1	0	0.071	0.083	0.059	250
S_2	0.091	0.100	0.167	0.100	350
S_3	0.083	0.125	0.067	0.143	400
demand	180	320	120	380	

Table 4 Step 03 & step 04 – Table showing the steps used in the improved Vogel's method

	D_1	D_2	D_3	D_4	supply	Penalty value					
S_1	0	$0.071 \, (250)^3$	0.083	0.059	250 (0)	0.012	0.012	**0.071**	-----	------	-------
S_2	$0.091 \, (180)^6$	$0.100 \, (50)^5$	$0.167 \, (120)^1$	0.100	350 (230) (180)	0.067	0	0.009	0.009	0.009	**0.091**
S_3	0.083	$0.125 \, (20)^4$	0.067	$0.143 \, (380)^2$	400 (20) (0)	0.018	0.018	0.042	**0.042**	------	-----

demand	180	320 (70) (50) (0)	120 (0)	380 (0)
Penalty value	0.008	0.025	**0.084**	0.043
	0.008	0.025	-------	**0.043**
	0.008	0.025	-------	-------
	0.008	0.025	-------	-------
	0.091	**0.100**	------	-------
	0.091	------	------	-------

Table 5 Step 05 - The corresponding assignment values were assigned to the appropriate cells of the transportation problem, and the corresponding total cost was calculated.

	D_1	D_2	D_3	D_4	supply
S_1	∞	14 (250)	12	17	250
S_2	11 (180)	10 (50)	6 (120)	10	350
S_3	12	8 (20)	15	7 (180)	400
demand	180	320	120	380	

Transportation cost $=(11 \times 180) + (14 \times 250) + (10 \times 50) + (8 \times 20) + (6 \times 120) + (7 \times 380) = 9520$

Problem 2 (Prah et al., 2022)

Table 6 Balance prohibited route transportation problem Table.

	D_1	D_2	D_3	D_4	supply
S_1	9	14	12	17	250
S_2	11	10	6	∞	350
S_3	12	8	15	7	400
demand	180	320	120	380	

Table 7 Step 02 - Table of Calculating the reciprocal value of unit costs for each cell

	D_1	D_2	D_3	D_4	supply
S_1	0.111	0.071	0.083	0.059	250
S_2	0.091	0.100	0.167	0	350
S_3	0.083	0.125	0.067	0.143	400
demand	180	320	120	380	

Table 8 Step 03 & step 04 - Table showing the steps used in the improved Vogel's method

	D_1	D_2	D_3	D_4	supply	Penalty value
S_1	0.111 (180)[5]	0.071 (70)[6]	0.083	0.059	250 (70) (0)	0.028 0.028 0.040 0.04 0.04 **0.071**
S_2	0.091	0.100 (230)[4]	0.167 (120)[2]	0	350 (230) (0)	0.067 0.067 0.009 0.09 ----- -----
S_3	0.083	0.125 (20)[3]	0.067	0.143 (380)[1]	400 (20) (0)	0.018 0.042 **0.042** ---- ----- -----
demand	180 (0)	320 (300) (70) (0)	120 (0)	380 (0)		
Penalty value	0.020	0.025	0.084	**0.084**		
	0.020	0.025	**0.084**	-----		
	0.020	0.025	------	-----		
	0.020	**0.029**	-----	-----		
	0.111	0.071	-----	-----		
	------	**0.071**	-----			

Table 9 Step 05 - The corresponding assignment values were assigned to the appropriate cells of the transportation problem, and the corresponding total cost was calculated.

	D_1	D_2	D_3	D_4	supply
S_1	9 (180)	14 (70)	12	17	250
S_2	11	10 (230)	6 (120)	∞	350
S_3	12	8 (20)	15	7 (380)	400
demand	180	320	120	380	

Transportation cost $=(9 \times 180) + (14 \times 70) + (10 \times 230) + (8 \times 20) + (6 \times 120) + (7 \times 380) = 8440$

Problem 3 (Prah et al., 2022)

Table 10 Balance prohibited route transportation problem table

	D_1	D_2	D_3	D_4	supply
S_1	9	14	12	17	250
S_2	11	10	6	10	350
S_3	12	∞	15	7	400
demand	180	320	120	380	

Table 11 Step 02 - Table of Calculating the reciprocal value of unit costs for each cell

	D_1	D_2	D_3	D_4	supply
S_1	0.111	0.071	0.083	0.059	250
S_2	0.091	0.100	0.167	0.1	350
S_3	0.083	0	0.067	0.143	400
demand	180	320	120	380	

Table 12 Step 03 & step 04 - Table showing the steps used in the improved Vogel's method

	D_1	D_2	D_3	D_4	supply	Penalty value					
S_1	0.111 (160)[4]	0.071 (90)[6]	0.083	0.059	250 (90) (0)	0.028	0.040	0.040	**0.040**	0.071	**0.071**
S_2	0.091	0.100 (230)[5]	0.167 (120)[1]	0.1	350 (230) (0)	0.067	0	0.009	0.009	**0.1**	------
S_3	0.083 (20)[3]	0	0.067	0.143 (380)[2]	400 (20) (0)	0.060	**0.060**	0.083	-----	----	------
demand	180 (160) (0)	320 (90) (0)	120 (0)	380 (0)							
Penalty value	0.020	0.025	**0.084**	0.043							
	0.020	0.029	------	0.043							
	0.020	0.029	------	------							
	0.020	0.029	------	------							
	------	0.029	------	-----							
	------	**0.071**	------	-----							

Table 13 Step 05 - The corresponding assignment values were assigned to the appropriate cells of the transportation problem, and the corresponding total cost was calculated.

	D_1	D_2	D_3	D_4	supply
S_1	9 (160)	14 (90)	12	17	250
S_2	11	10 (230)	6 (120)	10	350
S_3	12 (20)	∞	15	7 (380)	400
demand	180	320	120	380	

Transportation cost $=(9 \times 160) + (14 \times 90) + (10 \times 230) + (12 \times 20) + (6 \times 120) +$
$(7 \times 380) = 8620$

Problem 4   (Prah et al., 2022)

Table 14 Balance prohibited route transportation problem table

	D_1	D_2	D_3	D_4	supply
S_1	9	∞	12	17	250
S_2	11	10	∞	10	350
S_3	12	8	15	7	400
demand	180	320	120	380	

Table 15 Step 02 - Table of Calculating the reciprocal value of unit costs for each cell

	D_1	D_2	D_3	D_4	supply
S_1	0.111	0	0.083	0.059	250
S_2	0.091	0.100	0	0.1	350
S_3	0.083	0.125	0.067	0.143	400
demand	180	320	120	380	

Table 16 Step 03 & step 04 - Table showing the steps used in the improved Vogel's method

	D_1	D_2	D_3	D_4	supply	Penalty value					
S_1	0.111 (130)[5]	0	0.083 (120)[4]	0.059	250 (130) (0)	0.028	0.028	0.028	0.028	**0.111**	-----
S_2	0.091 (50)	0.100 (300)[3]	0	0.1	350 (50) (0)	0	0.009	0.009	0.009	0.091	**0.091**
S_3	0.083	0.125 (20)[2]	0.067	0.143 (380)[11]	400 (20) (0)	0.018	**0.042**	-----	-----	------	------
demand	180 (50) (0)	320 (300) (0)	120 (0)	380 (0)							
Penalty value	0.020	0.025	0.016	**0.043**							
	0.020	0.025	0.016	------							
	0.020	**0.1**	0.083	------							
	0.020	------	**0.083**	------							
	0.020	------	------	------							
	0.091										

Table 17 Step 05 - The corresponding assignment values were assigned to the appropriate cells of the transportation problem, and the corresponding total cost was calculated.

	D_1	D_2	D_3	D_4	supply
S_1	9 (130)	∞	12 (120)	17	250
S_2	11 (50)	10 (300)	∞	10	350
S_3	12	8 (20)	15	7 (380)	400
demand	180	320	120	380	

Transportation cost $=(9 \times 130) + (11 \times 50) + (10 \times 300) + (8 \times 20) + (12 \times 120) +$
$(7 \times 380) = 8980$

Problem 05 (Ekanayake et al., 2022)

Table 18 Unbalance prohibited route transportation problem table

	D_1	D_2	D_3	D_4	Supply
S_1	14	15	16	17	40
S_2	∞	16	17	18	50
S_3	∞	∞	15	16	30
S_4	∞	∞	∞	17	50
demand	20	30	50	40	

Table 19 Step 01 - Since the transportation problem is unbalanced, balance the problem using a dummy column.

	D_1	D_2	D_3	D_4	Dummy column	Supply
S_1	14	15	16	17	0	40
S_2	∞	16	17	18	0	50
S_3	∞	∞	15	16	0	30
S_4	∞	∞	∞	17	0	50
demand	20	30	50	40	30	

Table 20 Step 02 - Table of Calculating the reciprocal value of unit costs for each cell

	D_1	D_2	D_3	D_4	Dummy column	Supply
S_1	0.071	0.067	0.063	0.059	0	40
S_2	0	0.063	0.059	0.056	0	50
S_3	0	0	0.067	0.063	0	30
S_4	0	0		0.059	0	50
demand	20	30	50	40	30	

Table 21 Step 03 & 04 - Table showing the steps used in the improved Vogel's method.

	D_1	D_2	D_3	D_4	Dummy column	Supply	Penalty value
S_1	0.071 $(20)^1$	0.067 $(20)^5$	0.063	0.059	0	40 (20) (0)	0.004 0.004 0.004 0.004 0.004 ----- -----
S_2	0	0.063 $(10)^6$	0.059 $(20)^7$	0.056	0 $(20)^8$	50 (40) (20)	0.004 0.004 0.004 0.004 0.004 0.004 **0.059**
S_3	0	0	0.067 $(30)^4$	0.063	0	30 (0)	0.004 0.004 0.004 **0.067** ----- ------ -----
S_4	0	0		0.059 $(40)^2$	0 $(10)^3$	50 (10) (0)	0.059 **0.059 0.059** ------ ----- ------ -----
demand	20 (0)	30 (10) (0)	50 (20) (0)	40 (0)	30 (20)		
Penalty value	**0.071** ------ ------ ----- ----- ----- ----	0.004 0.004 0.004 0.004 **0.004** **0.063** ------	0.004 0.004 0.004 0.004 0.004 0.059 0.059	0.004 0.004 ------ ------ ----- ----- -----			

Table 22 Step 05 - The corresponding assignment values were assigned to the appropriate cells of the transportation problem, and the corresponding total cost was calculated.

	D_1	D_2	D_3	D_4	Dummy column	Supply
S_1	14 (20)	15 (20)	16	17	0	40
S_2	∞	16 (10)	17 (20)	18	0 (20)	50
S_3	∞	∞	15 (30)	16	0	30
S_4	∞	∞	∞	17 (40)	0 (10)	50
demand	20	30	50	40	30	

Transportation cost $= (14 \times 20) + (15 \times 20) + (16 \times 10) + (17 \times 20) + (15 \times 30) + (17 \times 40) + (0 \times 20) + (0 \times 10) = 2210$

Problem 06 (Ekanayake., 2022)

Table 23 Unbalance prohibited route transportation problem table

	D_1	D_2	D_3	D_4	supply
S_1	10	13	16	19	700

S₂	∞	10	13	16	700
S₃	∞	15	18	21	200
S₄	∞	∞	15	18	700
S₅	∞	∞	20	23	200
S₆	∞	∞	∞	15	700
demand	300	700	900	800	

Table 24 Step 01 - Since the transportation problem is unbalanced, balance the problem using a dummy column.

	D₁	D₂	D₃	D₄	Dummy column	supply
S₁	10	13	16	19	0	700
S₂	∞	10	13	16	0	700
S₃	∞	15	18	21	0	200
S₄	∞	∞	15	18	0	700
S₅	∞	∞	20	23	0	200
S₆	∞	∞	∞	15	0	700
demand	300	700	900	800	500	

Table 25 Step 02 - Table of Calculating the reciprocal value of unit costs for each cell

	D₁	D₂	D₃	D₄	Dummy column	supply
S₁	0.100	0.077	0.063	0.053	0	700
S₂	0	0.100	0.077	0.063	0	700
S₃	0	0.067	0.056	0.048	0	200
S₄	0	0	0.067	0.056	0	700
S₅	0	0	0.050	0.043	0	200
S₆	0	0	0	0.067	0	700
demand	300	700	900	800	500	

Table 26 Step 03 & step 04 - Table showing the steps used in the improved Vogel's method

	D₁	D₂	D₃	D₄	Dummy column	supply	Penalty value
S₁	0.100 (300)¹	0.077	0.063 (200)⁵	0.053 (100)⁶	0 (100)	700 (400) (200) (100) (0)	0.023 0.014 0.014 0.01 **0.010 0.053**
S₂	0	0.100 (700)³	0.077	0.063	0	700 (0)	0.023 0.023 **0.023** ----- ------
S₃	0	0.067	0.056	0.048	0 (200)	200 (0)	0.011 0.011 0.011 0.008 0.008 0.048
S₄	0	0	0.067 (700)⁴	0.056	0	700 (0)	0.011 0.011 0.011 **0.011** ------
S₅	0	0	0.050	0.043	0 (200)	200 (0)	0.007 0.007 0.007 0.007 0.007 0.043
S₆	0	0	0	0.067 (700)²	0	700 (0)	0.067 **0.067** ------ ------ ------
demand	300 (0)	700 (0)	900 (200) (0)	800 (100) (0)	500 (0)		
Penalty value	**0.100**	0.023	0.010	0.004			
	-----	0.023	0.010	0.004			
	-----	0.023	0.010	0.007			
	-----	-----	0.004	0.003			
	-----	-----	0.007	0.005			
	-----	------	------	0.005			

Table 27 Step 05 - The corresponding assignment values were assigned to the appropriate cells of the transportation problem, and the corresponding total cost was calculated.

	D_1	D_2	D_3	D_4	Dummy column	supply
S_1	10 (300)	13	16 (200)	19 (100)	0 (100)	700
S_2	∞	10 (700)	13	16	0	700
S_3	∞	15	18	21	0 (200)	200
S_4	∞	∞	15 (700)	18	0	700
S_5	∞	∞	20	23	0 (200)	200
S_6	∞	∞	∞	15 (700)	0	700
demand	300	700	900	800	500	

Transportation cost $=(10 \times 300) + (10 \times 700) + (16 \times 200) + (15 \times 700) + (15 \times 700) + (0 \times 100) + (0 \times 200) + (0 \times 200) = 36100$

The following table is intended to evaluate the performance of the proposed method. It is compared with the new algorithm created to find the basic solution to this special transportation problem. The proposed method has been checked with the initial solution as well as the optimal solution. The proposed method has been tested for its suitability in finding the initial solution to transportation problems.

Table 28 Comparison of the solution proposed method with the other research's algorithm

Problem no.	01	02	03	04
A 2-phase method for solving transportation (Ackora-Prah et al., 2022)	9520	8440	8620	8980
Optimum solution (Ackora-Prah et al., 2022)	9520	8440	8620	8980
Proposed algorithm	9520	8440	8620	8980

Table 29 Comparison of the solution proposed method with the other research's algorithm

| Problem	05	06
an improved ant colony algorithm to solve prohibited transportation problems (Ekanayake., 2022)	2210	36100
Optimal solution	2210	36100
Proposed method	2210	36100
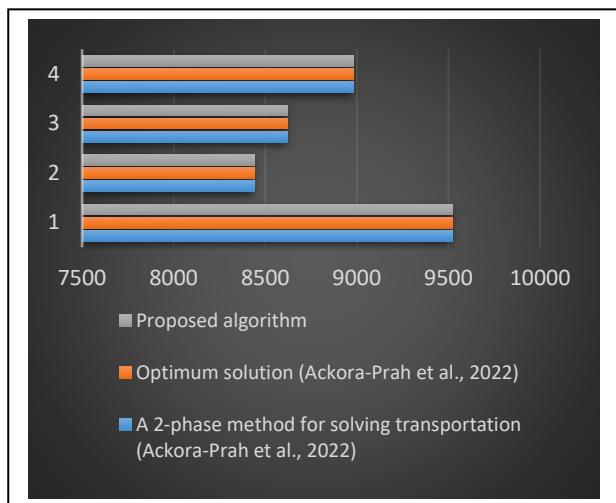
Figure 1 Comparison graph exiting method With proposed method
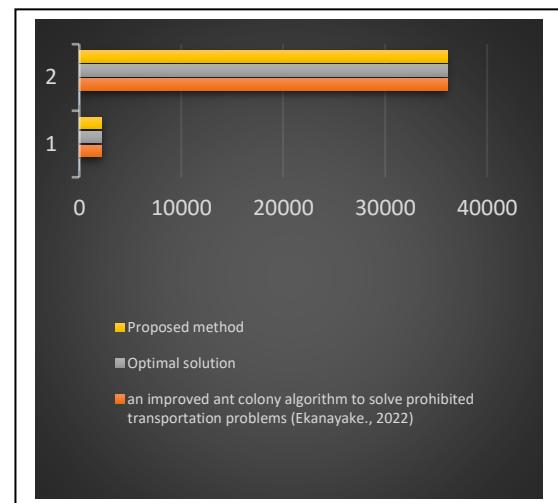
Figure 2 Comparison graph exiting with the proposed method

Problem 07 (Taha, 2011)

Table 30 Balance prohibited route transportation problem table

	D_1	D_2	D_3	supply
S_1	∞	3	5	4
S_2	7	4	9	7
S_3	1	8	6	19
demand	5	6	19	

Table 31 Step 02 - Table of Calculating the reciprocal value of unit costs for each cell

	D_1	D_2	D_3	supply
S_1	0	0.333	0.2	4
S_2	0.143	0.25	0.111	7
S_3	1	0.125	0.167	19
demand	5	6	19	

Table 32 Step 03 & step 04 - Table showing the steps used in the improved Vogel's method

	D_1	D_2	D_3	supply	Penalty value
S_1	0	0.333	$0.2 (4)^3$	4 (0)	0.133 0.133 **0.200** ------ ------
S_2	0.143	$0.25 (6)^2$	$0.111 (1)^5$	7 (1) (0)	0.107 **0.139** 0.111 0.111 **0.111**
S_3	$1 (5)^1$	0.125	$0.167 (14)^4$	19 (14) (0)	0.833 0.042 0.167 **0.167** ------
demand	5 (0)	6 (0)	19 (15) (1) (0)		
Penalty value	**0.857**	0.083	0.033		
	------	0.083	0.033		
	------	------	0.033		
	------	------	0.056		
	------	------	**0.111**		

Table 33 Step 05 - The corresponding assignment values were assigned to the appropriate cells of the transportation problem, and the corresponding total cost was calculated.

	D$_1$	D$_2$	D$_3$	supply
S$_1$	∞	3	5 (4)	4
S$_2$	7	4 (6)	9 (1)	7
S$_3$	1 (5)	8	6 (14)	19
demand	5	6	19	

Transportation cost $=(1 \times 5) + (4 \times 6) + (5 \times 4) + (9 \times 1) + (6 \times 14) = 142$

Problem 8

Table 34 Balance prohibited route transportation problem table

	D$_1$	D$_2$	D$_3$	D$_4$	D$_5$	supply
S$_1$	10	2	3	15	9	25
S$_2$	5	10	15	2	4	30
S$_3$	15	5	14	7	15	20
S$_4$	20	15	13	∞	8	30
demand	30	10	20	25	20	

Table 35 Step 02 - Table of Calculating the reciprocal value of unit costs for each cell

	D$_1$	D$_2$	D$_3$	D$_4$	D$_5$	supply
S$_1$	0.100	0.500	0.333	0.067	0.111	25
S$_2$	0.2	0.100	0.067	0.500	0.250	30
S$_3$	0.067	0.200	0.071	0.143	0.067	20
S$_4$	0.050	0.067	0.077	0	0.125	30
demand	30	10	20	25	20	

Table 36 Step 03 & step 04 - Table showing the steps used in the improved Vogel's method

	D$_1$	D$_2$	D$_3$	D$_4$	D$_5$	supply	Penalty value
S$_1$	0.100	0.500 (10)^2	0.333 (15)^3	0.067	0.111	25 (15) (0)	0.167 0.167 0.222 ----- ------ ------ ----- ---- -
S$_2$	0.2 (5)^4	0.100	0.067	0.500 (25)^1	0.250	30 (5) (0)	0.250 0.050 0.050 0.050 ------ ----- ----- ----
S$_3$	0.067 (20)^7	0.200	0.071	0.143	0.067	20 (0)	0.057 0.129 0.004 0.004 0.004 0.004 **0.067** --- -
S$_4$	0.050 (5)^8	0.067	0.077 (5)^6	0	0.125 (20)^5	30 (10) (5) (0)	0.048 0.048 0.048 0.048 0.048 **0.027** 0.050 **0.050**
demand	30 (25) (5) (0)	10 (0)	20 (0)	25 (0)	20 (0)		
Penalty value	0.100	0.300	0.256	**0.357**	0.125		
	0.100	**0.300**	0.250	------	0.125		
	0.100	------	**0.250**	------	0.125		
	0.133	------	0.006	------	0.125		
	0.017	------	0.006	------	**0.058**		
	0.017	------	0.006	------	------		
	0.017	-----	------	------	------		
	0.050						

Table 37 Step 05 - The corresponding assignment values were assigned to the appropriate cells of the transportation problem, and the corresponding total cost was calculated.

	D$_1$	D$_2$	D$_3$	D$_4$	D$_5$	supply
S$_1$	10	2 (10)	3 (15)	15	9	25
S$_2$	5 (5)	10	15	2 (25)	4	30
S$_3$	15 (20)	5	14	7	15	20
S$_4$	20 (5)	15	13 (5)	∞	8 (20)	30
demand	30	10	20	25	20	

Transportation cost $=(5 \times 5) + (15 \times 20) + (20 \times 5) + (2 \times 10) + (3 \times 15) + (13 \times 5) + (2 \times 25) + (8 \times 20) = 765$

Problem 09 (Srirangacharyulu & Srinivasan, 2010)

Table 38 Unbalance prohibited route transportation problem table

	D_1	D_2	D_3	supply
S_1	30	40	25	3
S_2	30	∞	30	5
S_3	∞	40	40	4
S_4	∞	∞	25	12
demand	3	4	5	

Table 39 Step 01- Since the transportation problem is unbalanced, balance the problem using a dummy column.

	D_1	D_2	D_3	Dummy column	supply
S_1	30	40	25	0	3
S_2	30	∞	30	0	5
S_3	∞	40	40	0	4
S_4	∞	∞	25	0	12
demand	3	4	5	12	

Table 40 Step 02 - Table of Calculating the reciprocal value of unit costs for each cell

	D_1	D_2	D_3	Dummy column	supply
S_1	0.033	0.025	0.040	0	3
S_2	0.033	0	0.033	0	5
S_3	0	0.025	0.025	0	4
S_4	0	0	0.040	0	12
demand	3	4	5	12	

Table 41 Step 03 & step 04 - Table showing the steps used in the improved Vogel's method

	D_1	D_2	D_3	Dummy column	supply	Penalty value
S_1	0.033	0.025 (3)[3]	0.040	0	3 (0)	0.007 0.008 **0.025** ----
S_2	0.033 (3)[2]	0	0.033	0 (2)	5 (2)	0.000 **0.033** 0.000 ----
S_3	0	0.025 (1)[4]	0.025	0 (3)	4 (3)	0.000 0.025 0.025 **0.025**
S_4	0	0	0.040 (5)[1]	0 (7)	12 (7)	**0.040** 0.000 0.000 0.000
demand	3 (0)	4 (1)	5 (0)	12		
Penalty value	0.000	0.000	0.000			
	0.000	0.000	-----			
	-----	**0.025**	-----			
	-----	**0.025**	-----			

Table 42 Step 05 - The corresponding assignment values were assigned to the appropriate cells of the transportation problem, and the corresponding total cost was calculated.

	D_1	D_2	D_3	Dummy column	supply
S_1	30	40 (3)	25	0	3
S_2	30 (3)	∞	30	0 (2)	5
S_3	∞	40 (1)	40	0 (3)	4
S_4	∞	∞	25 (5)	0 (7)	12
demand	3	4	5	12	

Transportation cost $=(30 \times 3) + (40 \times 3) + (40 \times 1) + (25 \times 5) + (0 \times 2) + (0 \times 3) + (0 \times 7) = 375$

Problem 10 (Srirangacharyulu & Srinivasan, 2010)

Table 43 Unbalance prohibited route transportation problem table

	D_1	D_2	D_3	D_4	supply
S_1	3	6	5	2	40
S_2	4	∞	5	5	50
S_3	3	4	4	4	30
demand	50	30	40	20	

Table 44 Step 01 - Since the transportation problem is unbalanced, balance the problem using a dummy row.

	D_1	D_2	D_3	D_4	supply
S_1	3	6	5	2	40
S_2	4	∞	5	5	50
S_3	3	4	4	4	30
Dummy row	0	0	0	0	20
demand	50	30	40	20	

Table 45 Step 02 - Table of Calculating the reciprocal value of unit costs for each cell

	D_1	D_2	D_3	D_4	supply
S_1	0.333	0.167	0.200	0.500	40
S_2	0.250	0	0.200	0.200	50
S_3	0.333	0.250	0.250	0.250	30
Dummy row	0	0	0	0	20
demand	50	30	40	20	

Table 46 Step 03 & 04 - Table showing the steps used in the improved Vogel's method.

	D_1	D_2	D_3	D_4	supply	Penalty value
S_1	0.333 $(20)^2$	0.167	0.200	0.500 $(20)^1$	40 (20) (0)	0.167 **0.133** ----- ------- -----
S_2	0.250 $(30)^4$	0	0.200 $(20)^5$	0.200	50 (20) (0)	0.050 0.050 0.050 0.050 **0.200**
S_3	0.333	0.250 $(30)^3$	0.250	0.250	30 (0)	0.083 0.083 0.083 ----- ------
Dummy row	0	0	0 (20)	0	20 (0)	
demand	50 (30) (0)	30 (0)	40 (20) (0)	20 (0)		
Penalty value	0.000 0.000 0.083 **0.250** ------	0.083 0.083 **0.250** ----- -----	0.050 0.050 0.050 0.200 **0.200**	**0.250** ------ ------ ------		

Table 47 Step 05 - The corresponding assignment values were assigned to the appropriate cells of the transportation problem, and the corresponding total cost was calculated.

	D_1	D_2	D_3	D_4	supply
S_1	3 (20)	6	5	2 (20)	40
S_2	4 (30)	∞	5 (20)	5	50
S_3	3	4 (30)	4	4	30
Dummy row	0	0	0 (20)	0	20
demand	50	30	40	20	

Transportation cost $=(3 \times 20) + (4 \times 30) + (5 \times 20) + (2 \times 20) + (0 \times 20) = 440$

The following table shows the compression of the initial solutions obtained by the traditional "VAM method" and the newly proposed "improved VAM method" used in constructing this new algorithm. Also, the initial solution obtained is compared with the optimal solution.

Table 48 Comparison of the solution proposed method with the other research's algorithm

Problem	07	08	09	10
VAM method	142	∞	∞	440
improved VAM method	142	765	375	440
Optimal solution	142	665	375	440

$()^1$, $()^2$, $()^3$ represent – the order of the allocations in assignment values for each cell.

4. CONCLUSION

This paper proposes a method to find the basic solution through a new approach to the prohibited route transportation problem, a special type of transportation problem. The traditional Vogels method can be pointed out as a method that can successfully obtain essential solutions to transportation problems. However, instead of the traditional Vogel method, this research aims to improve it differently and present it to suit the particular transportation problem. A new method has been proposed to improve the method and is suitable for the prohibited road transport problem. This study found basic solutions for balanced and unbalanced transportation problems using the improved Vogels method. We compared the primary solution to the transportation problem obtained by the proposed method with the essential solutions obtained by other research methods and checked its efficiency. Consequently, the solutions obtained from the problems solved by the proposed method were compared with those obtained from the existing methods. It was concluded that the solutions of the proposed method give similar or more accurate answers than those of other methods. Moreover, many of those solutions can be considered the best. Furthermore, through this proposed method, it is possible to check the basic solution of special transportation problem models. It can confirm the accuracy of this proposed method. Accordingly, this proposed method can be concluded as an approach that can obtain high-basic solutions efficiently to the prohibited route transportation problem.

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