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Epistemological obstacles in combinatorics as praxeological gaps among pre-service mathematics teachers

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Abstract

Combinatorics is widely regarded as difficult, yet most studies stop at classifying student errors without explaining their internal structures. This study reframed epistemological obstacles in combinatorial counting principles as praxeological gaps. Drawing on Brousseau's notion of obstacle and Chevallard's four-component praxeology (task, technique, technology, theory), it asks not where a difficulty comes from but which part of a learner's knowledge has failed to develop. Within the prospective phase of Didactical Design Research, we collected written responses to two counting problems from 83 pre-service mathematics teachers who had completed a Discrete Mathematics course, interviewed 17 of them, and read their textbooks praxeologically. Triangulating these sources, we identified three recurring gaps: a technique whose justification had collapsed into a surface keyword rule ("and" means multiply, "or" means add); a technique fitted only to familiar problems; and a technique applied with no justification at all. Across all three, the practical block was secure while the theoretical block remained thin, an asymmetry mirrored in the textbook, which defined the rules through keywords rather than independence and disjointness. Reading obstacles this way turns a general diagnosis into a specific one, giving teacher educators a tool for designing instruction that targets the precise failed component of reading.

Keywords: anthropological theory of the didactic; combinatorics; counting principles; epistemological obstacles; praxeology; pre-service mathematics teachers

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Introduction

Combinatorics is one of those topics that almost every mathematics teacher recognises as troublesome. Its questions sound simple like how many ways can a set of objects be arranged or chosen, yet learners across all levels, from school children to university students, repeatedly struggle to answer them reliably (Batanero et al., 1997; Lockwood, 2013). The simplicity of the phrasing hides a real difficulty, and because counting principles underpin discrete mathematics, probability, and computer science (Niven, 1965; Rosen, 2012), trouble here ripples outward into other areas.

More worrying is that the difficulty does not disappear with advanced training. Undergraduates in combinatorics courses make systematic (Eizenberg & Zaslavsky, 2004; Lockwood et al., 2020; Salavatinejad et al., 2021), and the same confusions: misapplying the combination formula, blurring the line between ordered and unordered selections, treating the surface look of a problem as a cue for choosing a technique, persist among pre-service mathematics teachers who are themselves preparing to teach the topic (Desfitri, 2024; Matitaputty et al., 2022; Sukoriyanto et al., 2016). The concern is not only pedagogical. If those who will soon teach have not secured a firm grasp of counting principles, the difficulty is likely to be handed down to their future students. A flexible, deep command of the subject has long been treated as a basic requirement for teaching well (Ball & Forzani, 2011), and the combinatorics evidence suggests this requirement is often unmet. This concern is sharper for counting principles specifically. Because the sum and product rules are the foundation on which permutations, combinations, and later probability are built, a pre-service teacher whose grasp of these rules rests on memorised keywords rather than on the underlying logic is likely to transmit the same fragile foundation to their own students, reproducing the difficulty one generation further down.

Two research traditions have addressed difficulties of this kind, each with its own vocabulary. The first, grounded in Brousseau's (2002) Theory of Didactical Situations, treats them as learning obstacles and traces their sources to the learner’s readiness (ontogenic), the way content is taught (didactical), or the structure of the knowledge itself (epistemological). Of these, the epistemological obstacle is the most closely tied to the mathematics itself: it is a piece of knowledge that works in one context yet misleads in another, and so recurs across learners regardless of individual ability. In Indonesia the obstacle lens has been applied widely, including, more recently, to combinatorics (Nopriana et al., 2023). The second tradition, Chevallard's (2019) Anthropological Theory of the Didactic, models knowledge as a praxeology with four components: a type of task (T), a technique (τ), a technology (θ) that justifies the technique, and a theory (Θ) that organises the justification. Praxeological analyses of textbooks have grown into a productive line of work, including in Indonesian settings, and have repeatedly found that such materials foreground procedural know-how while leaving the justifying discourse thin (Chandra et al., 2025; Marhami et al., 2024).

To make these components concrete, consider a familiar counting task. In the praxeological model (Chevallard, 2019), the type of task (T) might be "find the number of ways to travel from one city to another through several connecting points." A technique (τ) is the procedure used to solve it-here, multiplying the number of options at each stage. The technology (θ) is the discourse that justifies this technique: an explanation that the stages are independent, so each choice can be combined with every other, which is why they are multiplied rather than added. The theory (Θ) is the broader body of knowledge that underwrites this justification-in this case, the fundamental counting principle and the logic of independent events. Together, T and τ form the practical block (knowing what to do and how), while θ and Θ form the theoretical block (knowing why it works). This distinction is central to our analysis: as we will show, our participants often commanded the practical block while the theoretical block remained underdeveloped.

The two traditions rarely meet, and we think the separation comes at a cost. Obstacle studies often stop once a difficulty has been labelled epistemological, as though naming it were the same as explaining it. Saying that a student who applies a combination formula to a route problem has an epistemological obstacle does not reveal which part of the underlying knowledge has failed, nor how that failure relates to the textbooks and lectures the knowledge came from. Praxeological analysis supplies exactly that structural vocabulary, but applied only to textbooks it loses touch with what learners actually do. Bringing the two together is the gap this study addresses.

We propose to read the epistemological obstacles that arise in combinatorics as praxeological gaps: missing, broken, or underdeveloped links within the four-component structure of a combinatorial praxeology. This builds on the notion of praxeological incompleteness (Bosch & Gascón, 2014), extending it from the analysis of institutions to the diagnosis of individual learners. A student who can execute a technique but cannot justify it is displaying a praxeology whose practical block is intact while its theoretical block (θ and Θ) has not formed. A student who recognises only a narrow band of problem types is displaying a praxeology whose type of task (T) has been constrained by teaching. Read this way, an obstacle stops being a mere category and becomes a diagnosis: it tells us which component to address. It also connects the individual learner to the institution that taught them, and points toward interventions aimed at a specific structural weakness rather than at "teaching better" in general.

Combinatorics is an especially revealing place to look. Its formulas are compact and easy to compute, so a learner can produce an answer without engaging the reasoning that warrants it, an observation Lockwood (2013, 2014) makes in different language when she argues for a set-oriented perspective that reconnects formulas to the outcomes they count. Its problems are often surface-similar but structurally different, so a rule built on surface cues works on familiar cases and fails on new ones (Batanero et al., 1997), and its justifications are combinatorial-logical rather than algorithmic, making them easy to leave out.

Working with Indonesian pre-service mathematics teachers engaging with counting principles, this study pursues three questions. First, what epistemological obstacles do these students encounter? Second, how can these obstacles be characterised as praxeological gaps, as particular configurations of missing or disrupted components within a combinatorial praxeology? Third, what does reading obstacles in this way imply for designing combinatorics instruction in teacher education?

Methods

This study used a qualitative interpretive design, suited to research questions about the meaning and structure of students' reasoning rather than frequencies or effect sizes sizes (Creswell & Poth, 2018). It sits within the prospective phase of Didactical Design Research (Suryadi, 2019), which seeks to understand the obstacles produced by existing arrangements before a new design is proposed. Consistent with our focus, the analysis centres on epistemological obstacles, those rooted in the structure of the knowledge itself; the teaching and textbook are treated not as separate findings but as sources that help explain how such obstacles form.

Participants and context

The study took place at a public university in Aceh Province, Indonesia, whose Discrete Mathematics course treats the sum and product rules, permutations, and combinations, following standard tertiary texts (Rosen, 2012). Participants had already completed this course, since our interest lay in the praxeologies acquired through instruction. Eighty-three students completed the written task, and from these, 17 were selected for interview using maximum-variation sampling (Patton, 2015), based primarily on the type of error exhibited in their written work so that the full range of distinct error patterns (rather than a representative sample) would be probed. Ethical clearance was obtained, participants consented to the use of their written work, and pseudonyms (M1, M2, and so on) are used throughout. The study was conducted in Indonesian: students' written work is presented in its original form as authentic data, with English translations of the relevant elements. To ensure that no epistemological meaning was lost, interview excerpts were translated into English and independently back-translated into Indonesian, and the two Indonesian versions were compared and reconciled before the English translation was finalised.

Data collection

Three sources were used. The first was a written Combinatorial Task of two essay problems on counting principles, both easy to read but demanding a non-trivial identification of the task type and a justified choice of technique. **Problem 1** set a travel-route context for the sum and product rules: Putri travels from Bandung to Aceh, with six land-transport options from Bandung to Soekarno-Hatta Airport (CGK), then three mutually exclusive air routes, direct domestic (four airlines CGK–BTJ), transit domestic (four CGK–KNO and three KNO–BTJ), and international (seven CGK–KUL and two KUL–BTJ). Students found the number of ways to travel one way, to make an international round trip, and to travel out internationally and back domestically. **Problem 2** set a constrained-counting context: Tika's six-digit PIN encodes a nephew's birthday (two digits each for day, month, and the last two digits of the year), with no zero digit, an odd month, and a year between 2000 and 2020. Because these rules bind the digits together, any scheme that simply multiplies per-digit choices fails. Participants worked individually and were asked to explain their choice of technique in their own words, so that traces of the justifying discourse would surface

The second source was semi-structured interviews of about 40–60 minutes with the selected participants, designed to reach what written work could not, the technology (θ) and the sense of underlying theory (Θ). Following the Socratic questioning (Paul, 1990) tradition used in Indonesian DDR research, the interviewer asked participants to walk through their solutions and then probed more deeply: why a technique was chosen, whether the same approach would hold for a structurally different variant, and whether another route existed. Questions were adapted to what participants said (Brinkmann & Kvale, 2015). Interviews were audio-recorded, transcribed verbatim, and anchored to each participant's written work.

The third source was a praxeological analysis of the textbook most used in the course, focused on the counting principles chapters. The rationale is theoretical: an epistemological obstacle in a learner usually reflects, and is sustained by, the praxeological structure of the institution that taught them (Bosch & Gascón, 2014; Wijayanti & Winsløw, 2017). Following the reference-model method (Wijayanti & Winsløw, 2017), we examined how the principles were presented, the tasks posed, the techniques offered, and above all whether the justifying discourse (θ and Θ) was explicit, implicit, or omitted, seeking the overall pattern rather than cataloguing every example.

Data analysis

Analysis ran in two layers. The first followed the standard Brousseau-informed approach (Brousseau, 2002; Suryadi, 2019): reading written work and interviews together, we coded recurring difficulties and retained those rooted in the structure of the knowledge (epistemological obstacles). The second asked, of each obstacle, which of T, τ, θ, or Θ was functioning and which was absent, weak, or misaligned, using the mapping scheme in Table 1, drawing on written work for T and τ and interview evidence for θ and Θ. Each obstacle was then characterised as one or more of three gap types: logos-deprived praxeology (τ present, θ and Θ absent), misaligned technology (τ present, θ surface-based), and constrained type of task (T narrow, Θ absent), with the textbook analysis checked for a structural counterpart. These three sources together provided the triangulation on which each characterisation rests.

Table 1. Praxeological mapping scheme for analysing student responses

Component	Guiding question	Evidence base
T (task)	Is the task type correctly identified?	Written work; opening of interview
τ (technique)	Which technique is used? Is it appropriate and correctly executed?	Written work
θ (technology)	Can the student justify why the technique applies-structurally, by surface cue, or not at all?	Interview: why/when questions
Θ (theory)	Can the student connect the technique to general principles and adapt it to variants?	Interview: generalisation/transfer

Trustworthiness was addressed through Lincoln and Guba's (1985) criteria: credibility through triangulation across the three sources and sustained engagement, with divergences between written and spoken accounts treated as data rather than noise; transferability through detailed description of context and tasks; dependability through a documented audit trail and review with the supervisory team; and confirmability by actively seeking cases that resisted the typology, which was refined in response. The first author has taught combinatorics in a similar setting; a reflexive journal and challenge from co-authors guarded against over-familiar readings.

Results

We report the epistemological obstacles as three types of praxeological gap, each illustrated through the triangulation of students' written work and interviews. Table 2 first summarises how the three gaps were distributed across the 17 interviewed participants; we then examine each gap in turn, and close with the textbook analysis that locates the institutional source of these gaps. Problem 1 (the travel-route problem) tested the sum and product rules; Problem 2 (the PIN problem) tested constrained counting under the product rule.

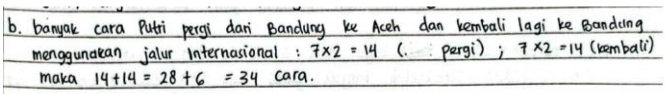
Table 2. Distribution of the three praxeological gaps

Praxeological gap	Number of participants
Misaligned technology	14
Constrained type of task	8
Logos-deprived praxeology	5

As Table 2 shows, the most frequently observed gap was the misaligned technology, in which a technique was driven by surface keywords rather than by the structure of the problem; the logos-deprived praxeology, in which a technique was applied with no justification at all, appeared least often. These counts are indicative rather than statistical: the categories overlap, several participants exhibited more than one gap, and our aim is to characterise the structure of each gap rather than to measure its prevalence.

Misaligned technology: Techniques driven by surface keywords

The most pervasive gap was a technology block that was present but tied to surface keywords rather than to the structure of the problem. M2's written work on Problem 1 (Figure 1) shows this clearly in part (b). M2 correctly multiplied the segments within each one-way trip (7 × 2 = 14) but then added the outbound and return trips (14 + 14 = 28), where a round trip requires multiplication, and finally added the six land-transport options (28 + 6 = 34) as though they were a separate route rather than part of every journey.


b. banyak cara Putri pergi dari Bandung ke Aceh dan kembali lagi ke Bandung menggunakan jalur Internasional : 7×2 = 14 (pergi) ; 7×2 = 14 (kembali) maka 14+14 = 28+6 = 34 cara.

Translation

b. The number of ways Putri travels from Bandung to Aceh and returns to Bandung via the international route: 7 × 2 = 14 (going); 7 × 2 = 14 (returning), so 14 + 14 = 28 + 6 = 34 ways

Figure 1. M2’s written response to Problem 1

The interview revealed why M2 was applying a keyword rule (excerpts translated from Indonesian):

I : Why did you use the sum rule for 14 + 14 = 28 and 28 + 6 = 34?

M2: Because there are 14 ways to go and 14 to return; then 28 + 6 because there are 6 kinds of transport.

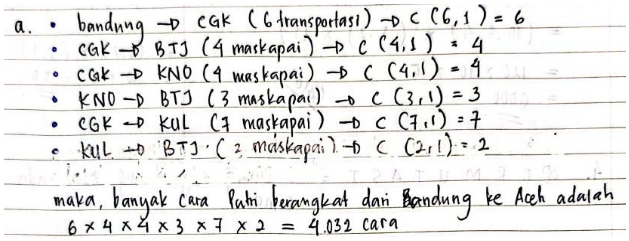
I: How do we tell the sum rule from the product rule?

M2: The sum rule has the word "or", and the product rule we use when there is the word "and".

I: So you used addition because there is an "or". Where is the "or" in the problem?

M2: Hmm… there isn't one, is there? Hehe.

Probed with a simple variant, choosing one representative from 15 boys and 12 girls, M2 answered "15 × 12, because there is the word 'and'." The technique (τ) was available and executable, but the technology (θ) had collapsed into a lexical rule ("and" → multiply, "or" → add) detached from the structural conditions of independence and disjointness, with no theory (Θ) to fall back on when the keyword was absent. This is a misaligned technology in the precise sense of our framework: the justification exists but points at the wrong thing. A related misalignment appeared in M1 (Figure 2), who rendered every leg of the journey as a combination.



Translation

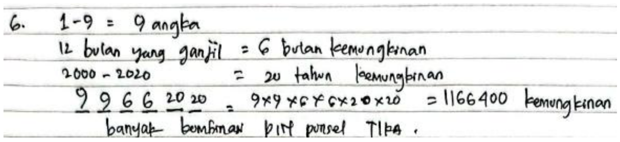
Bandung → CGK (6 transports)→ C(6,1)=6
CGK → BTJ (4 airlines) → C(4,1) = 4
CGK → KNO (4 airlines) → C(4,1) = 4
KNO → BTJ (3 airlines) → C(3,1) = 3
CGK → KUL (7 airlines) → C(7,1) = 7
KUL → BTJ (2 airlines) → C(2,1) = 2.
So the number of ways Putri travels from Bandung to Aceh is 6 × 4 × 4 × 3 × 7 × 2 = 4,032 ways

Figure 2. M1's written response to Problem 1

In interview M1 explained that "*all selection problems are combinations*". Numerically C(6,1) = 6 is harmless, and the final multiplication is defensible, but casting every simple choice as a combination signals that M1 did not distinguish a genuine combinatorial selection from a direct count, a sound technique resting on a miscalibrated technology inherited from procedurally framed prior experience.

Constrained type of task: Schemes that fit only familiar problems

A second gap involved a praxeology built around too narrow a range of task types, with no theory to support adaptation. On Problem 2, M14 (Figure 3) transferred a per-digit independence scheme from simpler PIN or licence-plate problems. The scheme treats each of the six digit-positions as an independent choice, but the PIN's digits are not independent: the day, month, and year are bound together by calendar rules and the no-zero condition, so many of the 1,166,400 combinations are not valid dates at all.



Translation

6. 1–9 = 9 digits.
12 months, the odd ones = 6 possible months.
2000–2020 = 20 possible years.
$\underline{9}\ \underline{9}\ \underline{6}\ \underline{6}\ \underline{20}\ \underline{20}$ = 9×9×6×6×20×20 = 1,166,400 the number of possible PINs for Tika's phone."

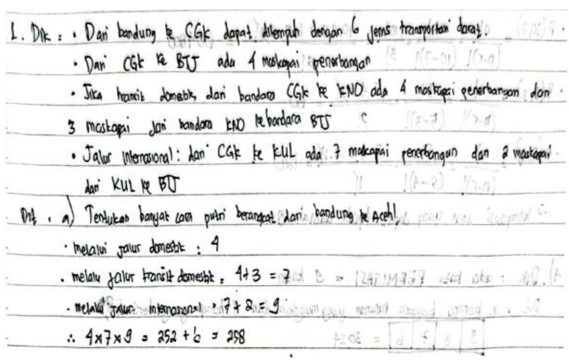
Figure 3. M14's written response to Problem 2

When asked for a single valid PIN, the scheme broke down:

I : Can you give one example of a valid PIN for Tika?
M14 : Oh, I never tried… 240304. The 24th, month 03, year 2004.
I : But the problem says no zeros.
M14: Oh right, so 241314.
I : Is 13 a month? Is there a month 13?
M14: No… so how, Bu? Oh, month 11, so 241114. That fits.

M14's technique (multiplying per-digit options) was correct for the task type it was built for, but Problem 2 is a different type: its digits are not independent, since calendar rules and the no-zero condition constrain them jointly. The type of task (T) recognised by M14 was too narrow, and there was no theoretical organisation (Θ) that would have flagged the new problem as structurally different. The same constraint showed in M3 and M4, who inserted numbers straight into permutation or combination formulas: M3 computing C(6,2) "because route A–B is the same as B–A," M4 using P(n,r) "because there is an order", treating the formula as a template to be filled rather than a model to be matched to structure.

Logos-deprived praxeology: Techniques without justification



Translation

Known: ● from Bandung to CGK can be reached by 6 kinds of land transport
● From CGK to BTJ there are 4 airlines
● If transit-domestic, from CGK to KNO there are 4 airlines and 3 airlines from KNO to BTJ
● International route: from CGK to KUL there are 7 airlines, with 2 airlines from KUL to BTJ.
Asked: a) determine the number of ways Putri travels from Bandung to Aceh!
● Via the domestic route: 4
● Via the transit-domestic route: 4 + 3 = 7
● Via the international route: 7 + 2 = 9; so 4 × 7 × 9 = 252 + 6 = 258

Figure 4. M5's written response to Problem 1

In a third gap the justifying discourse was not merely misaligned but largely absent. M5 (Figure 4) set out the problem correctly but then handled the rules inconsistently. Two things are striking here. Within the transit-domestic and international routes, M5 added the consecutive flight segments ($4 + 3 = 7$ and $7 + 2 = 9$) where the segments are sequential and should be multiplied; yet across the three mutually exclusive routes, M5 multiplied ($4 \times 7 \times 9$) where the routes should be added, before appending a final "+ 6" whose role M5 could not explain. The two principles were applied in exactly the wrong places.

Asked when each rule applies, M5 offered “if it is one experiment, add; if there are two experiments, multiply”, a self-made rule with no grounding in the independence or disjointness of the events. Here the technique was deployed, but the technology offered for it was improvised and the underlying theory was not available, a praxeology whose practical block functioned while its logos block stayed empty.

The textbook: An implicit, keyword-bound logos block

Triangulating with the textbook supplied a striking institutional counterpart to these learner-level gaps. A praxeological reading of its counting-principles chapters showed that the two rules were defined through the keywords “and” and “or” rather than through the conditions of independence and disjointness that Rosen (2012) and Niven (1965) make explicit. The justifying discourse was thus present but thin and lexically framed, precisely the form that reappeared in the learners as a misaligned technology. The keyword rule that defeated M2, who searched for an “or” that was not there, is in this sense not an individual lapse but a faithful reproduction of the way the book itself presents the concept.

Two further features of the presentation reinforced this. The worked examples were short and routine, solved by direct multiplication or addition with little prior analysis of the situation, and no single example required both rules together in one contextual problem; this offers learners little practice in deciding which rule a situation calls for. And the two principles were introduced and revisited in a back-and-forth order rather than being clearly contrasted, so the conditions distinguishing them were never brought into sharp relief. Both features are plausible institutional sources of the epistemological gaps reported above, and together they suggest that strengthening the logos block in the materials themselves is part of any remedy.

Discussion

The first research question asked what epistemological obstacles these pre-service teachers encounter. Reading each obstacle through the praxeology located the failure within a specific component, and three configurations recurred. In the misaligned-technology configuration, seen most sharply in M2 and M1, the technique was sound but its justification had collapsed into a lexical rule (“and” means multiply, “or” means add) detached from the conditions of independence and disjointness. In the constrained-task configuration, seen in M14, M3, and M4, the technique fit only a narrow, familiar task type and there was no theory to signal when a new problem fell outside it. In the logos-deprived configuration, seen in M5, the technique ran on an improvised justification with no underlying theory at all. What unifies the three is an asymmetry: the practical block (T and τ) was comparatively secure while the theoretical block (θ and Θ) was thin, broken, or missing.

This asymmetry is not peculiar to our participants. It is precisely what Lockwood et al., (2015) observed when their undergraduates could reinvent counting formulas yet struggled to justify them, and what Lockwood et al. (2018) captured in arguing that students often reason with formulas and expressions while losing sight of the sets of outcomes those formulas are meant to count. Tatira (2024), studying first-year undergraduates' ways of thinking in combinatorics, found that students were skilled at the step-by-step application of counting formulae but remained at an "action" level of conception, rarely reaching the object-level understanding that would let them handle novel counting situations. The convergence across independently developed frameworks (Lockwood's cognitive model, the APOS lens Tatira (2024) applies, and Chevallard's praxeology) strengthens the claim that the difficulty is structural rather than incidental. The keyword strategy in particular has a long history: Batanero et al., (1997) showed decades ago that students attend to surface features rather than underlying structure, and our M2, who searched for an "or" that was not there, reproduces that pattern.

What our analysis adds is an account of where the asymmetry comes from. Because epistemological obstacles, in Suryadi's (2019) sense, are sustained by the institution that taught the knowledge, the textbook analysis is not a separate finding but part of the explanation. The book defined the two rules through "and" and "or" rather than through independence and disjointness, the conditions Rosen (2012) and Niven (1965) make explicit, so the lexical rule that defeated M2 is a faithful reproduction of the book's own framing rather than an individual lapse. This is consistent with Lockwood et al., (2017) analysis of how the multiplication principle is stated across combinatorics textbooks, which found that statements frequently omit the conditions under which the principle validly applies. When the logos block is thin in the taught knowledge, it tends to remain thin in the learner. Topphol (2023), analysing a different topic through the same praxeological lens, describes exactly this outcome, a didactic transposition that yields "a poor logos block" in the material, and more recent praxeological analyses report the same imbalance, Chandra et al. (2025), examining the treatment of the derivative in Indonesian textbooks, found technical components (T/τ) foregrounded while the epistemic justification (θ/Θ) remained weak and fragmented. Our data suggest the counting principles in this setting have undergone a similar transposition.

The stakes are higher because our participants are pre-service teachers. Following the logic of didactic transposition (Chevallard, 2019), a teacher who has internalised an incomplete praxeology is positioned to transpose that incompleteness onward to the next generation of learners. This is why we read the gaps not merely as individual shortcomings but as a potential link in a reproductive chain. The point connects to a wider conversation about teacher knowledge: Pansell (2023) argues for treating mathematical knowledge for teaching itself as a didactic praxeology, in which a teacher's know-how and know-why are two faces of one structure rather than separable competencies; subsequent work in teacher education extends this view, with Mensah & Pansell (2024) analysing how the practical and theoretical blocks of a praxeology are jointly shaped by the institutional setting in which teachers are prepared. Read through that lens, a pre-service teacher with a robust τ but a thin θ does not simply "lack understanding"; they hold a praxeology whose logos block has not been built, and which is therefore not yet ready to be taught with justification. Studies of prospective teachers' praxeologies in Indonesian contexts, such as Putra's (2020) analysis of how rational-number knowledge is transposed in a teacher-education, and Suryadi et al. (2023) close analysis of a prospective teacher's task design from an ATD perspective, similarly show that the gap between knowledge to be taught and knowledge actually held by prospective teachers is a recurring concern, not an isolated one.

Turning to the third research question (what reading obstacles as praxeological gaps implies for designing combinatorics instruction in teacher education), the value of the framework is that it makes the remedy specific rather than generic. Each gap points to a different instructional response. A misaligned technology calls not for more practice but for tasks designed to break the keyword rule, so that the lexical cue is exposed as unreliable and replaced by the structural conditions of independence and disjointness.

Such tasks are most effective when the surface keyword points to the wrong operation. Consider a task in which the word "and" appears but addition is required: "Budi has 2 red shirts *and* 3 blue shirts. How many shirt choices does Budi have to wear?" A keyword-driven student reads "and" and multiplies ($2 \times 3 = 6$), yet the task asks for a single shirt chosen from a combined set of disjoint options, so the answer is $2 + 3 = 5$. Conversely, consider a task in which the word "or" appears but multiplication is required: "A PIN consists of 4 digits, and each digit may be 0, 1, *or* 2. How many PINs can be formed?" Here "or" may suggest addition, but each of the four positions is filled independently from three options, so the answer is $3 \times 3 \times 3 \times 3 = 81$. In both cases the connective contradicts the operation the structure demands, forcing the surface cue to fail visibly and pushing students to reconstruct their reasoning on the conditions of independence and disjointness rather than on keywords; this is precisely the kind of task M2's difficulty suggests is needed.

The other two gaps call for their own responses. A constrained type of task calls for deliberately varying the surface of a problem while holding its structure constant, and the reverse, so that learners build a theory (Θ) capable of recognising a new problem as an instance of a known type; this resonates with Lockwood's (2014) argument for cultivating a set-oriented perspective that ties techniques back to the outcomes they enumerate. A logos-deprived praxeology calls for making justification a routine part of solving rather than an afterthought, much as Lockwood et al., (2015) found that pressing students to justify their formulas was what moved them toward genuine combinatorial understanding. The broader implication is that designing combinatorics instruction for pre-service teachers should begin not from the techniques, which our participants already commanded, but from the logos block they lacked: instruction should target the specific component a diagnosis reveals to be missing. In the present Didactical Design Research, each identified gap thus becomes a design specification for the didactical situation intended to close it.

Several limitations bound these claims. The study used one cohort at a single institution with two tasks, so the patterns are illustrative rather than exhaustive, and other settings may reveal further configurations. The three-way typology is an analytical device; real responses often show more than one gap at once, as M14's did. Finally, the praxeological reading is interpretive, resting on inferences about θ and Θ drawn from what students said and wrote; we have disciplined those inferences through triangulation, but they remain interpretations. These limits notwithstanding, the close correspondence between the learner-level gaps and the textbook's praxeological structure gives us some confidence that what we have described is a structural feature of how these counting principles are taught and learned in this setting, not merely a collection of isolated mistakes.

Conclusion

This study set out to understand the epistemological obstacles that pre-service mathematics teachers meet in combinatorial counting principles, and to characterise them through the structure of mathematical knowledge rather than only by their source. Bringing Brousseau's notion of obstacle together with Chevallard's praxeology, we read each difficulty as a praxeological gap and identified three recurring forms: misaligned technology, constrained type of task, and logos-deprived praxeology. Across all three, the practical block was relatively secure while the theoretical block remained thin, an asymmetry mirrored in the textbook from which students had learned.

This reframing makes a twofold contribution. Theoretically, reading obstacles as praxeological gaps converts classification into diagnosis, naming not just where a difficulty comes from but which component of the knowledge has failed. Practically, that diagnosis yields specific design implications that we carry into the design phase of this Didactical Design Research. The study is limited by its single setting, two tasks, and interpretive method, and the typology will need refinement against other contexts. Even so, the close correspondence between what learners did and how their textbook was structured points to a task for the wider community: strengthening the logos block cannot be the work of instructors alone, but calls on textbook authors and curriculum designers, in Indonesia and elsewhere, to treat the justification of the counting principles not as optional elaboration but as the core of teaching counting with understanding.

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Declarations

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