



Comparative Analysis of Endowment Life Insurance Premium Reserves Using the Canadian Method Based on the 2019 TMI and Gompertz's Law

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Abstract

Life uncertainty, particularly mortality risk, creates the need for financial protection through life insurance. Endowment life insurance is a product that provides both death benefits and survival benefits at the end of the coverage period; therefore, insurance companies must establish adequate premium reserves. This study aims to calculate premium reserves for endowment life insurance using the Canadian method and to compare the effects of different mortality models based on the 2019 Indonesian Mortality Table (TMI) and Gompertz's law. The results show that premium reserves calculated using Gompertz's law are higher than those calculated using the 2019 Indonesian Mortality Table (TMI). This difference indicates that the choice of mortality model affects the magnitude of the resulting premium reserves.

Keywords: endowment life insurance, premium reserves, Canadian method, Gompertz's law

1. Introduction

Human life is constantly faced with various uncertainties, one of which is mortality risk, which may cause financial consequences for surviving dependents, particularly in the absence of adequate financial planning. One effort to anticipate such risk is through life insurance ownership. According to Ekawati and Fardinah (2020), life insurance is a form of insurance that provides financial benefits to the beneficiaries or family of the insured upon the insured's death, in accordance with the terms stated in the policy.

In general, life insurance products consist of term life insurance, whole life insurance, and endowment life insurance. Endowment life insurance is selected as the focus of this study because it provides two benefits simultaneously, a death benefit if the insured dies during the coverage period and a maturity benefit if the insured survives until the end of the policy term. (Bowers et al., 1997)

In managing life insurance products, insurance companies are required to calculate premium reserves as funds set aside to fulfill future benefit payments. Ghifari dan Sudding (2023) state that premium reserves can be calculated using various methods, one of which is the Canadian method, which equalizes modified initial premiums with net premiums through adjustments between net premiums and natural premiums. In addition to the calculation method, the amount of premium reserves is also influenced by the mortality model used. Karakaş and Bulut (2025) explain that Gompertz's law has long been used to describe aging and mortality processes due to its ability to represent a hazard function that increases with age.

Previous studies indicate that both calculation methods and mortality models significantly affect premium reserves. Fitriyani et al. (2021) show that the Canadian method produces higher premium reserves than the Commissioners method for endowment life insurance. Meanwhile, Jannah et al. (2023) demonstrate that Gompertz's law can generate mortality probabilities based on the 2019 Indonesian Mortality Table (TMI).

Based on these studies, this research examines the calculation of endowment life insurance premium reserves using the Canadian method by comparing two mortality models, 2019 TMI and Gompertz's law. This comparison aims to analyze the effect of mortality model selection on the resulting premium reserves.

2. Literature Review

2.1. Endowment Life Insurance

Nadilia et al. (2020) state that various risks related to health, property, and life safety may arise. One common way to minimize the impact of these risks is by participating in insurance programs. According to Adelia et al. (2024), insurance aims to transfer the risk borne by individuals to insurance companies as insurers, thereby providing financial protection and security against various risks of loss that may occur unexpectedly.

Vahabi and Najafabadi (2022) explain that life insurance can be divided into three types based on benefit payment methods and coverage periods, namely term life insurance, whole life insurance, and endowment life insurance. Each type of life insurance is designed with different benefits to adjust to diverse needs and varying levels of financial capability in society.

According to Sadli and Sari (2025), endowment life insurance is an insurance product that combines life protection with a savings component, so that insurance benefits will be provided both when the insured dies and when the insured is still alive until the end of the coverage period.

2.2. Interest

According to Yulita (2025), interest is a payment made by borrowers as compensation for the use of borrowed funds over a certain period. The amount of interest income obtained depends on the principal amount, the length of the investment period, and the interest rate used.

There are two methods of interest calculation, namely simple interest and compound interest. Futami (1993) states that simple interest is an interest rate calculated based on the ratio between the principal and the investment period, while compound interest is a calculation method in which the principal invested in the subsequent period is the sum of the previous principal and the interest earned.

2.3. Mortality Table

Yulita (2025) states that a mortality table is a statistical representation that shows mortality rates based on age and gender. In insurance practice, mortality tables are used to estimate the life expectancy of the insured, determine the length of premium payment periods, and determine the timing of benefit payments. Mortality tables consist of several columns, namely x, l_x, d_x, p_x, q_x (Bowers et al., 1997).

$$l_x = l_{x-1}p_{x-1} \quad (1)$$

$$d_x = l_x - l_{x+1} \quad (2)$$

$$p_x = 1 - q_x \quad (3)$$

$$q_x = \frac{l_x - l_{x+1}}{l_x}$$

$$q_x = \frac{d_x}{l_x} \quad (4)$$

where x : age, l_x : the number of people alive at age x years, d_x : the number of people who die before reaching $x + 1$ years, p_x : the probability that a person aged x years survives, and q_x : probability that a person aged x years dies.

2.4. Commutation Functions

According to Kele et al. (2024), commutation functions are values defined to simplify insurance calculations and the writing of formulas in mortality tables. Commutation functions are used in calculating single premiums, annual premiums, and other insurance calculations. The commutation functions are as follows:

$$D_x = v^x l_x \quad (5)$$

$$N_x = D_x + D_{x+1} + D_{x+2} + \dots + D_\omega \quad (6)$$

$$C_x = v^{x+1} d_x \quad (7)$$

$$M_x = C_x + C_{x+1} + C_{x+2} + \dots + C_\omega \quad (8)$$

where v : the present value of a payment of 1 made one year later, D_x : a commutation function representing the product of the discount factor raised to age x and the number of people alive at age x , N_x : cumulative value of D_x , C_x : a commutation function representing the product of the discount factor raised to age $x + 1$ and the number of people who die before reaching age $x + 1$, and M_x : cumulative value of C_x .

2.5. Annuities

According to Wettstein et al. (2021), an annuity is a series of payments of a certain amount of money made periodically over a certain period of time. Based on its type, annuities can be divided into two, namely certain annuities and life annuities. Siswono et al. (2021) state that payments of certain annuities are made without specific conditions, whereas payments of life annuities depend on the living or death condition of a person.

Life annuities are divided into two types, namely whole life annuities that are paid until the recipient dies, and temporary life annuities that are paid for a certain period of time. The equation for a temporary life annuity-due can be written as follows:

$$\ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x} \quad (9)$$

2.6. Net Single Premium of Endowment Life Insurance

The net single premium for endowment life insurance is denoted by $A_{x:\overline{n}|}$, which is for a person aged x years with a coverage period of n years and a sum assured of 1 unit (Futami, 1993), and can be expressed by the following equation:

$$A_{x:\overline{n}|} = A_{x:\overline{n}|}^1 + A_{x:\overline{n}|}^{\frac{1}{}}$$
(10)

The net single premium of endowment life insurance can be written as:

$$A_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x} \quad (11)$$

2.7. Net Annual Premium of Endowment Life Insurance

The net annual premium for endowment life insurance is denoted by $P_{x:\overline{n}|}$, which is the the annual premium for a person aged x years with a sum assured of 1 unit that is paid at the end of the coverage period for n years (Futami, 1993), and can be expressed by the following equation:

$$P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \quad (12)$$

based on equation (9) and (11), then:

$$P_{x:\overline{n}|} = \frac{\frac{M_x - M_{x+n} + D_{x+n}}{D_x}}{\frac{N_x - N_{x+n}}{D_x}}$$

$$P_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}} \quad (13)$$

2.8. Premium Reserves

According to Adam et al. (2024) and Saragih et al. (2024), premium reserves are funds that must be set aside by insurance companies, which originate from premiums paid by policyholders. Premium reserves play an important role in maintaining the financial stability of insurance companies and ensuring the ability of companies to meet their obligations for future claims.

Yulita (2025) explains that the calculation of premium reserves using actuarial calculations can be carried out through two methods, namely the prospective method and the retrospective method. In the prospective method, premium reserves are calculated as the difference between the estimated future expenditures of the company and the estimated future premium income. Meanwhile, the retrospective method calculates premium reserves as the difference between the premiums that have been received along with accumulated interest and the value of benefits that must be paid.

2.9. Regression Analysis

Khairiyah et al. (2018) explain that regression analysis is an analytical method used to describe the mathematical relationship between independent variables and dependent variables. For the i -th observation data, the simple linear regression model can be expressed as follows:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, \quad i = 1, 2, \dots, n \quad (14)$$

where Y_i : dependent variable (response), X_i : independent variable (regressor), β_0 : intercept, β_1 : regression coefficient, ε : error term.

The parameters β_0 and β_1 are estimated using the Ordinary Least Squares (OLS) method, so that the following equation is obtained:

$$Y_i = b_0 + b_1 X_i + e_i, \quad i = 1, 2, \dots, n \quad (15)$$

where b_0 and b_1 are estimators of the parameters β_0 and β_1 , and e_i denotes the residual.

2.10. Canadian Method

According to Kusumawati et al. (2024), the Canadian method is a premium reserve calculation method carried out by combining the modified initial premium of the Canadian method with the net premium through the difference between the net premium for a life insurance policy and the natural premium. The calculation of premium reserves using the Canadian method can be expressed by the following equation:

$$\beta^{(can)} = {}_m P_{x:\overline{n}|} + \frac{P_{x:\overline{n}|} - \frac{C_x}{D_x}}{\ddot{a}_{x:\overline{m}|} - 1} \quad (16)$$

$${}_t V_{x:\overline{n}|}^{(can)} = \begin{cases} A_{x+t:\overline{n-t}|} - \beta^{(can)} \ddot{a}_{x+t:\overline{m-t}|}, & t < m \\ A_{x+t:\overline{n-t}|}, & t \geq m \end{cases} \quad (17)$$

2.11. Gompertz's Law

According to Yang and Zhou (2003), mortality acceleration based on the Gompertz distribution is defined by the following equation:

$$\mu(x) = Bc^x \quad (18)$$

where $B > 0$, $c > 1$, and $x \geq 0$.

The probability that a person aged x years survives to age $x + t$ years under Gompertz's mortality law can be expressed as:

$${}_t p_x = \exp \left[-\frac{Bc^x}{\ln c} (c^t - 1) \right] \quad (19)$$

To complete the mortality table based on Gompertz's law, the following equations are obtained by setting $t = 1$.

$$l_x = l_0 \cdot \exp \left[-\frac{B}{\ln c} (c^x - 1) \right] \quad (20)$$

$$d_x = l_0 \cdot \exp \left[-\frac{B}{\ln c} (c^x - 1) \right] - l_0 \cdot \exp \left[-\frac{B}{\ln c} (c^{x+1} - 1) \right] \quad (21)$$

$$p_x = \exp \left[-\frac{Bc^x}{\ln c} (c - 1) \right] \quad (22)$$

$$q_x = 1 - \exp \left[-\frac{Bc^x}{\ln c} (c - 1) \right] \quad (23)$$

3. Materials and Methods

3.1. Materials

The object used in this study is the analysis of premium reserves of endowment life insurance using the Canadian method based on the 2019 TMI and Gompertz's law. This study uses simulation data with the assumptions that the policyholder is a 30 year old male, with a premium payment period of 20 years, a coverage period of 30 years, an interest rate of 5%, and a sum assured of IDR500,000,000. The mortality data used refers to 2019 TMI, which is then

constructed using Gompertz's law. The calculation and data analysis processes were performed numerically using Python and Microsoft Excel software.

3.2. Methods

The stages of data analysis in this study are as follows:

- 1) Constructing a mortality table based on Gompertz's law from the 2019 TMI using equations (20)-(23)
- 2) Calculating the temporary life annuity-due using equation (9).
- 3) Calculating the net single premium of endowment life insurance using equation (11).
- 4) Calculating the net annual premium of endowment life insurance using equation (13).
- 5) Calculating premium reserve values using the Canadian method based on the 2019 TMI and Gompertz's law using equation (17).
- 6) Comparing the results of premium reserve values from both mortality models.

4. Results and Discussion

The calculation begins with the construction of the mortality table using equations (20)-(23). Next, the value of the temporary life annuity-due is calculated using equation (9), then the net single premium and net annual premium are calculated using equations (11) and (13). Based on these results, the premium reserves of endowment life insurance are calculated using the Canadian method based on the 2019 TMI and Gompertz's law using equation (17). The results of the premium reserve calculations are presented in Table 1.

Table 1: Comparison of premium reserves using the Canadian method based on TMI 2019 and Gompertz's law

t	Canadian Reserves (2019 TMI) (IDR)	Canadian Reserves (Gompertz's Law) (IDR)	t	Canadian Reserves (2019 TMI) (IDR)	Canadian Reserves (Gompertz's Law) (IDR)
1	23,458,951.37	23,752,373.37	16	228,328,390.13	228,412,995.72
2	33,111,921.49	33,453,063.91	17	247,750,407.91	247,890,355.85
3	43,228,328.10	43,603,916.05	18	268,121,949.94	268,347,481.10
4	53,833,011.09	54,226,422.69	19	289,506,386.02	289,846,515.92
5	64,952,166.18	65,343,315.35	20	311,973,103.22	312,455,609.39
6	76,604,943.67	76,978,673.07	21	326,691,350.44	327,091,044.15
7	88,816,419.79	89,158,045.38	22	342,148,262.30	342,472,206.38
8	101,609,486.72	101,908,591.40	23	358,393,290.56	358,650,683.77
9	115,013,031.06	115,259,237.44	24	375,482,422.85	375,683,746.29
10	129,046,907.21	129,240,856.11	25	393,482,159.29	393,635,196.27
11	143,741,126.61	143,886,469.98	26	412,465,621.00	412,576,364.43
12	159,128,502.84	159,231,483.94	27	432,517,323.78	432,587,279.94
13	175,241,651.84	175,313,950.45	28	453,727,664.40	453,758,047.99
14	192,119,945.39	192,174,873.08	29	476,190,476.19	476,190,476.19
15	209,800,606.19	209,858,554.47	30	500,000,000.00	500,000,000.00

To clarify the results of the comparison of premium reserves using the Canadian method based on the 2019 TMI and Gompertz's law, a comparison graph of premium reserves for each year of payment is presented.

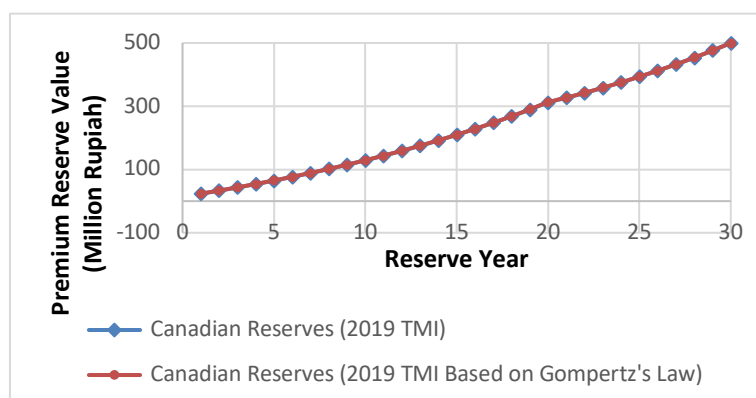


Figure 1: Comparison graph of premium reserves

5. Conclusion

The calculation results show that premium reserves calculated based on Gompertz's law are higher than premium reserves calculated based on the 2019 TMI. This difference indicates that the selection of a mortality model affects the magnitude of the resulting premium reserves. However, at the end of the coverage period, both models produce the same reserve value, which is equal to the sum assured.

Based on the research that has been conducted, future studies may be applied to other types of life insurance, such as term life insurance, whole life insurance, or joint life insurance, revise the mortality table used by considering other mortality models such as De Moivre, Makeham, or Weibull, and use primary data as the object of research.

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