Jurnal Elemen, 12(3), 672-688, July 2026
https:/doi.org/10.29408/jel.v12i3.34070

e- ISSN: 2442 - 4226

Check for updates

Students' geometric thinking and problem-solving in circle topic: Mixed-method research

Mirna Mirna [*], Hamdani Syaputra, Dewi Murni, Nur 'Aini, Ainil Mardhiah

Faculty of Mathematics and Natural Sciences, Universitas Negeri Padang, West Sumatra, Indonesia

[*] Correspondence: mirnabahar@fmipa.unp.ac.id

Abstract

Despite the central role of geometry in the senior high school curriculum, students continue to struggle with abstract concepts in circle topics, often relying on rote memorization rather than conceptual understanding. This study employed a mixed-methods sequential explanatory design involving 69 senior high school students selected through purposive sampling. Data were collected using the Van Hiele geometry test to measure geometric thinking skills and a circle problem-solving test based on Polya's stages. Quantitative data were analyzed using Pearson product–moment correlation, while qualitative data were examined through document analysis of students' written responses. The results revealed a statistically significant but weak positive correlation between geometric thinking skills and mathematical problem-solving abilities ($r = 0.277$, $p = 0.21$), with geometric thinking contributing only 7.7% to the variance ($R^2 = 0.077$). Qualitative findings showed notable discrepancies: some students with low geometric thinking achieved high problem-solving scores through procedural approaches, while others with high geometric thinking performed poorly due to difficulties in execution and representation. These findings imply that improving geometric thinking alone is insufficient to enhance problem-solving performance. Mathematics instruction should move beyond rote memorization by integrating visualization activities and explicit heuristic training to better connect conceptual understanding with effective problem-solving strategies.

Keywords: circle geometry; geometric thinking; mathematical problem-solving; mixed-methods; Polya's stages; senior high school; Van Hiele levels

How to cite: Mirna, M., Syaputra, H., Murni, D., 'Aini, N., & Mardhiah, A. (2026). Students' geometric thinking and problem-solving in circle topic: Mixed-method research. *Jurnal Elemen, 12*(3), 672-688. https://doi.org/10.29408/jel.v12i3.34070

Received: 3 February 2026 | Revised: 3 April 2026
Accepted: 13 May 2026 | Published: 13 July 2026

Introduction

In the landscape of 21st-century education, mathematical problem-solving is no longer viewed as a peripheral skill but as the core of mathematical literacy. International frameworks such as the Programme for International Student Assessment (PISA) emphasize students' ability to apply mathematical knowledge in unfamiliar, real-world contexts, particularly within the "Space and Shape" domain. However, the PISA 2022 results reveal that Indonesian students continue to perform at a low level in this domain, indicating persistent difficulties in geometric reasoning and modeling (OECD, 2023; Wijaya et al., 2024). At the national level, the Indonesian curriculum (*Kurikulum Merdeka*) explicitly mandates the development of Higher Order Thinking Skills (HOTS), yet classroom practices often remain dominated by procedural instruction and rote memorization (Ambarita et al., 2025; Saputra et al., 2025). This discrepancy highlights a critical issue between curricular expectations and actual student competencies in geometry learning.

Among geometry topics, circles present a particularly complex cognitive challenge. Circle geometry requires students to integrate multiple abstract relationships, such as tangency, secants, cyclic quadrilaterals, and angle theorems, which demand not only procedural knowledge but also structural and relational understanding (Mifetu, 2023). Recent studies indicate that students often resort to procedural memorization of formulas (e.g., $A = \pi r^2$) but fail to grasp the structural relationships required to solve complex proofs or modeling tasks (Hermiati et al., 2021; Sulistiowati et al., 2019). This failure is often not computational but conceptual, stemming from a lack of visualization skills (Tiwari et al., 2021).

To explain this conceptual limitation, the Van Hiele Theory of Geometric Thinking provides a robust theoretical framework. According to Van Hiele (1986), students' progress through hierarchical levels of geometric understanding: Visualization (L0), Analysis (L1), Abstraction (L2), Deduction (L3), and Rigor (L4). Recent studies indicate that many secondary students remain at lower levels (L0–L1), which restricts their ability to reason about relationships and solve complex geometric problems (Celik & Yilmaz, 2022; Naufal et al., 2024). The majority of Indonesian senior high school students in circle topics remain at Level 1 (Analysis) (Mirna et al., 2020; Pujawan et al., 2020). More recent perspectives, such as the commognitive approach, further emphasize that students' geometric reasoning is shaped by their discourse and ability to construct mathematical meaning, reinforcing the importance of higher-level thinking in geometry (Mahlaba & Mudaly, 2022; Sfard, 2007; Sihlangu et al., 2025).

The relationship between geometric thinking and mathematical problem-solving can be more precisely understood by mapping Van Hiele levels onto Polya's four problem-solving stages: understanding the problem, devising a plan, carrying out the plan, and looking back (Polya, 1973; Van Hiele, 1986). At the Visualization level (L0), students are able to recognize geometric shapes but struggle to interpret problem conditions meaningfully, limiting their effectiveness in the understanding stage. At the Analysis level (L1), students can identify properties of geometric figures, which supports basic comprehension but remains insufficient for strategic planning. The transition to Abstraction (L2) marks a critical threshold, as students begin to understand relationships between properties; this level is particularly essential in the devising a plan stage, where solving circle problems often requires constructing auxiliary elements (e.g., radii to points of tangency, chords, or central angles). At higher levels, Deduction (L3) enables students to logically justify procedures during the carrying out the plan stage, while Rigor (L4) supports reflection, generalization, and verification in the looking back stage. This mapping indicates that students' success in each phase of problem-solving is cognitively constrained by their level of geometric thinking. Consequently, students at lower Van Hiele levels may successfully perform routine procedures yet fail to generate appropriate strategies or evaluate their solutions in non-routine contexts.

Despite this theoretical linkage, previous research has examined the relationship between Van Hiele levels and mathematical problem-solving; however, these studies have predominantly relied on quantitative approaches and treated the relationship as linear and assumed homogeneity across learners (Abidin & Retnawati, 2019; Naufal et al., 2024), thereby providing limited insight into the underlying cognitive processes involved in problem solving. Recent theoretical perspectives suggest that geometric thinking and problem-solving are mediated by students' discourse and strategy use, which cannot be fully captured through correlational analysis alone; for example, Mahlaba & Mudaly (2022) demonstrate that students' progression across Van Hiele levels is closely linked to the nature of their mathematical discourse, highlighting the importance of qualitative analysis in understanding how students engage with geometric tasks. However, despite this recognition, no prior studies have employed a mixed-methods approach to systematically investigate discrepant cases—students who deviate from the expected correlation, such as those with low geometric thinking but high problem-solving performance, or vice versa—particularly within the theorem-rich context of circle geometry. This gap is critical, as such cases may reveal alternative cognitive pathways, including procedural recall or algebraic compensation, that remain obscured in purely quantitative models; therefore, this study addresses this gap by integrating quantitative correlation analysis with qualitative document analysis to examine not only the relationship between geometric thinking and problem-solving but also the inconsistencies within that relationship, specifically how students at different Van Hiele levels navigate Polya's problem-solving stages.

Grounded in Van Hiele theory and Polya's problem-solving framework, this study provides a more nuanced understanding of how students at different levels of geometric thinking engage with circle problem-solving tasks, particularly in discrepant cases. Accordingly, the purpose of this study is to analyze the relationship between students' geometric thinking skills and their mathematical problem-solving abilities, and to qualitatively investigate how students with different Van Hiele levels navigate Polya's problem-solving stages, particularly in cases that deviate from expected patterns.

Methods

Research design

This study employed a mixed-methods sequential explanatory design (Palinkas et al., 2019), which consists of two phases: a quantitative phase followed by a qualitative phase. In the quantitative phase, the Van Hiele Geometry Test (VHGT) and the Circle Problem-Solving Test (CPST) were administered to 69 students to examine the relationship between geometric thinking skills and mathematical problem-solving abilities. The data were analyzed using correlation and regression techniques to identify the strength and contribution of the relationship between variables. Based on the quantitative results, purposive sampling was used to select students for the qualitative phase. The qualitative phase involved an in-depth document analysis of students' written responses, focusing on how students at different Van Hiele levels navigated Polya's problem-solving stages and identifying discrepancies between geometric thinking and problem-solving performance. The overall research design is illustrated in Figure 1, which presents the sequential flow from quantitative analysis to qualitative exploration.

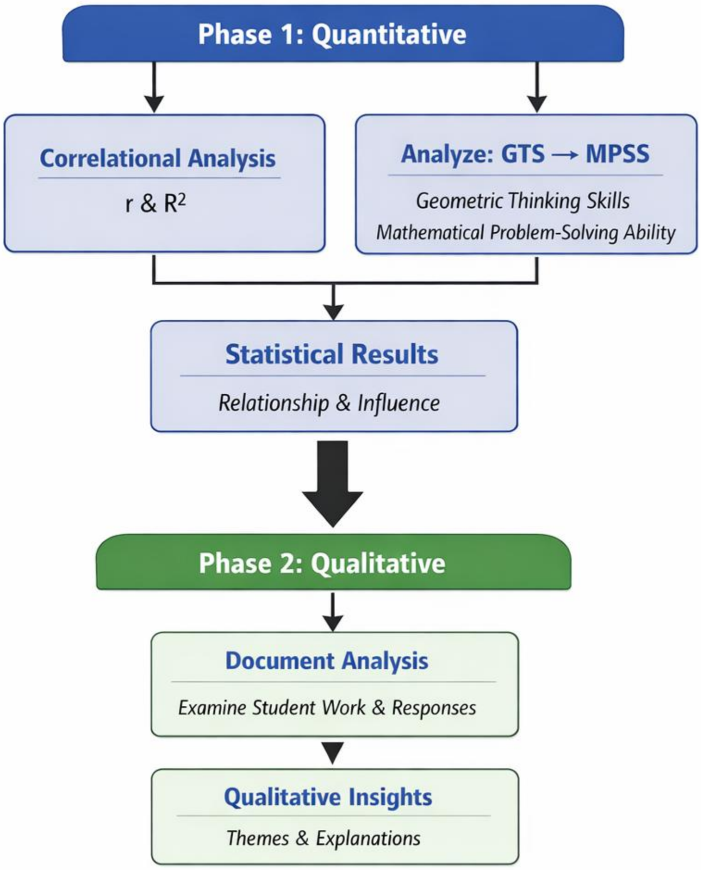


Figure 1. Research design of mixed methods sequential explanatory

Population and sample

The population of this study consisted of all Grade 11 science (*IPA*) students in senior high schools in Padang, Indonesia. In the quantitative phase, a sample of 69 students from *Sekolah Menengah Atas Negeri 7 Padang* was selected using purposive sampling, based on specific criteria, namely the similarity of students' prior mathematics achievement and the implementation of the same curriculum. This sample size was considered adequate for correlation analysis, as previous methodological guidelines suggest that samples exceeding 30 participants are sufficient for detecting medium effect sizes using Pearson correlation (Lachenbruch & Cohen, 1989).

In the qualitative phase, a subset of students was selected through purposive sampling based on their Van Hiele levels of geometric thinking. The selection represented different levels (e.g., low, medium, and high) to enable an in-depth analysis of students' problem-solving processes in solving circle-related problems.

Instrumentation

Two instruments were used in this study. The Van Hiele Geometry Test (VHGT) consisted of 25 multiple-choice items adapted from the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project (Mirna et al., 2026; Usiskin, 1982). In addition, the Circle Problem-Solving Test (CPST) comprised three essay questions addressing the circumference of a circle, the length of a circular arc, and the area of a circular sector. Students' responses on the CPST were assessed using a scoring rubric based on Polya's four problem-solving steps, with scores ranging from 0 to 4. CPST has been reviewed by two mathematics education experts (Prof. Dr. Armiati, M.Pd. – Professor in Mathematics Education assessment and Prof. Dr. Yerizon, M.Si. – Professor in Mathematics Education) and tested to 31 different students to sample with similar sampling criteria (similarity of students' prior mathematics achievement and the implementation of the same curriculum). Therefore, the validity of the instrument was confirmed through corrected item–total correlation, with items considered valid if the correlation coefficient exceeded the critical value ($r > 0.355$, $p < 0.05$). The analysis showed that all items met this criterion and were thus empirically valid. Reliability was assessed using Cronbach's Alpha, with a coefficient of 0.842, which exceeds the acceptable threshold of 0.70, indicating high internal consistency. The detailed description of the CPST is presented in Table 1.

Table 1. The detail of CPST

Topic	Description	Indicators			
		1	2	3	4
The circumference of a circle	Students are given a word problem involving a circular tablecloth that extends beyond the edge of a table and is decorated with ribbon around its perimeter. Students are expected to calculate the cost of purchasing the ribbon based on the price per meter and to determine an appropriate type of ribbon that fits within the available budget.	✓	✓	✓	✓

Topic	Description	Indicators			
		1	2	3	4
The length of a circular arc	Students are given a word problem related to a circular fountain. Two statues are placed along the edge of the fountain, forming an arc with a specific central angle. Students are expected to determine the length of the fountain's edge corresponding to the arc and calculate the amount of paint required (in liters).	✓	✓	✓	✓
The area of a circular sector	Students are given a word problem involving a circular pizza that is divided among several children with different central angles. Students are expected to determine the area of each pizza slice based on its central angle and analyze the number of calories contained in each slice.	✓	✓	✓	✓

Indicators:

1. Understand the Problem
2. Devise a Plan
3. Carry Out the Plan
4. Look Back

Data analysis

Quantitative data were analyzed using SPSS, including tests of normality (Kolmogorov–Smirnov and Shapiro–Wilk), linearity, and hypothesis testing using Pearson's product–moment correlation and simple linear regression. Although the normality test indicated deviations from normal distribution, parametric analysis was still employed based on the Central Limit Theorem, which states that for sample sizes greater than 30 (N = 69), the sampling distribution of the mean approximates normality. Therefore, Pearson correlation and regression analyses were considered appropriate. Next, qualitative data were analyzed using the interactive model proposed by Miles et al. (2016), which consists of data condensation, data display, and conclusion drawing.

Results

The quantitative data for this study were obtained from 69 Grade 11 Science students (N=69). The variables analyzed were Students' Geometric Thinking Skills (independent variable) and Mathematical Problem-Solving Skills (dependent variable). Descriptive statistics of students' Geometric Thinking Skills (GTS) and Mathematical Problem-Solving Skills (MPSS) are presented in Table 2.

Table 2. The students' geometric thinking skills (GTS) and mathematical problem-solving skills (MPSS)

	Means	Variance	Std. Deviation	Min.	Max.	Range	Skw.	Kurt.
GTS	12.19	29.56	5.43	4	23	19	0.73	-0.48
MPSS	67.1	321.2	17.92	26.67	93.33	66.66	-0.24	-0.97

Table 2 indicates that students' Geometric Thinking Skills (GTS) are at a moderate level, as reflected by the mean score of 12.19. The relatively small standard deviation (5.43) suggests that students' scores are moderately clustered around the mean. The positive skewness value (0.73) indicates a slight concentration of lower scores, suggesting that a larger proportion of students still experience difficulties in geometric reasoning. The slightly negative kurtosis value (−0.48) indicates a flatter-than-normal distribution, reflecting a spread of scores across different levels of geometric thinking.

In contrast, students' Mathematical Problem-Solving Skills (MPSS) exhibit a broader distribution, with a mean of 67.10 and a standard deviation of 17.92, indicating substantial variability in performance. The negative skewness value (−0.24) suggests a slight concentration of higher scores, meaning that more students performed above the mean. The negative kurtosis value (−0.97) further indicates a relatively flat distribution. This contrast between GTS and MPSS distributions provides an early indication that strong problem-solving performance may not always be accompanied by high geometric thinking skills.

Before hypothesis testing, normality and linearity tests were conducted. The Shapiro–Wilk test indicated that the data distribution for both variables deviated slightly from normality ($p < 0.05$). However, given the sample size ($N = 69$), which exceeds the threshold ($N > 30$) for the Central Limit Theorem, it is assumed that the sampling distribution approximates normality, thus permitting parametric analysis. The linearity test yielded a deviation from linearity value of $p > 0.05$, confirming that the relationship between the variables is linear.

To determine the relationship between geometric thinking skills and mathematical problem-solving ability, a Pearson Product-Moment Correlation analysis was conducted. The results are summarized in Table 3.

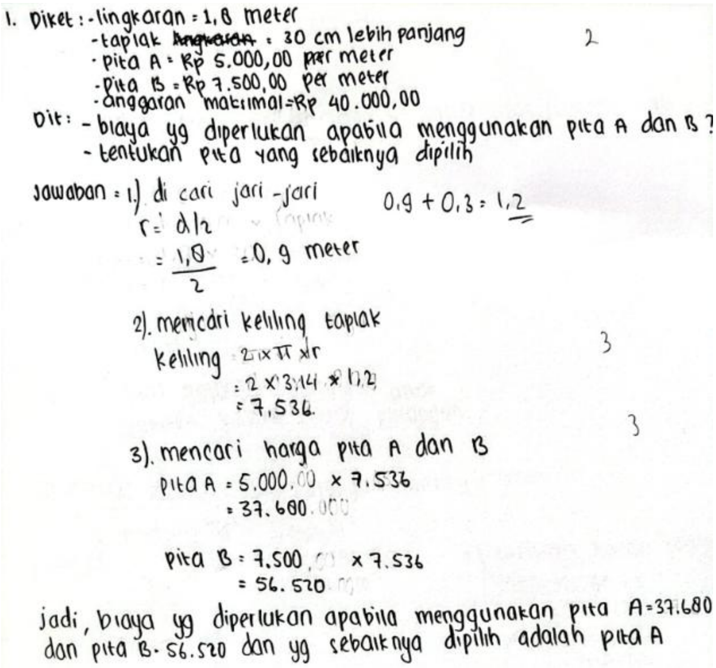
Table 3. The result of Pearson product-moment correlation analysis

Statistics	Value
Pearson correlation	0.277
R^2	0.077
Sig. (2-tailed)	0.021

The analysis reveals a positive correlation coefficient (r) of 0.277. The significance value (Sig.) is 0.021, which is less than the significance level $(\alpha = 0.05)$. Therefore, the null hypothesis (H_0) is rejected, and the alternative hypothesis (H_a) is accepted. This indicates a statistically significant, albeit weak, positive relationship between students' geometric thinking skills and their mathematical problem-solving ability. Furthermore, this finding suggests that the relationship between geometric thinking and problem-solving is not only weak but also potentially non-uniform across students. Based on Cohen's criteria, this correlation falls within the weak to moderate range, indicating a limited practical effect despite statistical significance.

Furthermore, the Coefficient of Determination (R^2) is 0.077. This implies that geometric thinking skills contribute only 7.7% to the variance in students' problem-solving abilities, while the remaining 92.3% is influenced by other factors not measured in this quantitative model. These findings highlight that, although geometric thinking is a statistically significant predictor, its practical contribution is relatively limited. The vast majority (over 92%) of the variance in problem-solving performance is attributable to other variables, such as algebraic fluency, reading comprehension, or strategic knowledge. This relatively low contribution further justifies the need for qualitative analysis to explore how students with different geometric thinking levels approach problem-solving tasks and to identify potential discrepancies that are not captured by quantitative measures alone.

To further explain this weak relationship and the low explanatory power ($R^2 = 0.077$), qualitative analysis was conducted to examine individual student responses, particularly those exhibiting discrepant patterns. One illustrative case is student HH, who demonstrated very low geometric thinking skills (score: 6) yet achieved a very high mathematical problem-solving score (90.00). Figure 2 presents HH's response to question number 1 of the mathematical problem-solving test, illustrating this discrepant (low-high) case.



1. Diket : -lingkaran = 1,8 meter
-taplak ~~lingkaran~~ = 30 cm lebih panjang
-pita A = Rp 5.000,00 per meter
-pita B = Rp 7.500,00 per meter
-anggaran maksimal = Rp 40.000,00
Dit : - biaya yg diperlukan apabila menggunakan pita A dan B ?
- tentukan pita yang sebaiknya dipilih
Jawaban : 1.) di cari jari-jari 0,9 + 0,3 = 1,2
r = d/2
= 1,8/2 = 0,9 meter
2). mencari keliling taplak
keliling 2×π×r
= 2×3,14×1,2
= 7,536
3). mencari harga pita A dan B
pita A = 5.000,00 × 7,536
= 37.680,00
pita B = 7.500,00 × 7,536
= 56.520,00
jadi, biaya yg diperlukan apabila menggunakan pita A = 37.680 dan pita B = 56.520 dan yg sebaiknya dipilih adalah pita A

Translation

1. Given: - circle = 1,8 meters
- tablecloth = 30 cm longer
- Ribbon A = Rp 5.000,00 per meter
- Ribbon B = Rp 7.500,00 per meter
- maximum budget = Rp 40.000,00

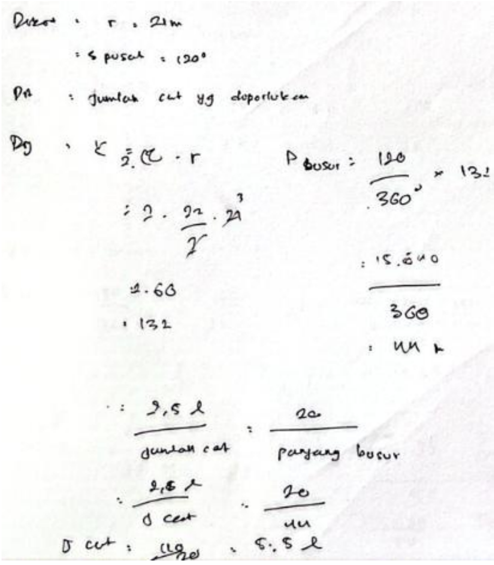
Questions:
- the cost needed if using ribbon A and B?
- determine which ribbon should be chosen

Answer:
1.) finding the radius 0,9+0,3=<u>1,2</u>
$r = \frac{d}{2} = \frac{1,8}{2} = 0,9$ meter
2.) finding the circumference of the tablecloth
Circumference $2 \times \pi \times r$
$= 2 \times 3,14 \times 1,2$
$= 7,536$
3.) finding the price of ribbon A and B
Ribbon A : $5.000 \times 7,536$
$= 37.680$
Ribbon B: $7.500 \times 7,536$
$=56.520$
So, the cost needed if using ribbon A is 37.680 and ribbon B is 56.520 and the ribbon that should be chosen is ribbon A.

Figure 2. HH's answer for problems number 1

Based on the student's response in Figure 2, it is evident that the student understood the problem. This is demonstrated by their ability to explicitly write down the knowns and the unknowns of the question. Regarding the indicator of devising a plan, the student successfully determined the appropriate formula for the given problem. Furthermore, the solution steps were executed precisely, allowing the student to solve the problem accurately. For the indicator of looking back and drawing conclusions, the student correctly wrote the final conclusion and answer. Consequently, the student achieved the maximum score across every mathematical problem-solving indicator for question number 1. A similar performance was observed for questions 2 and 3. This pattern suggests that the student's success is likely driven more by procedural fluency or memorized strategies than by conceptual geometric reasoning.

In contrast, another student, identified as NGS, possessed high geometric thinking skills (score: 23) but demonstrated low mathematical problem-solving skills (score: 46.67), identified as NGS. Figure 3 presents NGS's response to question number 2 of the mathematical problem-solving test, serving as an example of this discrepant (high-low) category.



Translation

Given: $r = 21\ m$

central angle $= 120°$

Question: Amount of paint needed

Answer: $k = 2 \cdot \pi \cdot r$

$= 2 \cdot \frac{22}{\cancel{7}} \cdot \cancel{21}^{3}$

$= 132$

$p_{arc} = \frac{120}{360°} \times 132$

$= \frac{15.840}{360}$

$= 44\ m$

$$\frac{2{,}5\ l}{amount\ of\ paint} = \frac{20}{arch\ length}$$

$$\frac{2{,}5\ l}{j\ paint} = \frac{20}{44}$$

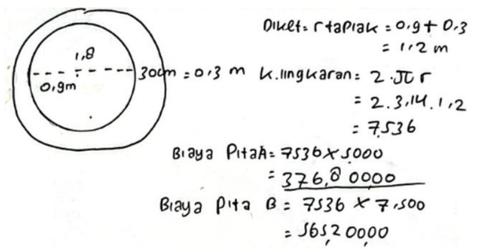
$$j\ paint = \frac{110}{20} = 5{,}5\ l$$

Figure 3. NGS's answer for problems number 2

Based on the student's response in Figure 3, it is evident that the student attempted to identify the knowns and unknowns, yet the information provided was incomplete. Regarding the indicator of devising a plan, the student successfully determined the appropriate formula for the given problem. Furthermore, the execution of the problem-solving steps was accurate, leading to a correct calculation. However, the student failed to write down the final conclusion and the specific answer required by the question.

Similar deficiencies were observed in questions 1 and 3. In both instances, the student listed the knowns and unknowns, but the responses remained incomplete. Regarding the planning indicator: for question 1, the student attempted to draft a plan but included errors or omitted the formula; meanwhile, for question 3, the devised plan was entirely incorrect. This error significantly affected the execution phase for question 3, where the student's solution was wrong, unlike in questions 1 and 2 where the calculations were performed correctly. Finally, consistent with the error in question 2, the student failed to provide a conclusion or final answer for both questions 1 and 3. This indicates that high geometric thinking skills does not guarantee successful problem-solving, particularly when students fail to engage in reflective and metacognitive processes such as verification and conclusion drawing.

While discrepant cases exist, there are also students who demonstrate consistent abilities. One such student is MHO, who exhibited low geometric thinking skills (score: 7) and low mathematical problem-solving skills (score: 33.33). Figure 4 presents MHO's response to question number 1 of the mathematical problem-solving test, illustrating this consistent (low-low) category.



Translation

Given: r tablecloth $= 0,9 + 0,3 = 1,2\ m$

$k.circle = 2 \cdot \pi \cdot r$

$= 2 \cdot 3,14 \cdot 1,2$

$= 7,536$

Cost of Ribbon $A = 7,536 \times 5000 = \underline{376.800.000}$

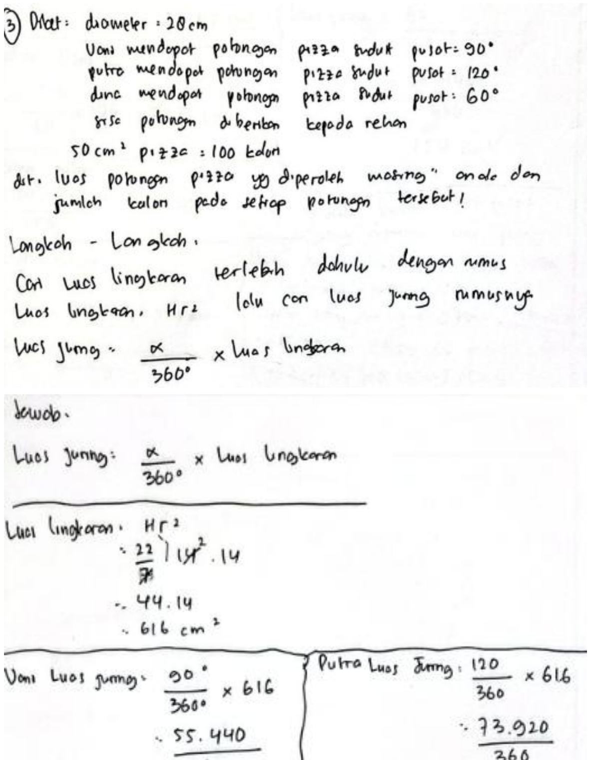
Cost of Ribbon $B = 7,536 \times 7500 = 565.200.00$

Figure 4. MHO's answer for problems number 1

Based on the student's response in Figure 4, it is evident that the student wrote down the knowns, yet they were incomplete, and omitted the unknowns (what is asked). Regarding the indicator of devising a plan, the student correctly noted the formula for the circumference of a circle; however, the subsequent problem-solving steps were incorrect, resulting in a failure to solve the problem. Additionally, the student did not provide a conclusion or final answer.

A different pattern was observed in questions 2 and 3. In question 2, regarding the understanding indicator, the student failed to write down both the knowns and unknowns. However, for the planning indicator, the student successfully identified the correct formula. The execution of the steps was also accurate, allowing the student to solve the problem correctly, though they still failed to write a conclusion or final answer. In contrast, for question 3, the student provided no response at all.

Finally, there are also students who demonstrate consistent high abilities in both domains. One such student is KZZ, who achieved a high Van Hiele score and high mathematical problem-solving skills. Figure 5 presents KZZ's response to question number 3 of the mathematical problem-solving test, illustrating this consistent (high-high) category.



Translation

Given: diameter = 28 cm

Vani gets a slice of pizza with a central angle of 90°

Putra gets a slice of pizza with a central angle of 120°

Dina gets a slice of pizza with a central angle of 60°

The remaining pizza slice is given to Rehan

$50\ cm^2$ pizza = 100 calories

Question:

the area of the pizza slice obtained by each child and the number of calories in each slice!

Steps:

Find the area of the circle first with the formula for the area of a circle, then use the sector area formula.

$$\text{Sector area} = \frac{\alpha}{360°} \times \text{Area of circle}$$

Answer:

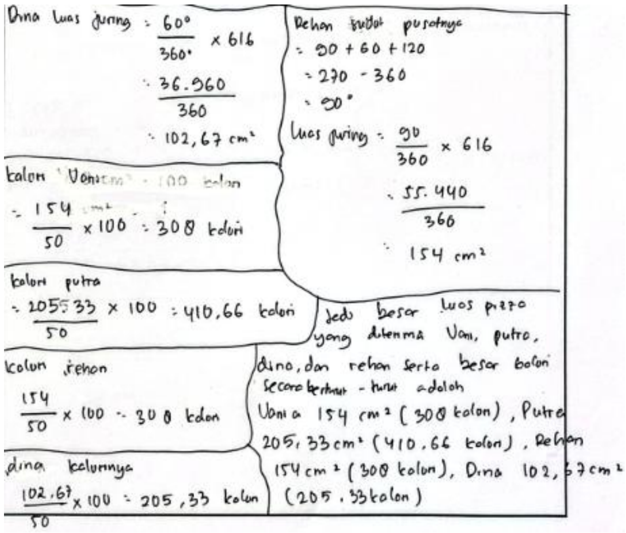
$$\text{Sector area} = \frac{\alpha}{360°} \times \text{Area of circle}$$

Area of circle $= \pi r^2 = \frac{22}{\cancel{7}} \cdot \cancel{14}^2 \cdot 14 = 44 \cdot 14 = 616\ cm^2$

Vani sector area $= \frac{90°}{360°} \times 616 = \frac{55.440}{360} = 154\ cm^2$

Putra sector area $= \frac{120}{360} \times 616 = \frac{73.920}{360} = 205,33\ cm^2$

Figure 5. KZZ's answer for problems number 3



Vani sector area $= \frac{60°}{360°} \times 616 = \frac{36.960}{360} = 102,67\ cm^2$
Rehan's central angle $= 90 + 60 + 120 = 270 - 360 = 90°$
Sector area $= \frac{90°}{360°} \times 616 = \frac{55.440}{360} = 154\ cm^2$
Vani's calories $= \frac{154}{50} \times 100 = 308\ calories$
Putra's calories $= \frac{205,33}{50} \times 100 = 410,66\ calories$
Rehan's calories $= \frac{154}{50} \times 100 = 308\ calories$
Dina's calories $= \frac{102,67}{50} \times 100 = 205,33\ calories$
So the area of the pizza received by Vani, putra, dina, and rehan as well as the amount of calories respectively are Vani 154 cm² (308 calories), Putra 205,33 cm² (410,66 calories), Rehan 154 cm² (308 calories), Dina 102,67 cm² (205,33 calories)

Figure 5. KZZ's answer for problems number 3 (continued)

Based on the student's response in Figure 5, it is evident that the student fully understood the problem. This is demonstrated by their ability to explicitly write down the knowns and the unknowns of the question. Regarding the indicator of devising a plan, the student successfully determined the appropriate formula for the given problem. Furthermore, the solution steps were executed precisely, allowing the student to solve the problem accurately. For the indicator of looking back and drawing conclusions, the student correctly wrote the final conclusion and answer. Consequently, the student achieved the maximum score across every mathematical problem-solving indicator for question number 3. A similar flawless performance was observed for questions 1 and 2.

To provide a systematic comparison, the performance of selected students across Polya's four problem-solving stages for each question is summarized in Table 4, illustrating both consistent (high–high, low–low) and discrepant (low–high, high–low) patterns between geometric thinking level and mathematical problem-solving performance.

Table 4. The students' mathematical problem-solving performance

	Geometric Thinking Level	Mathematical Problem-Solving Score												Total	Category
		Question 1				Question 2				Question 3					
		1	2	3	4	1	2	3	4	1	2	3	4		
HH	9 (Low)	2	3	3	2	2	3	2	1	1	3	3	2	27 (High)	Discrepant (Low-High)
NGS	23 (High)	1	2	2	1	1	2	2	0	1	1	1	0	14 (Low)	Discrepant (High-Low)
MHO	7 (Low)	1	1	1	0	1	2	2	0	1	1	0	0	10 (Low)	Consistent (Low-Low)
KZZ	23 (High)	1	3	3	2	1	3	3	2	2	3	3	2	28 (High)	Consistent (High-High)

As shown in Table 4, HH demonstrates consistently high performance across all problem-solving stages despite having low geometric thinking skills, indicating a discrepant (low–high) pattern. In contrast, NGS, who possesses high geometric thinking skills, shows inconsistent performance, particularly in the “looking back” stage, resulting in a discrepant (high–low) pattern. Meanwhile, MHO and KZZ exhibit consistent patterns, with low–low and high–high performance respectively across both domains. These patterns further explain the weak correlation (r = 0.277), demonstrating that the relationship between geometric thinking and problem-solving is not uniform across students but varies depending on how they engage with different stages of the problem-solving process.

Discussion

From a theoretical standpoint, this result aligns with Van Hiele’s theory of geometric thinking, which posits that students’ geometric reasoning develops hierarchically and constrains how they interpret and solve problems. Students operating at lower Van Hiele levels tend to rely on visual recognition or isolated properties, thereby limiting their ability to engage in the relational reasoning required in complex geometric contexts. Recent studies further confirm that students’ Van Hiele levels significantly influence their ability to analyze geometric relationships and apply concepts in non-routine problems (Andira et al., 2022; Pebrianti & Suhendra, 2023; Yi et al., 2020). In the context of circle geometry, this limitation becomes more pronounced when students are required to integrate multiple properties, such as angle relationships, tangency conditions, and cyclic structures, all of which demand higher-order geometric reasoning.

However, the relatively small effect size observed in this study suggests that the relationship between geometric thinking and problem-solving is neither linear nor deterministic. This finding can be better understood through the lens of Polya’s problem-solving framework, which conceptualizes problem solving as a multidimensional process involving understanding, planning, execution, and reflection. While geometric thinking primarily supports the initial stages—particularly problem comprehension and strategy formulation—it does not guarantee successful execution or reflective evaluation. Recent studies on mathematical cognition reinforce this view, indicating that domain-specific knowledge alone is insufficient unless accompanied by strategic competence and metacognitive regulation (Ishak et al., 2025; Krawitz et al., 2025; Schoenfeld, 2016). Thus, geometric thinking should be understood as one component within a broader constellation of interdependent problem-solving competencies.

A particularly important insight emerging from this study is the presence of discrepant cases, where students with low geometric thinking demonstrate high problem-solving performance, and vice versa. These findings point to the existence of alternative cognitive pathways in mathematical problem solving. For students in the low–high category, success appears to be driven by procedural fluency and algorithmic recall, enabling them to solve routine or well-structured problems without deep conceptual understanding. This suggests that, for standard tasks—such as calculating circumference, arc length, or sector area—procedural knowledge can partially compensate for limited geometric reasoning. However, such compensation is unlikely to sustain performance in more complex problems requiring relational understanding, such as those involving tangents, cyclic quadrilaterals, or geometric proofs, where the construction of auxiliary elements and deductive reasoning is indispensable.

Conversely, students in the high–low category reveal a different limitation: possessing advanced geometric thinking does not necessarily translate into effective problem-solving performance. These students tend to struggle in the later stages of Polya's framework, particularly in executing solution strategies and reflecting on their results. This pattern indicates the presence of metacognitive failure or strategic deficits, where students are unable to regulate their cognitive processes, verify the correctness of their solutions, or communicate conclusions coherently. Prior research supports this interpretation, emphasizing that successful problem solving depends not only on conceptual understanding but also on self-regulation, representation skills, and strategic decision-making (Tambychik & Meerah, 2010).

The findings of this study are also closely associated with the persistence of rote learning practices, which remain prevalent in many mathematics classrooms, particularly in the Indonesian context. Existing literature indicates that instructional practices often prioritize procedural repetition and formula memorization over conceptual understanding and reasoning (Klau et al., 2020; Siregar et al., 2025; Verina & Juandi, 2022). This instructional orientation may explain why some students achieve high problem-solving scores despite low geometric thinking levels, as they rely on memorized procedures to address familiar problem types. However, such reliance may hinder students' ability to engage with non-routine problems, thereby weakening the observed relationship between geometric thinking and problem-solving performance. In this regard, the weak correlation identified in this study should not be interpreted as evidence of a negligible relationship, but rather as an artifact of instructional practices that privilege procedural fluency over conceptual reasoning.

From an instructional perspective, these findings suggest that improving geometric thinking alone is insufficient to substantially enhance students' problem-solving performance. Effective instruction should adopt an integrated approach that combines Van Hiele–based geometry learning with explicit problem-solving scaffolds aligned with Polya's stages. This includes systematically encouraging students to articulate their reasoning, justify the selection of solution strategies, and engage in reflective evaluation of their answers. In addition, the use of dynamic visualization tools, such as GeoGebra, can play a critical role in bridging the gap between conceptual understanding and procedural application, particularly in complex circle problems that require the construction and manipulation of auxiliary elements.

Despite its contributions, this study has several limitations that should be acknowledged. The sample was drawn from a single senior high school, which may limit the generalizability of the findings across diverse educational contexts. Furthermore, the study focused exclusively on circle geometry; therefore, the observed relationship may not extend to other domains, such as three-dimensional or transformation geometry. Finally, as a correlational study, it does not establish causal relationships between variables. Future research should employ experimental designs to examine the effects of Van Hiele phase-based instruction integrated with metacognitive scaffolding, as well as longitudinal approaches to track the development of geometric thinking and problem-solving performance over time.

Conclusion

There is a statistically significant but weak positive relationship ($r = 0.277$) between students' geometric thinking skills and their mathematical problem-solving abilities in the topic of circles, with geometric thinking accounting for only 7.7% of the variance. This finding indicates that while geometric thinking constitutes a necessary cognitive foundation, it is not sufficient to ensure successful problem-solving performance. The qualitative findings further reveal the presence of discrepant cases, demonstrating that students with low geometric thinking can achieve high problem-solving scores through procedural fluency and algorithmic recall, particularly in routine and well-structured tasks, whereas students with high geometric thinking may underperform due to weaknesses in execution, strategic regulation, or reflective evaluation. These results reinforce the argument that mathematical problem solving is inherently multidimensional, involving not only conceptual understanding but also strategic competence, metacognitive regulation, and representational skills.

From a pedagogical perspective, this implies that mathematics instruction should move beyond procedural drilling toward an integrated approach that simultaneously develops conceptual visualization and problem-solving processes. In practical terms, teachers can implement dynamic visualization activities using tools such as GeoGebra, for example by allowing students to manipulate inscribed and central angles to observe invariant relationships, construct tangents and explore radius–tangent perpendicularity, or dynamically partition sectors to connect angle measures with area formulas. In parallel, instruction should explicitly scaffold Polya's problem-solving stages, particularly by modeling how to devise solution plans, justify the selection of formulas, and systematically verify results through reflective questioning.

Despite these contributions, the findings are limited by the relatively small and context-specific sample drawn from a single school, as well as the focus on circle geometry, which may not generalize to other domains such as three-dimensional or transformation geometry. Furthermore, the correlational design does not permit causal inference regarding the impact of geometric thinking on problem-solving performance.

Future research should employ experimental designs to examine the effects of Van Hiele phase-based instruction combined with metacognitive scaffolding, conduct in-depth qualitative investigations of students exhibiting discrepant profiles (particularly low Van Hiele–high problem-solving) to uncover compensatory strategies such as pattern matching, and implement longitudinal studies to track the development of geometric thinking and problem-solving abilities over time in order to better establish causal relationships.

Acknowledgment

Thanks to *Sekolah Menengah Atas Negeri 7 Padang*, Indonesia and Department of Education and Culture in Padang that give permission to this study with permission letter number 000.9.2/4747/PSMASLB/DISDIK-2025.

Declarations

Conflicts of Interest	:	The authors declare no conflict of interest.
Generative AI Statement	:	Generative AI tools, Grammarly were employed solely for language editing and minor phrasing enhancements. All conceptualization, analysis, and scholarly content were independently developed and verified by the authors.
Funding Statement	:	This work received no specific grant from any public, commercial, or not-for-profit funding agency.
Author Contributions	:	**Mirna Mirna:** Conceptualization, writing - original draft, editing, and visualization; **Hamdani Syaputra:** Writing - review & editing, formal analysis, and methodology; **Dewi Murni:** Validation; **Nur 'Aini:** Resources; **Ainil Mardhiah:** Formal analysis.

References

Abidin, M., & Retnawati, H. (2019). A diagnosis of difficulties in answering questions of circle material on junior high school students. *Jurnal Penelitian dan Evaluasi Pendidikan*, *23*(2), 144–155. https://doi.org/10.21831/pep.v23i2.16454

Ambarita, J., Purnamasari, U., & Siahaya, A. (2025). Deep learning as a pathway to pedagogical transformation in Indonesia. *Jurnal Penelitian Kebijakan Pendidikan*, *18*(1). https://doi.org/10.24832/jpkp.v18i1.1229

Andira, A., Darwis M, M., & Syam, H. (2022). Exploring of students' ability to solve geometry problems based on Van Hiele's level of thinking. *ETDC: Indonesian Journal of Research and Educational Review*, *1*(2). https://doi.org/10.51574/ijrer.v1i2.181

Celik, H. S., & Yilmaz, G. K. (2022). Analysis of Van Hiele geometric thinking levels studies in Turkey: A meta-synthesis study. *International Journal of Curriculum and Instruction*, *14*(1). https://ijci.net/index.php/IJCI/article/view/845

Hermiati, K., Suprihatiningsih, S., & Annurwanda, P. (2021). Visual thinking ability of mathematics education students on geometry transformation learning material. *AlphaMath: Journal of Mathematics Education*, *7*(2), 84. https://doi.org/10.30595/alphamath.v7i2.12034

Ishak, M., Oderinde, I., & Ahmad, S. (2025). The role of metacognitive strategies in enhancing learning outcomes and educational efficiency: A systematic review of quantitative, qualitative and mixed-method studies. *International Journal of Academic Research in Business and Social Sciences*, *15*(4). https://doi.org/10.6007/ijarbss/v15-i4/24964

Klau, K. Y., Siahaan, M. M. L., & Simarmata, J. E. (2020). An identification of conceptual and procedural understanding: Study on preservice secondary mathematics teacher. *Al-Jabar: Jurnal Pendidikan Matematika*, *11*(2), 339–350. https://doi.org/10.24042/ajpm.v11i2.7310

Krawitz, J., Schukajlow, S., Yang, X., & Geiger, V. (2025). A systematic review of international perspectives on mathematical modelling: Modelling goals and task characteristics. *ZDM – Mathematics Education*, *57*(2–3), 193–212. https://doi.org/10.1007/s11858-025-01683-2

Lachenbruch, P. A., & Cohen, J. (1989). Statistical power analysis for the behavioral sciences (2nd ed.). *Journal of the American Statistical Association*, *84*(408). https://doi.org/10.2307/2290095

Mahlaba, S. C., & Mudaly, V. (2022). Exploring the relationship between commognition and the Van Hiele theory for studying problem-solving discourse in Euclidean geometry education. *Pythagoras*, *43*(1). https://doi.org/10.4102/pythagoras.v43i1.659

Mifetu, R. K. (2023). Using activity method to address students' problem-solving difficulties in circle geometry. *Contemporary Mathematics and Science Education*, *4*(1). https://doi.org/10.30935/conmaths/13079

Miles, M. B., Huberman, A. M., & Saldaña, J. (2016). Qualitative data analysis: A methods sourcebook (3rd ed.). In *Educacao e Sociedade* (Vol. 1, Number 1). Safe Publications. https://www.metodos.work/wp-content/uploads/2024/01/Qualitative-Data-Analysis.pdf

Mirna, Fauzan, A., Yarman, Arnawa, I. M., & Lufri. (2026). Geometric thinking skills of prospective mathematics teachers in the transformation geometry course: Effects and obstacles. *Mathematics Education Journal*, *20*(1), 225–240. https://doi.org/10.22342/mej.v20i1.pp225-240

Mirna, M., Dwina, F., & Khairani, K. (2020). Students' characteristics, learning outcomes and needs of geometry media tools in junior high school at Padang. *Journal of Physics: Conference Series*, *1554*(1). https://doi.org/10.1088/1742-6596/1554/1/012051

Naufal, M. A., Abdullah, A. H., Zainal, Z., Ihsan, H., Zaki, A., & Alshaye, I. A. (2024). The trend of geometric thinking studies: A systematic review. *ITM Web of Conferences*, *58*, 03010. https://doi.org/10.1051/itmconf/20245803010

OECD. (2023). *PISA 2022 results (volume I)*. OECD Publishing. https://doi.org/10.1787/53f23881-en

Palinkas, L. A., Mendon, S. J., & Hamilton, A. B. (2019). Innovations in mixed methods evaluations. In *Annual Review of Public Health* (Vol. 40). https://doi.org/10.1146/annurev-publhealth-040218-044215

Pebrianti, A., & Suhendra, S. (2023). Ways of thinking senior high school student to solve geometri Van Hiele problem use reversible thinking ability. *Al-Jabar: Jurnal Pendidikan Matematika*, *14*(2). https://doi.org/10.24042/ajpm.v14i2.18116

Polya, G. (1973). *How to solve it: A new aspect of mathematical method*. NJ: Princeton University Press. https://www.hlevkin.com/hlevkin/90MathPhysBioBooks/Math/Polya/George_Polya_How_To_Solve_It_.pdf

Pujawan, I. G. N., Suryawan, I. P. P., & Prabawati, D. A. A. (2020). The effect of Van Hiele learning model on students' spatial abilities. *International Journal of Instruction*, *13*(3). https://doi.org/10.29333/iji.2020.13332a

Saputra, F. A. D. W. P., Yayuk, E., & Tinus, A. (2025). Challenges and success factors in digital classroom implementation. *Academia Open*, *10*(1). https://doi.org/10.21070/acopen.10.2025.11144

Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (reprint). *Journal of Education*, *196*(2). https://doi.org/10.1177/002205741619600202

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *Journal of the Learning Sciences*, *16*(4). https://doi.org/10.1080/10508400701525253

Sihlangu, S., Maphutha, K., & Mokwana, L. (2025). A commognitive analysis of learners' mathematical thinking on mathematics vocabulary used during classroom discourse. *Journal on Mathematics Education*, *16*(4). https://doi.org/10.22342/jme.v16i4.pp1389-1406

Siregar, T., Fauzan, A., Yerizon, Y., & Syafriandi, S. (2025). Designing mathematics teaching through deep learning pedagogy: toward meaningful, mindful, and joyful learning. *Journal of Deep Learning*, 188–202. https://doi.org/10.23917/jdl.v1i2.11969

Sulistiowati, D. L., Herman, T., & Jupri, A. (2019). Student difficulties in solving geometry problem based on Van Hiele thinking level. *Journal of Physics: Conference Series*, *1157*, 042118. https://doi.org/10.1088/1742-6596/1157/4/042118

Tambychik, T., & Meerah, T. S. M. (2010). Students' difficulties in mathematics problem-solving: What do they say? *Procedia - Social and Behavioral Sciences*, *8*. https://doi.org/10.1016/j.sbspro.2010.12.020

Tiwari, S., Obradovic, D., Rathour, L., Narayan Mishra, L., & Mishra, V. N. (2021). Visualization in mathematics teaching. *Journal of Advances in Mathematics*, *20*, 431–439. https://doi.org/10.24297/jam.v20i.9136

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry* [University of Chicago]. https://eric.ed.gov/?id=ED220288

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. FL: Academic Press. https://archive.org/details/structureinsight0000hiel

Verina, I., & Juandi, D. (2022). Indonesian students' conceptual understanding in mathematics based on learning style: Systematic literature review (SLR). *Symmetry: Pasundan Journal of Research in Mathematics Learning and Education*, *7*(2), 160–170. https://doi.org/10.23969/symmetry.v7i2.6409

Wijaya, T. T., Hidayat, W., Hermita, N., Alim, J. A., & Talib, C. A. (2024). Exploring contributing factors to PISA 2022 mathematics achievement: Insights from Indonesian teachers. *Infinity Journal*, *13*(1), 139–156. https://doi.org/10.22460/infinity.v13i1.p139-156

Yi, M., Flores, R., & Wang, J. (2020). Examining the influence of Van Hiele theory-based instructional activities on elementary preservice teachers' geometry knowledge for teaching 2-D shapes. *Teaching and Teacher Education*, *91*, 103038. https://doi.org/10.1016/j.tate.2020.103038