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Analysis of Students' Difficulties Based on Epistemological Aspects In Function Limit Proving Problem

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Abstract
This study aims to analyse the mathematical proof ability and learning difficulties of students in understanding Limit Function material, focusing on epistemological aspects. Success in proving mathematics is at the core of understanding concepts in mathematics, but this ability has not been fully internalised in students. The descriptive method was used in this study, involving nine prospective mathematics teachers at Pancasakti Tegal University who took the Real Analysis course. Data were collected through: (1) mathematical proof ability tests; (2) observation; (3) interviews; and (4) documentation. The results of the analysis show that there are five types of student difficulties in the context of epistemology, namely: a) Ability to understand and apply ideas; b) Difficulty visualising objects; c) Difficulty determining principles; d) Difficulty understanding problems; and e). Inability to prove math. In particular, students experience challenges in starting the proof process, using known definitions and principles, and tend to focus on what must be proven without a clear initial step. These findings are expected to provide insights into the development of learning processes and teaching materials, as well as improve mathematical proof skills among mathematics education students.

Keywords: Learning Difficulties, Ability to Prove, Epistemology

INTRODUCTION

Real Analysis is one of the courses that fits into some of the college's math goals. States that students begin to learn reasoning and practice formal mathematical statements with Real Analysis as their first subject (Avalos et al., 2015). Although real analysis is a very important subject, most math students still consider it a difficult subject (Isnani et al., 2020; Moore, 1994). It also explains that many students face difficulties in understanding Real Analysis courses because most of the material in Real Analysis is considered abstract. Said that students face many problems during the learning process, including difficulty in providing counterexamples, difficulty in understanding implications, difficulty in understanding proofs using definitions, and difficulties in understanding algebraic forms. However, students face some problems in building evidence (Isnani et al., 2021; Kartini & Suanto, 2016; Selden & Selden, 2012; Sundawan et al., 2018). These include a lack of understanding of various proofing techniques, an inability to use premises, an inability to use existing definitions or theorems, difficulties in writing linguistic sentences and mathematical notation, difficulties in algebraic manipulation, and a lack of understanding of sets and logic.

Learning Real Analysis needs to learn the functions of evidence and proof are verification, explanation, systematisation, discovery, communication, construction, exploration, and incorporation (Sundawan et al., 2018). Since both are the result of a very mature process of developing mathematical thinking, proof verification and proof are considered the most important functions in proof. Explanation provides the reason why the statement is true, while verification refers to the truth of a statement.

Limit Function is a Real Analysis lecture topic (Bartle,2000). Students have a fine conceptual picture of limits, as shown by their understanding of the concept of limits (Anthony, 2017; Lesseig, 2016; Oktaviyanthi et al., 2018). Most students face difficulties in applying intuitive limit concepts to formal concepts. Many of them do not understand the formal definition of limit as a statement equivalent to what they have intuitively learned (Karsten et al., 2023; Saavedra & Gutierrez, 2020). Students have difficulty proving skills in this material for several reasons. One is that they don't have enough learning experience, and they don't have a strategy to prove their skills.

This study examines students' epistemology of the ability to prove mathematics in the Real Analysis course for Limit Function material and their learning difficulties about Limit Function material.

METHODS

This research method is a descriptive approach that aims to explain students' epistemology about mathematical proof errors in the Real Analysis for Function Limit Material course.

Participants in this study are students who take Real Analysis courses at private universities on the west coast of the north coast of Central Java. Furthermore, nine students were taken from the participants. The participants have various proving abilities, namely high, medium, and low abilities. Then the participants were taken as many as nine students, with details of 3 people with high ability, 3 people with medium ability, and 3 people with low ability. The initial knowledge of the participation is based on achievements in the Basic Calculus course.

Five indicators were used to overcome students' learning difficulties about the epistemology of transformation materials: concepts, visualisations, principles, comprehension problems, and mathematical proofs.

RESULTS AND DISCUSSION

Solving problems on function limits in real analysis, especially in error analysis, is a multifaceted problem that involves understanding the types of errors students make as well as the factors that contribute to those errors. This study emphasises the various dimensions of error analysis, such as conceptual errors, procedural errors, and translation problems between mathematical and natural languages.

The results of the study show that Mathematics Education students on the westernmost north coast, precisely in the city of Tegal, have different abilities to prove the limit material of Functions in the Real Analysis course. According to this evidence, some individuals have a high, medium, or low level of ability.

In working on Real Analysis questions for the Function limit material, students face difficulties in understanding the concept of the function limit from an epistemological perspective:

1. Ability to understand and apply ideas

   Limitations are not understood by students, namely students memorise the definition of the limit function and the concept of limit, but they do not know how to use the definition of the limit function. This happens to students who have moderate or low math skills at first. To take the delta to be used, students with high, middle, and low abilities face difficulties. Nonetheless, people with high abilities can solve it to completion, while people with medium and low abilities can solve incomplete problems. Some students are confused when answering questions because they don't understand the concept of limits correctly. This supports the findings of the research (Häggström et al., 2018; Isnani et al., 2025; Oktaviyanthi et al., 2018). Most students were found to have difficulty applying the concept of limits intuitively to formal concepts. Many of them do not understand the formal definition of limit as an equivalent statement of what they intuitively learn. This is in line with research that found that students often do not have a deep understanding of the concept of limits, what they mean and how they are visualised (Khairani et al., 2019; Slavíčková & Vargová, 2023). It was also found that the most easily found errors were misunderstandings related to the use of incorrect formulas or theorems. In one study, each student showed conceptual errors (Cusmariu, 2022; Hodiyanto, 2017).

2. Difficulty visualising objects

   Difficulty seeing objects beyond the functional limit. This means that students have difficulty finding the position of the domain area and the result area when drawing in Cartesian coordinates. High-ability students can visualise according to the questions asked correctly, while low-ability students have difficulties. Don't draw in this visualisation if you're not proficient. Because students don't have more formal representations, visualisation becomes difficult. Writing Evidence (Raman, 2003). Likewise, in functional limit learning errors, they cannot convert their visual knowledge into a language of proof. (Zazkis & Villanueva, 2016). Demikian pula, mempertimbangkan contoh klaim yang spesifik dapat membantu dalam penulisan bukti (Stolet & Holmowaia, 2020; Zazkis et al., 2015). Errors that occur when converting a problem into a solvable form, such as factoring or rationalisation errors(Dj Pomalato et al., 2020; Hodiyanto, 2017).

3. Difficulty in determining principles

This problem is a problem faced by students when they determine what principles will be used to solve the problem of the limit function and the concept of limit. The delta used is still in the domain area. Determining delta is difficult for students with medium and low ability. Students don't have the experience and strategies to prove it. This is also following the explanation given by (Melhuish et al., 2019),(Wasserman et al., 2018). Similarly, a person who has the experience of proving one day can create a proofing strategy and a proofing strategy in one day. Those who have the experience of proving one day can also make a proof strategy in one day (Pimm, 2010). Errors in mathematical processes, such as errors in the simplification and manipulation of algebra (Dj Pomalato et al., 2020; Khairani et al., 2019). Likewise, errors were found in the process of addition, subtraction, multiplication, and division (Hodiyanto, 2017; Pg. Johari & Shahrill, 2020).

4. Difficulty understanding the problem

It is difficult for students to understand the problem and use the resolution procedure that corresponds to the definition of the limit function. High ability is defined as the ability to complete existing procedures correctly; Medium ability is defined as the ability to complete a procedure but is not correct; and low ability is defined as the ability to complete the procedure and is not precise. Students do not understand the concept and definition of boundaries, which leads to this difficulty. As observed in the study (Ndemo et al., 2017; Utari & Hartono, 2019), our Concept and Definition of Limits. These problems have shown the complex relationship between the phases of understanding, planning, and executing plans. The above plan is intended to understand some of the aspects of the problem that confuse them (Savic, 2015, 2016).

5. Inability to prove mathematically

Students' difficulty in building question-proof. Students who receive a certificate of high ability can build evidence correctly, but there are writing errors. At the end of the section, there is an error in giving reasons; Moderate ability can build partial evidence. Those who cannot use definitions cannot build evidence. The ability to construct mathematical statements based on definitions, principles, and theorems and write them down in the form of complete proofs is called proof-building (Sumarmo, 2014). Students were also recorded to make mistakes in the problem-solving process, such as misspelling "lim", using the wrong grade, and failing to simplify the final result. Major mistakes include using incomplete steps or failing to write procedures (Hodiyanto, 2017; Pg. Johari & Shahrill, 2020). Errors in basic arithmetic operations such as addition, subtraction, multiplication, and division (Hodiyanto, 2017; Pg. Johari & Shahrill, 2020). Errors that occur when converting a problem into a solvable form, such as factoring or rationalisation errors (Dj Pomalato et al., 2020; Hodiyanto, 2017).

Based on the errors found above, after tracing the cause of the error is in line with previous studies, namely; (1) Students' understanding is still lacking, students often do not have a deep understanding of the concept of limit, what it means and how it is visualized (Khairani et al., 2019; Slavíčková & Vargová, 2023); (2) Students have low initial abilities after being traced to inadequate basic knowledge about related topics such as trigonometry and quadratic functions (Cusmariu, 2022; Hodiyanto, 2017; Pg. Johari & Shahrill, 2020); (3) Persistent misconceptions mean Constant misconceptions about limits, such as assuming a limit equals the value of a function (Martínez-miraval, 2025; Nagle et al., 2017); (4) Misunderstandings can arise from the way the content is presented in the textbook or by the lecturer (Inarejos et al., 2022); (5) Inadequate connection skills difficulty connecting different mathematical concepts and techniques (Shahrill & Prahmana, 2018). Considering that in learning, the problem of proving the concepts that have been learned can be used to prove theorems and the next problem.

Based on the findings of errors and their causes, it is recommended that these errors can be minimized, so that the following efforts are needed according to the results of the relevant research, namely:1) The need for scaffolding to provide structured support to help students understand and apply the concept of boundaries (Dj Pomalato et al., 2020; Lestari, 2021; Shahrill & Prahmana, 2018); (2) Dual Representation: Improve understanding of concepts and problem-solving skills by using a variety of representations and tools (Tarida et al., 2025); dan (3) A focused teaching method that serves to change the teaching approach to deal with certain errors and misunderstandings (Hodiyanto, 2017; Pg. Johari & Shahrill, 2020).

CONCLUSION

The ability of students to prove varies, namely there are high, medium, and low, which were obtained in this study. In the Real Analysis course for the Function Limit material, students face five epistemological problems, namely a) Ability to understand and apply ideas; b) Difficulty visualising objects; c) Difficulty determining principles; d) Difficulty understanding problems; and e). Inability to prove math.

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