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Mathematical Communication Skills Analysis through the Discovery Learning Approach with the Help of the Quizizz Application

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***Abstract*:**
Mathematical communication skills are crucial in the digital era, but students often experience cognitive barriers in abstract composition and inverse function materials. Evaluation of the impact of innovative digital learning is generally still dominated by a macro quantitative approach, thus ignoring specific cognitive dynamics based on students' initial abilities. This descriptive qualitative study aims to describe the profile of mathematical communication skills (drawing, math expression, written text) of students in the Quizizz-assisted Discovery Learning model in terms of their Prior Mathematical Ability (PMA). Through a purposive sampling technique, 6 subjects were selected from a total of 27 students in the experimental class to represent the high, medium, and low PMA clusters. Data were collected through triangulation of test achievement graphs, analysis of student worksheet results, and in-depth clinical interviews. The results of the study revealed three main findings: (1) The high groups showed established metacognition and fluency of layered representations across all indicators. (2) The medium groups manifested a cognitive bypass anomaly, where students tactically sacrificed the drawing aspect for the sake of efficient algebraic-verbal calculation time (cognitive trade-off). (3) The low groups experienced the paradox of digital gamification; The Quizizz feature helps with short-term memory retention in written text, but the pressure of the fast-paced game rhythm triggers impulsive cognitive styles and time-constrained anxiety, which leads to careless errors in algebraic expressions during manual written tests. In conclusion, the integration of gamification technology does not have a linear impact on all levels of student ability, requiring intervention in the form of time regulation, differentiation, and scaffolding to minimize cognitive barriers in lower groups.
Keywords: Discovery Learning, Mathematical communication skills, Prior Mathematical Ability, Quizizz

Introduction

The development of science and technology demands an improvement in the quality of mathematics learning so that students are not only able to master calculation procedures but also have high-level thinking skills (Erwin & Julie, 2025). One of the important competencies of the 21st century is mathematical communication skills, namely students' ability to convey ideas, reasoning, and mathematical solutions orally, in writing, as well as through symbolic and visual representations (OECD, 2023; Suhenda & Munandar, 2023; Wardhana et al., 2025). The National Council of Teachers of Mathematics (NCTM, 2000) asserts that communication is an integral part of mathematics education because it acts as an instrument for students to reflect, refine, and consolidate their mathematical thinking. This ability is the basis for a deep understanding of concepts and supports the problem-solving process in a real-life context (Astuti & Hidayat, 2020). These skills include the ability to articulate mathematical ideas, use proper mathematical notation, and present solutions in a clear and logical way (Tokada et al., 2017) . Communication is the process of conveying information, ideas, emotions, and skills using symbols, words, images, graphs, numbers, etc.(Inganah et al., 2023). This is in line with the purpose of mathematics learning as stated in the Permendikbud (Indarta et al., 2022), which emphasizes the importance of being able to communicate ideas with symbols, tables, diagrams, or other media to clarify a situation or problem. Mathematical communication skills are multidimensional, encompassing three main indicators: geometric visualization (Drawing), manipulation of formal algebraic expressions (Math Expression), and verbal and written explanation of logical arguments (Written Text). A recent study by Stylianou (2020) shows that the fluency of transitions between these representations (visual to symbolic, or vice versa) is a key indicator of deep conceptual understanding. When students are able to communicate mathematical objects in various formats, they are not simply memorizing algorithms but have built a robust network of cognitive schemas. If communication skills are not good, then students will have difficulty in translating real-world problems into mathematical models or vice versa, resulting in a low understanding of concepts in depth (Aprilia et al., 2024; Saragih et al., 2024).

Although the importance of mathematical communication has been globally agreed upon, the reality on the ground shows that this ability remains one of the biggest stumbling blocks for students, particularly in abstract topics such as Compositional Functions and Inverse Functions. The characteristics of function material require multi-level symbolic operations and complex spatial understanding. Students often fall into algebraic procedural errors or fail to interpret functional meanings in real-world contexts. This obstacle stems from variations in Prior Mathematical Ability (PMA) that students possess before entering the classroom. According to Sweller (2011), in ZDM, through Cognitive Load Theory, prior knowledge structures in long-term memory act as cognitive filters. Students with high PMA have well-established relational schemas, preventing their working memory from experiencing cognitive overload when confronted with new material. In addition, when students are able to articulate their mathematical thinking and are open to the perspectives of others, there is a process of social interaction that enriches their insights and broadens their perspective in solving problems. This is in line with Vygotsky's view (McLeod & Augustus, 2022), which speaks to the importance of social interaction in learning, where communication is a means to bridge the proximal development zone (ZPD). Conversely, students with low PMA often experience communication failures because their working memory capacity is depleted to process basic arithmetic operations that are not yet automatic. Therefore, mapping mathematical communication profiles cannot be generalized uniformly, but must be reviewed based on the student's initial ability cluster to obtain an accurate diagnostic portrait (Radford, 2014; Wandari & Fardillah, 2021).

To bridge this skill gap, a paradigm shift in learning is needed from teacher-centered to active, discovery-based learning. The Discovery Learning model is widely recognized for its ability to stimulate students' independent thinking through the syntax of stimulation, problem identification, data collection, data processing, and proof (Amalia et al., 2024; Junaila & Yerizon, 2021). However, the effectiveness of Discovery Learning in this modern era requires accelerators in the form of interactive learning media based on digital technology to maintain the cognitive engagement of the new generation of students. One of the most popular digital gamification platforms is Quizizz (Mahmud & Law, 2022; Zuhriyah & Pratolo, 2020). Research by Zainuddin et al. (2020) states that game elements such as leaderboards, penalty points, and instant feedback in Quizizz can significantly increase students' intrinsic motivation and conceptual retention. However, recent international literature has begun to detect side effects from the use of digital gamification. Attali & Arieli-Attali (2015) in the journal Computers & Education warned that the time-attack mechanism on digital educational platforms has the potential to trigger an impulsive cognitive style. Students are accustomed to chasing final results in order to achieve points, potentially sacrificing procedural accuracy and the neatness of manual motor skills on paper. This paradox requires further investigation: Does the digital environment of Quizizz support or degrade students' cognitive resilience in formal written communication?

Although numerous studies have been conducted on the use of Quizizz and Discovery Learning, the majority of previous research has relied on a quantitative-positivistic approach that only measures the effectiveness of the final product through N-gain tests or average class post-test scores at a macro level (Example Rosa et al., 2025). Qualitative literature exists that explores the granular "how" students' thinking processes work and "why" unique worksheet variations occur across different early ability groups. This research aims to fill this theoretical research gap. Its novelty lies in its triangulation of methods, which combines quantitative classroom data, physical analysis of student worksheets, and in-depth clinical interviews. Through this descriptive qualitative approach, researchers successfully uncovered hidden cognitive phenomena overlooked in quantitative studies, such as the phenomenon of Cognitive Bypass in medium-ability students, who choose to bypass visual representations for time efficiency, and the psychological impact of Time-Constrained Anxiety, which triggers careless algebra errors in lower-level students. Therefore, the purpose of this study is to in-depth describe the profile of students' mathematical communication skills (Drawing, Math Expression, and Written Text) in the Quizizz-assisted Discovery Learning model, based on their levels of High, Medium, and Low Prior Mathematical Ability (PMA). The results of this study are expected to provide theoretical contributions to the development of mathematics instructional design that is friendly to students' cognitive diversity in the digital age.

Research Methods

This study uses a qualitative approach with a descriptive method to analyze students' mathematical communication skills through Discovery Learning assisted by the Quizizz application. This design was chosen because the research focused on an in-depth description of students' mathematical communication processes, not on testing hypotheses or statistical influences. The researcher does not intend to test the influence or prove statistical hypotheses, but rather to analyze the "how" of communication patterns of students with high, medium, and low initial ability. The research was carried out at SMAN 9 Kendari in class XI A1 with a total of 27 students, in the odd semester of the 2025/2026 school year (August - December), starting from the instrument preparation stage to reporting results. The research subjects were selected using purposive sampling by grouping students based on Prior Mathematical Ability (PMA) into high, medium, and low categories. According to Sugiyono (2019) Purposive sampling is a sampling technique with certain considerations. The grouping of students' prior mathematical abilities is obtained from the pre-test covering prerequisite concepts (basic functions, domain/ range, and algebraic manipulation), which would yield a far more valid and content- aligned measure of PMA. Students are grouped into three categories: High, Medium, and Low based on average () and standard deviation (SD). From each category, two students were selected as in-depth interview subjects so that there were a total of six main subjects.

Table 1. Criteria for Grouping Prior Mathematical Ability (PMA)

Categories PMA	Score Criteria
High	Score $\geq (\bar{x} + SD)$
Medium	$(\bar{x} - SD) \leq Score < (\bar{x} + SD)$
Low	Score $< (\bar{x} - SD)$

(Arikunto, 2020).

Description: $\bar{x}$ = Class Average, SD = Standard deviation.

Table 2. List of Research Subject Codes

No.	Subject Code	Categories PMA	Remarks
1	S-T1	High	Interview Subject 1
2	S-T2	High	Interview Subject 2
3	S-S1	Medium	Interview Subject 1
4	S-S2	Medium	Interview Subject 2
5	S-R1	Low	Interview Subject 1
6	S-R2	Low	Interview Subject 2

The research procedure was carried out in four meetings on the composition of functions and inverse functions. Learning applies the Discovery Learning syntax, while Quizizz is used as a supporting medium, especially at the stimulation and verification stages. In the first three meetings, students engaged in LKPD-based discussions with the Discovery Learning syntax, while Quizizz was used as a medium of stimulation and verification to increase engagement and validation of concepts. The fourth meeting was used for the provision of the Mathematical Communication Ability Test (TKKM) and in-depth interviews. The research instruments include: (1) mathematical communication ability description test, (2) semi-structured interview guidelines, and (3) documentation in the form of Quizizz results and student answer sheets. The mathematical communication indicators analyzed include drawing skills, written text, and mathematical expression.

Table 3. Mathematical Communication Ability Indicator Grid

Communication Aspects	Measured Indicators	Description of Student Abilities
Drawing	Declaring mathematical situations into pictures/graphs	Able to paint pictures, diagrams, or graphs that are relevant to the problem of the story correctly and completely
Written text (Ekspresi Tertulis)	Explain mathematical ideas or situations in writing	Able to write down the steps to solve in a sequence and provide arguments or logical reasons for the answers given
Mathematical Expression	Use notation, symbols, and mathematical terms	Able to turn a problem into a mathematical model (mathematical sentence), using the symbols of the calculation operation correctly until a solution is obtained

(Pramuditya et al., 2021).

In qualitative analysis, student assignments and interview results are not assigned numerical scores but are classified into specific categories according to the level of mathematical communication (Agustin et al., 2024). In this study, data analysis refers to the Miles, Huberman, and Saldaña models (Miles et al., 2014), which include data condensation, data presentation, and drawing conclusions. The validity of the data is guaranteed through triangulation techniques, namely, by comparing the results of written tests and interviews. If what the student writes (Test) matches what is explained orally (Interview), then the data is considered valid (credible). If a discrepancy is found between the written test and the interview response, the researcher conducts further investigation to determine the source of the inconsistency.

Result and Discussion

Result

Based on the results of the analysis of students' mathematical communication skills test on composition and inverse function materials, as well as the results of interviews conducted on the research subjects. The following is a description of students' mathematical communication skills based on the grouping of Prior Mathematical Ability (PMA).

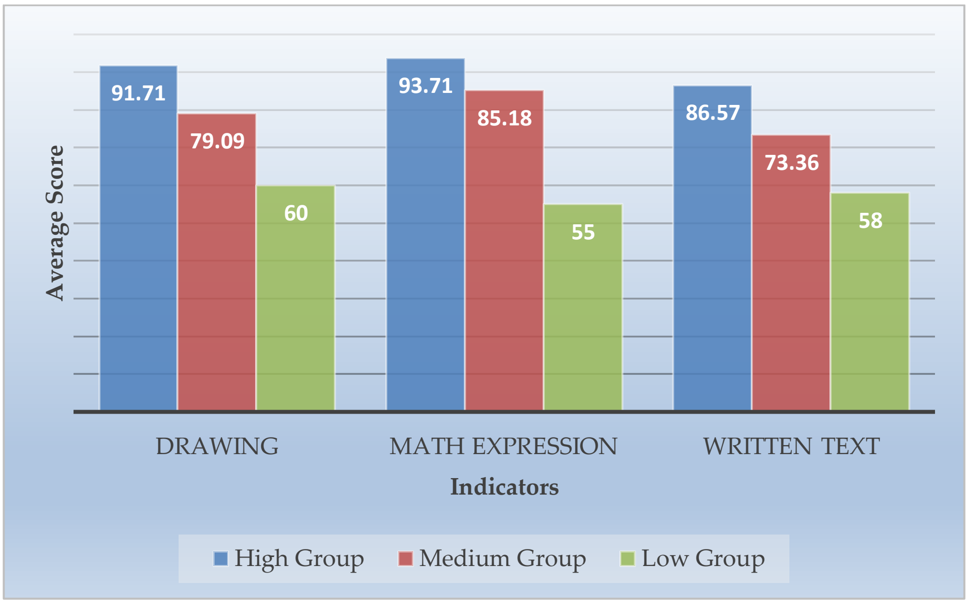
Classification of research subjects based on the value of Prior Mathematical Ability (PMA), from 27 students in the experimental class, the following groupings were obtained:

Table 4. Distribution of Research Subjects Based on PMA

Ability Categories	Number of Students	Percentage	Selected Subject Code (Interview)
High	7	26 %	S-T1, S-T2
Medium	11	41 %	S-S1, S-S2
Low	9	33 %	S-R1, S-R2

Based on the data in Table 4, of the 27 students in the experimental class who participated in the Discovery Learning series assisted by the Quizizz media, subject categorization was carried out using the Prior Mathematical Ability (PMA) instrument. The distribution of student abilities resulted in the following classification: High Group: Consisting of 7 students (26%). This group represents students who already have a mature basic mathematical cognitive scheme before the treatment was given. Medium Group: Consisting of 11 students (41%). This group dominates the experimental class population, reflecting the average ability of students under normal learning conditions. Low Group: Consisting of 9 students (33%). This group includes one-third of the total student body, indicating the presence of a significant group that requires special attention regarding the understanding of formal mathematical prerequisite concepts. To thoroughly examine the mental activities and cognitive obstacles experienced by students, the researcher referred to the Purposive Sampling technique. Based on this technique, 6 students were selected as the main interview subjects, who were divided proportionally into 2 subjects from each PMA cluster: S-T1 and S-T2 (High), S-S1 and S-S2 (Medium), and S-R1 and S-R2 (Low).

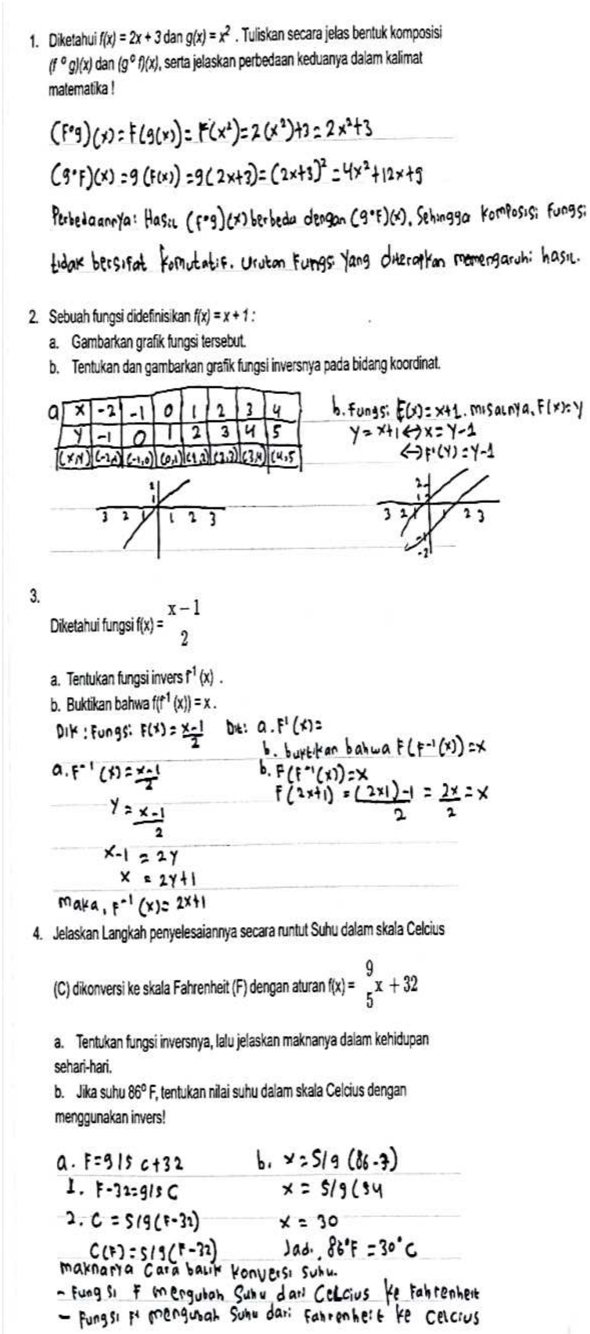
The following is a recapitulation of the average score of the mathematical communication indicator from the results of the description test at the four meetings:



Graph 1. Average Mathematical Communication Indicator Scores per Group

This graph reveals 3 important findings, namely: (1) Linearity of Initial Ability (KAM) to Mathematical Communication: The graph consistently shows a descending staircase pattern (Blue → Red → Green) in all indicators. This empirically proves that Initial Mathematical Ability is positively and significantly correlated with students' readiness to communicate their formal mathematical ideas. (2). Characteristics of Algebraic Excellence (Symbolic Dominance): The highest achievement of the upper and middle groups is in the Math Expression aspect. This indicates that the learning approach (Discovery Learning assisted by Quizizz) is very successful in strengthening students' algebraic reasoning and symbolic operations. However, this high symbolic aspect has not been fully balanced by the ability to describe the concept in a standard written language narrative (Written Text). (3). Cognitive Barriers of the Lower Group (Low Group): For the lower group, the greatest inequality is in the Math Expression aspect (55). They had significant difficulty with abstract manipulation, but uniquely scored slightly better on the visual (Drawing = 60) and verbal (Written Text = 58) aspects. This indicates that lower-group students are more easily helped using a concrete-visual (Iconic) representation approach before being forced into abstract, purely algebraic calculations (Symbolic).

Analysis of Mathematical Communication Skills of High-Ability Students (S-T1, S-T2)



1. Diketahui $f(x) = 2x + 3$ dan $g(x) = x^2$. Tuliskan secara jelas bentuk komposisi $(f \circ g)(x)$ dan $(g \circ f)(x)$, serta jelaskan perbedaan keduanya dalam kalimat matematika !

$(f \circ g)(x) = f(g(x)) = f(x^2) = 2(x^2)+3 = 2x^2+3$

$(g \circ f)(x) = g(f(x)) = g(2x+3) = (2x+3)^2 = 4x^2+12x+9$

Perbedaannya: Hasil $(f \circ g)(x)$ berbeda dengan $(g \circ f)(x)$. Sehingga komposisi fungsi tidak bersifat komutatif. Urutan fungsi yang diterapkan memengaruhi hasil.

2. Sebuah fungsi didefinisikan $f(x) = x + 1$:
a. Gambarkan grafik fungsi tersebut.
b. Tentukan dan gambarkan grafik fungsi inversnya pada bidang koordinat.

a

x	-2	-1	0	1	2	3	4
y	-1	0	1	2	3	4	5
(x,y)	(-2,-1)	(-1,0)	(0,1)	(1,2)	(2,3)	(3,4)	(4,5)

b. Fungsi: $f(x) = x+1$. misalnya, $f(x) = y$
$y = x+1 \Leftrightarrow x = y-1$
$\Leftrightarrow f^{-1}(y) = y-1$

3. Diketahui fungsi $f(x) = \frac{x-1}{2}$
a. Tentukan fungsi invers $f^{-1}(x)$.
b. Buktikan bahwa $f(f^{-1}(x)) = x$.

Dik : Fungsi: $f(x) = \frac{x-1}{2}$ Dit: a. $f^{-1}(x) =$
b. buktikan bahwa $f(f^{-1}(x)) = x$

a. $f^{-1}(x) = \frac{x-1}{2}$
$y = \frac{x-1}{2}$
$x-1 = 2y$
$x = 2y+1$
Maka, $f^{-1}(x) = 2x+1$

b. $f(f^{-1}(x)) = x$
$f(2x+1) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$

4. Jelaskan Langkah penyelesaiannya secara runtut Suhu dalam skala Celcius (C) dikonversi ke skala Fahrenheit (F) dengan aturan $f(x) = \frac{9}{5}x + 32$
a. Tentukan fungsi inversnya, lalu jelaskan maknanya dalam kehidupan sehari-hari.
b. Jika suhu 86° F, tentukan nilai suhu dalam skala Celcius dengan menggunakan invers!

a. $F = 9/5\ C + 32$
1. $F - 32 = 9/5\ C$
2. $C = 5/9(F-32)$
$C(F) = 5/9(F-32)$

b. $x = 5/9\ (86-32)$
$x = 5/9\ (54)$
$x = 30$
Jadi. 86°F = 30°C

Maknanya cara balik konversi suhu.
- Fungsi F mengubah suhu dari Celcius ke Fahrenheit
- Fungsi F^{-1} mengubah suhu dari Fahrenheit ke Celcius

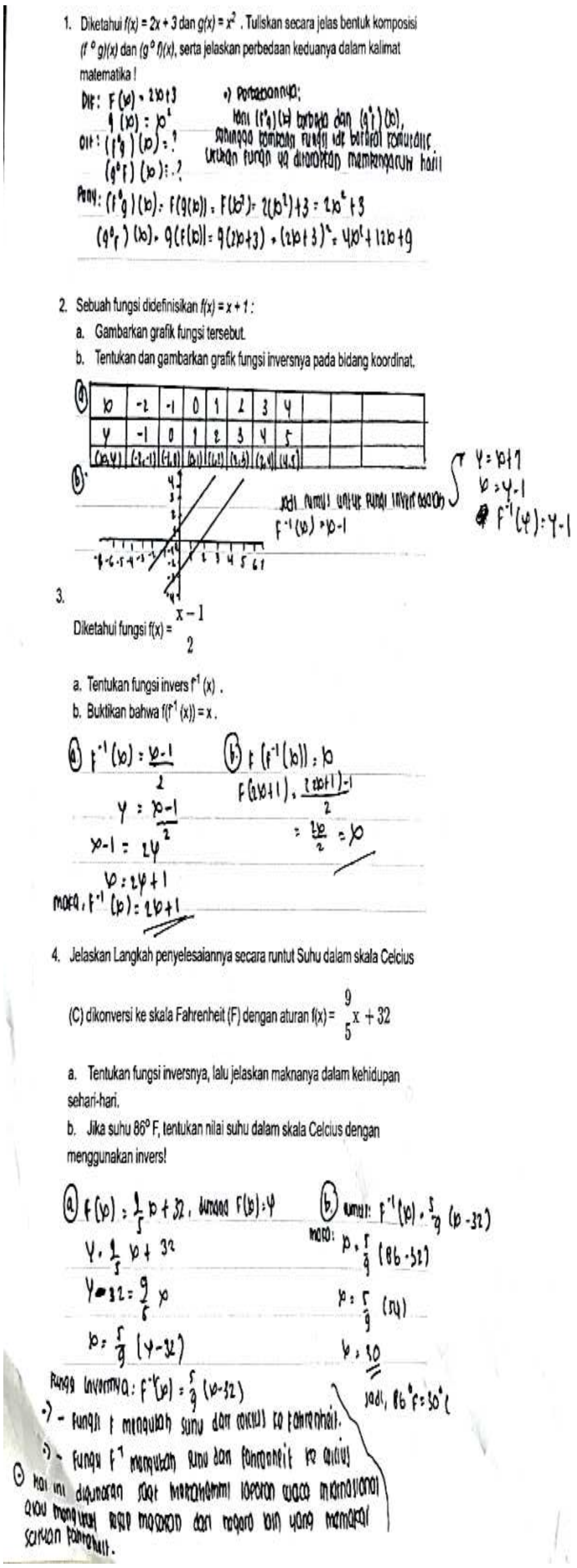
English Version Answered S-T1:

1. $(f \circ g)(x) = f(g(x)) = f(x^2) = 2(x^2)+3 = 2x^2 + 3$
   $(g \circ f)(x) = g(f(x)) = g(2x+3) = (2x + 3)^2 = 4x^2 + 12x + 9$
   Explanation of Difference: "The result of $(f \circ g)(x)$ is different $(g \circ f)(x)$. Therefore, function composition is not commutative. The order in which the functions are applied affects the final result."
2. *a. Coordinate Table for $f(x) = x + 1$:*

x	-2	-1	0	1	2	3
y	-1	0	1	2	3	4
(x, y)	(-2,-1)	(-1,0)	(0,1)	(1,2)	(2,3)	(3,4)

   b. Finding the Inverse Function:
   Function: $f(x) = x + 1$. Let $f(x) = y$
   $y = x + 1 \leftrightarrow x = y - 1$
   $\leftrightarrow f^{-1}(y) = y - 1$
   Therefore, the inverse function is $f^{-1}(x) = x - 1$.
3. *Given Function: $f(x) = \frac{x-1}{2}$*
   a. Finding $f^{-1}(x)$:
   $y = \frac{x-1}{2}$
   $x - 1 = 2y$
   $x = 2y + 1$
   Therefore, $f^{-1}(x) = 2x + 1$
   b. Proof that $f(f^{-1}(x)) = x$
   $f(f^{1}(x)) = x$
   $f(2x + 1) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$ *(Proven)*
4. *a. Finding the inverse and explaining its meaning:*
   $F = \frac{9}{5}C + 32 \Leftrightarrow F - 32 = \frac{9}{5}C \Leftrightarrow C = \frac{5}{9}(F - 32)$
   Therefore, the inverse function is: $C(F) = \frac{5}{9}(F - 32)$
   b. If temperature is 86°F, find the value in Celsius using the inverse:
   $x = \frac{5}{9}(86 - 32) \Leftrightarrow x = \frac{5}{9}(54) \Leftrightarrow x = 30$
   Therefore, 86°F = 30°C.
   Meaning in daily life: It represents the reverse process of temperature conversion.
   Function F converts temperature from Celsius to Fahrenheit.
   Function F^{-1} converts temperature from Fahrenheit to Celsius.

Figure 1. Results of Written Test for S-T1 Students


1. Diketahui $f(x) = 2x + 3$ dan $g(x) = x^2$. Tuliskan secara jelas bentuk komposisi $(f \circ g)(x)$ dan $(g \circ f)(x)$, serta jelaskan perbedaan keduanya dalam kalimat matematika !

2. Sebuah fungsi didefinisikan $f(x) = x + 1$:
   a. Gambarkan grafik fungsi tersebut.
   b. Tentukan dan gambarkan grafik fungsi inversnya pada bidang koordinat.

3. Diketahui fungsi $f(x) = \frac{x-1}{2}$
   a. Tentukan fungsi invers $f^{-1}(x)$.
   b. Buktikan bahwa $f(f^{-1}(x)) = x$.

4. Jelaskan Langkah penyelesaiannya secara runtut Suhu dalam skala Celcius (C) dikonversi ke skala Fahrenheit (F) dengan aturan $f(x) = \frac{9}{5}x + 32$
   a. Tentukan fungsi inversnya, lalu jelaskan maknanya dalam kehidupan sehari-hari.
   b. Jika suhu 86° F, tentukan nilai suhu dalam skala Celcius dengan menggunakan invers!

English Version Answered S-T2 :

1. ***Known (Dik):*** $f(x) = 2x + 3$
   $g(x) = x^2$
   Asked (Dit): $(g)(x) = \ldots ?$
   $(f)(x) = \ldots. ?$
   Solution (Peny):
   $(f \circ g)(x) = f(g(x)) = f(x^2) = 2(x^2) + 3 = 2x^2 + 3$
   $(g \circ f)(x) = g(f(x)) = g(2x + 3) = (2x + 3)^2 = 4x^2 + 12x + 9$
   The Difference (Perbedaannya):
   "The result of $(f \circ g)(x)$ is different from $(g \circ f)(x)$, so function composition is not commutative. The order of functions applied affects the result."
2. *a. Coordinate Table for $f(x) = x + 1$:*

x	-2	-1	0	1	2	3
y	-1	0	1	2	3	4
(x, y)	(-2,-1)	(-1,0)	(0,1)	(1,2)	(2,3)	(3,4)

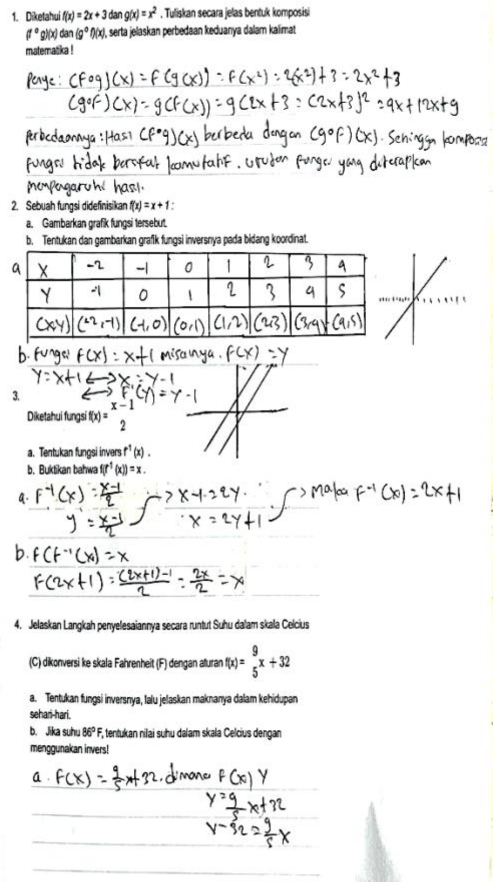
   b. Finding the Inverse Function:
   $y = x + 1$
   $x = y - 1$
   $f^{-1}(y) = y - 1$
   "So, the formula for the inverse function is $f^{-1}(x) = x - 1$"
3. *Given Function:* $f(x) = \frac{x-1}{2}$
   a. Finding $f^{-1}(x)$:
   $y = \frac{x-1}{2}$
   $x - 1 = 2y$
   $x = 2y + 1$
   Therefore, $f^{-1}(x) = 2x + 1$
   b. Proof that $f(f^{-1}(x)) = x$
   $f(f^{1}(x)) = x$
   $f(2x + 1) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$ *(Proven)*
4. *a. Finding the inverse and explaining its meaning:*
   $F = \frac{9}{5}C + 32 \leftrightarrow F - 32 = \frac{9}{5}C \leftrightarrow C = \frac{5}{9}(F - 32)$
   Its inverse function is: $C(F) = \frac{5}{9}(F - 32)$
   b. Converting 86°F:
   Formula: $f^{-1}(x) = \frac{5}{9}(x - 32)$
   Then : $x = \frac{5}{9}(86 - 32) \leftrightarrow x = \frac{5}{9}(54) \leftrightarrow x = 30$
   So, 86°F = 30°C.
   Meaning :
   "Function f converts temperature from Celsius to Fahrenheit."
   "Function f^{-1} converts temperature from Fahrenheit to Celsius."
   "This is used when understanding international weather reports or converting cooking recipes from other countries that use the Fahrenheit scale.

Figure 2. Results of Written Test for S-T2 Students

Based on the worksheets of questions from Students in the High Group (S-T1 and S-T2) demonstrated consistently superior performance across all mathematical communication indicators, with the highest achievement in the Math Expression aspect (93.71). The cognitive maturity of these two subjects is characterized by the ability to think structuredly from the initial phase of problem solving. Drawing Aspect: S-T1 and S-T2 did not immediately draw speculative graph lines in Question No. 2. Both consciously constructed a complete coordinate helper table for the domain x = -2 to x = 4. This step reflects strong metacognitive abilities. Based on the interview, S-T2 emphasized: "I deliberately made this coordinate table at the beginning, so that when plotting the points (x, y) of the inverse graph, the axes would not be swapped."

Math Expression & Written Text Aspect: Both subjects' algebraic manipulation abilities were very precise, as seen in the accuracy of proving the identity of the composition and inverse functions in Questions No. 1 and 3. This mature procedural ability had a positive impact on the Written Text aspect. Unencumbered by the complexity of calculations, S-T1 and S-T2 have the remaining cognitive capacity to write rich verbal arguments, such as describing the contextual meaning of temperature conversion in real life and formally deducing the non-commutative properties of the function ($f(g(x)) \neq g(f(x))$).

Analysis of Mathematical Communication Skills of Moderately Capable Students (S-S1, S-S2)



1. Diketahui $f(x) = 2x + 3$ dan $g(x) = x^2$. Tuliskan secara jelas bentuk komposisi $(f \circ g)(x)$ dan $(g \circ f)(x)$, serta jelaskan perbedaan keduanya dalam kalimat matematika!

Penye: $(f \circ g)(x) = f(g(x)) = f(x^2) = 2(x^2) + 3 = 2x^2 + 3$

$(g \circ f)(x) = g(f(x)) = g(2x+3) = (2x+3)^2 = 4x + 12x + 9$

Perbedaannya: Hasil $(f \circ g)(x)$ berbeda dengan $(g \circ f)(x)$. Sehingga komposisi fungsi tidak bersifat komutatif. Urutan fungsi yang diterapkan mempengaruhi hasil.

2. Sebuah fungsi didefinisikan $f(x) = x + 1$:
a. Gambarkan grafik fungsi tersebut.
b. Tentukan dan gambarkan grafik fungsi inversnya pada bidang koordinat.

a

X	-2	-1	0	1	2	3	4
Y	-1	0	1	2	3	4	5
(x,y)	(-2,-1)	(-1,0)	(0,1)	(1,2)	(2,3)	(3,4)	(4,5)

b. fungsi $f(x) : x+1$ misalnya $f(x) = y$

$Y = x+1 \leftrightarrow x = y-1$
$\leftrightarrow f^{-1}(y) = y-1$

3. Diketahui fungsi $f(x) = \frac{x-1}{2}$
a. Tentukan fungsi invers $f^{-1}(x)$.
b. Buktikan bahwa $f(f^{-1}(x)) = x$.

a. $f^{-1}(x) = \frac{x-1}{2}$, $y = \frac{x-1}{2}$ → $x - 1 = 2y$, $x = 2y + 1$ → Maka $f^{-1}(x) = 2x + 1$

b. $f(f^{-1}(x)) = x$

$f(2x+1) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$

4. Jelaskan Langkah penyelesaiannya secara runtut Suhu dalam skala Celcius (C) dikonversi ke skala Fahrenheit (F) dengan aturan $f(x) = \frac{9}{5}x + 32$
a. Tentukan fungsi inversnya, lalu jelaskan maknanya dalam kehidupan sehari-hari.
b. Jika suhu 86° F, tentukan nilai suhu dalam skala Celcius dengan menggunakan invers!

a. $f(x) = \frac{9}{5}x + 32$, dimana $f(x)$ Y

$Y = \frac{9}{5}x + 32$

$Y - 32 = \frac{9}{5}x$

English Version Answered S-S1 :

1. ***Solution (Peny):***

$(f \circ g)(x) = f(g(x)) = f(x^2) = 2(x^2) + 3 = 2x^2 + 3$

$(g \circ f)(x) = g(f(x)) = g(2x + 3) = (2x + 3)^2 = 4x + 12x + 9$

The Difference (Perbedaannya):

"The result of $(f \circ g)(x)$ is different from $(g \circ f)(x)$.. Therefore, function composition is not commutative. The order of functions applied affects the final result."

2. *a. Coordinate Table for $f(x) = x + 1$:*

x	*-2*	*-1*	*0*	*1*	*2*	*3*
y	*-1*	*0*	*1*	*2*	*3*	*4*
(x, y)	*(-2,-1)*	*(-1,0)*	*(0,1)*	*(1,2)*	*(2,3)*	*(3,4)*

b. Finding the Inverse Function:

Function $f(x) = x + 1$, for example, $f(x) = y$

$y = x + 1 \leftrightarrow x = y - 1$

$\leftrightarrow f^{-1}(y) = y - 1$

3. *a. Finding $f^{-1}(x)$:*

$f^{-1}(x) = \frac{x-1}{2} \rightarrow y = \frac{x-1}{2} \rightarrow x - 1 = 2y \rightarrow x = 2y + 1$

Therefore,$f^{-1}(x) = 2x + 1$

b. Proof that $f(f^{-1}(x)) = x$

$f(f^{1}(x)) = x$

$f(2x + 1) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$

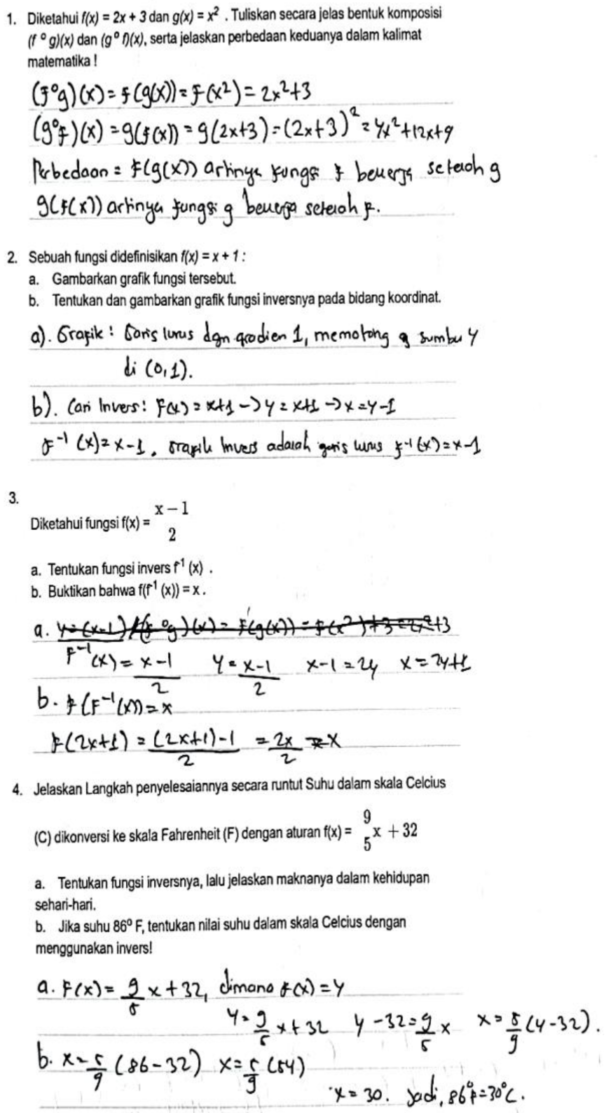
4. *a. Setting up the Inverse Function :*

$f(x) = \frac{9}{5}x + 32$ *, where $f(x) = y$*

$y = \frac{5}{9}x + 32$

$y - 32 = \frac{5}{9}$

Figure 3. Results of Written Test for S-S1 Students



1. Diketahui $f(x) = 2x + 3$ dan $g(x) = x^2$. Tuliskan secara jelas bentuk komposisi $(f \circ g)(x)$ dan $(g \circ f)(x)$, serta jelaskan perbedaan keduanya dalam kalimat matematika !

$(f \circ g)(x) = f(g(x)) = f(x^2) = 2x^2+3$

$(g \circ f)(x) = g(f(x)) = g(2x+3) = (2x+3)^2 = 4x^2+12x+9$

Perbedaan = f(g(x)) artinya fungsi f bekerja setelah g

g(f(x)) artinya fungsi g bekerja setelah f.

2. Sebuah fungsi didefinisikan $f(x) = x + 1$:
   a. Gambarkan grafik fungsi tersebut.
   b. Tentukan dan gambarkan grafik fungsi inversnya pada bidang koordinat.

a). Grafik : Garis lurus dgn gradien 1, memotong g sumbu Y di (0,1).

b). Cari Invers : $f(x) = x+1 \rightarrow y = x+1 \rightarrow x = y-1$

$f^{-1}(x) = x-1$, Grafik Invers adalah garis lurus $f^{-1}(x) = x-1$

3. Diketahui fungsi $f(x) = \frac{x-1}{2}$
   a. Tentukan fungsi invers $f^{-1}(x)$.
   b. Buktikan bahwa $f(f^{-1}(x)) = x$.

a. ~~y = (x-1) (f∘g)(x) = f(g(x)) = f(x²)+3 = 2x²+3~~

$f^{-1}(x) = \frac{x-1}{2}$ $y = \frac{x-1}{2}$ $x-1 = 2y$ $x = 2y+1$

b. $f(f^{-1}(x)) = x$

$f(2x+1) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$

4. Jelaskan Langkah penyelesaiannya secara runtut Suhu dalam skala Celcius (C) dikonversi ke skala Fahrenheit (F) dengan aturan $f(x) = \frac{9}{5}x + 32$
   a. Tentukan fungsi inversnya, lalu jelaskan maknanya dalam kehidupan sehari-hari.
   b. Jika suhu 86° F, tentukan nilai suhu dalam skala Celcius dengan menggunakan invers!

a. $f(x) = \frac{9}{5}x + 32$, dimana $f(x) = y$

$y = \frac{9}{5}x + 32$ $y - 32 = \frac{9}{5}x$ $x = \frac{5}{9}(y-32)$.

b. $x = \frac{5}{9}(86-32)$ $x = \frac{5}{9}(54)$

$x = 30$. Jadi, 86°F = 30°C.

English Version Answered S-S2 :

1. ***Solution (Peny):***

   $(f \circ g)(x) = f(g(x)) = f(x^2) = 2(x^2) + 3 = 2x^2 + 3$

   $(g \circ f)(x) = g(f(x)) = g(2x + 3) = (2x + 3)^2 = 4x + 12x + 9$

   The Difference (Perbedaannya):

   "f (g(x)) means function f operates after g".

   "g (f(x)) means function g operates after f".

2. *a. Graph : A straight line with gradient 1, intersecting the Y-axis at (0, 1).*

   b. Finding the invers function :

   "Find invers : $f(x) = x + 1 \leftrightarrow y = x - 1 \leftrightarrow x = y - 1$

   $f^{-1}(x) = x - 1$, *the inverse graph is a straight line* $f^{-1}(x) = x - 1$

3\. a. $f^{-1}(x) = \frac{x-1}{2}$ $y = \frac{x-1}{2}$ $x - 1 = 2y$ $x = 2y + 1$

   b. Proof that $f(f^{-1}(x)) = x$

   $f(f^{1}(x)) = x$

   $f(2x + 1) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$

4.a. Finding the inverse & meaning :

   $f(x) = \frac{9}{5}x + 32$, *where* $f(x) = y$

   $y = \frac{9}{5}x + 32$

   $y - 32 = \frac{9}{5}x$

   $x = \frac{9}{5}(y - 32)$

   b. $x = \frac{5}{9}(86 - 32)$

   $x = \frac{5}{9}(54)$

   $x = 30$

   So, 86 °F = 30 °C

Figure 4. Results of Written Test for S-S2 Students

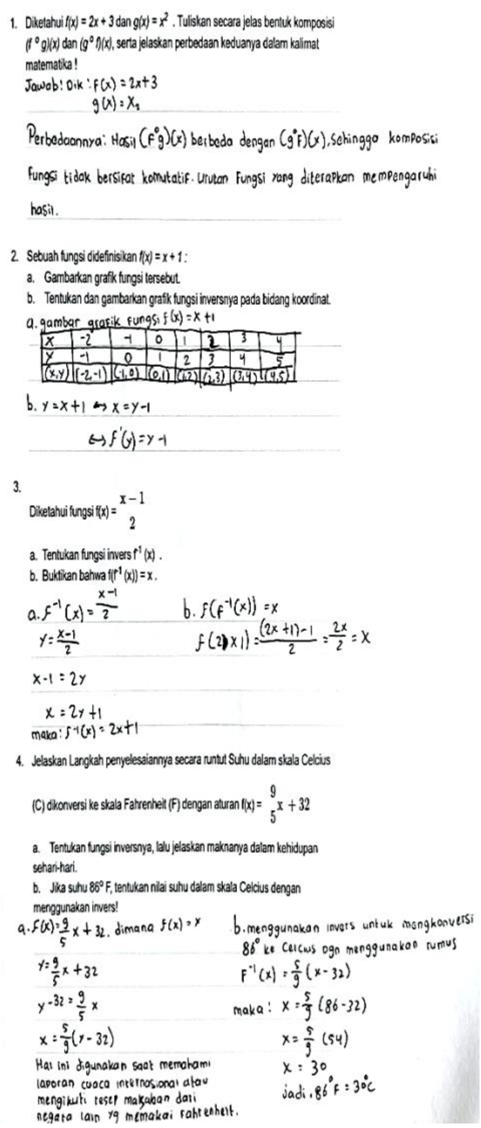
The Medium Group represented the most unique cognitive dynamics. The graph shows their Math Expression achievement was quite high (85.18), but they experienced a decline in the Drawing (79.09) and Written Text (73.36) aspects. The test results indicated a clash of strategies between S-S1 and S-S2 in dealing with time constraints. The Drawing vs. Math Expression aspect (Case S-S1) displayed a coherent mastery of algebra in Questions 1, 2, and 3. However, in the Drawing aspect, he engaged in extreme graphical reduction. He drew a Cartesian plane without including precise coordinate scale numbers. In the interview, S-S1 admitted:

"I know how to calculate it, so I just sketched the picture quickly, as long as the slope was correct. However, I was still confused about how to graph the inverse function, and time was tight, so I just drew the lines by guesswork."

Cognitive Diversion Strategy (Case S-S2) demonstrated a total anomaly. The Lagging Work Cycle Barrier (Case S-S2) showed the reverse pattern; his algebraic calculations from number 1 to 3 were neatly patterned, but he also completely emptied the Drawing aspect and explanation of contextual meaning in Question No. 4. S-R2 failed to complete the final instructions in Question No. 4. S-S2 confirmed that the time ran out because he was too focused on verifying the calculations in the initial numbers due to a lack of confidence.

In learning activities, the Medium Group often exhibits the phenomenon of cognitive trade-off. When faced with a written test deadline, medium-ability students will choose one of the representation domains they are most proficient in (pure algebra, verbal-analytical, or visual / pictorial) and deliberately sacrifice the other representation domains to ensure work efficiency.

Analysis of Mathematical Communication Ability of Low-Ability Students (S-R1, S-R2



1. Diketahui $f(x) = 2x + 3$ dan $g(x) = x^2$. Tuliskan secara jelas bentuk komposisi $(f \circ g)(x)$ dan $(g \circ f)(x)$, serta jelaskan perbedaan keduanya dalam kalimat matematika!

Jawab: Dik: $f(x) = 2x+3$
$g(x) = x^2$
Perbedaannya: Hasil $(f \circ g)(x)$ berbeda dengan $(g \circ f)(x)$, sehingga komposisi fungsi tidak bersifat komutatif. Urutan fungsi yang diterapkan mempengaruhi hasil.

2. Sebuah fungsi didefinisikan $f(x) = x + 1$:
a. Gambarkan grafik fungsi tersebut.
b. Tentukan dan gambarkan grafik fungsi inversnya pada bidang koordinat.

a. gambar grafik fungsi $f(x) = x+1$

x	-2	-1	0	1	2	3	4
y	-1	0	1	2	3	4	5
(x,y)	(-2,-1)	(-1,0)	(0,1)	(1,2)	(2,3)	(3,4)	(4,5)

b. $y = x+1 \leftrightarrow x = y-1$
$\leftrightarrow f^{-1}(y) = y-1$

3. Diketahui fungsi $f(x) = \frac{x-1}{2}$
a. Tentukan fungsi invers $f^{-1}(x)$.
b. Buktikan bahwa $f(f^{-1}(x)) = x$.

a. $f^{-1}(x) = \frac{x-1}{2}$
$y = \frac{x-1}{2}$
$x - 1 = 2y$
$x = 2y + 1$
maka: $f^{-1}(x) = 2x+1$

b. $f(f^{-1}(x)) = x$
$f(2x+1) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$

4. Jelaskan Langkah penyelesaiannya secara runtut Suhu dalam skala Celcius (C) dikonversi ke skala Fahrenheit (F) dengan aturan $f(x) = \frac{9}{5}x + 32$
a. Tentukan fungsi inversnya, lalu jelaskan maknanya dalam kehidupan sehari-hari.
b. Jika suhu 86° F, tentukan nilai suhu dalam skala Celcius dengan menggunakan invers!

a. $f(x) = \frac{9}{5}x + 32$, dimana $f(x) = y$
$y = \frac{9}{5}x + 32$
$y - 32 = \frac{9}{5}x$
$x = \frac{5}{9}(y - 32)$
Hal ini digunakan saat memahami laporan cuaca internasional atau mengikuti resep makanan dari negara lain yg memakai Fahrenheit.

b. menggunakan invers untuk mengkonversi 86° ke Celcius dgn menggunakan rumus
$F^{-1}(x) = \frac{5}{9}(x-32)$
maka: $x = \frac{5}{9}(86-32)$
$x = \frac{5}{9}(54)$
$x = 30$
Jadi, $86°F = 30°C$

English Version Answered S-R1 :

1. ***Solution (Peny):***
   $f(x) = 2x^2 + 3$
   $g(x) = x^2$
   The Difference (Perbedaannya):
   "The result of $(f \circ g)(x)$ is different from $(g \circ f)(x)$. Therefore, function composition is not commutative. The order of functions applied affects the final result."
2. *a. Coordinate Table for $f(x) = x + 1$:*

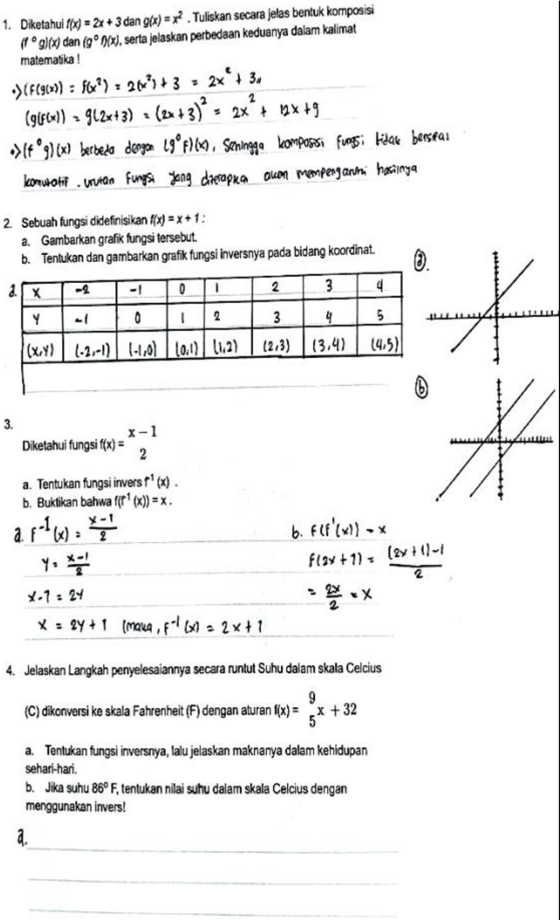
x	-2	-1	0	1	2	3
y	-1	0	1	2	3	4
(x, y)	(-2,-1)	(-1,0)	(0,1)	(1,2)	(2,3)	(3,4)

b. Finding the invers function :
$y = x + 1 \leftrightarrow x = y - 1$
$\leftrightarrow f^{-1}(y) = y - 1$

3. a. Finding $f^{-1}(x)$:
$f^{-1}(x) = \frac{x-1}{2}$
$y = \frac{x-1}{2}$
$x - 1 = 2y$
$x = 2y + 1$, *therefore*, $f^{-1}(x) = 2x + 1$
b. Proof that $f(f^{-1}(x)) = x$
$f(f^{1}(x)) = x$
$f(2x + 1) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$

4.a. Finding the inverse & meaning :
$f(x) = \frac{9}{5}x + 32$, *where* $f(x) = y$
$y = \frac{9}{5}x + 32$
$y - 32 = \frac{9}{5}x$
$x = \frac{9}{5}(y - 32)$
"This is used when understanding international weather reports or following cooking recipes from other countries that use Fahrenheit."
b. "Using the inverse to convert 86° to Celsius by using the formula $f^{-1}(x) = \frac{5}{9}(x - 32)$*"*
Then : $x = \frac{5}{9}(86 - 32)$
$x = \frac{5}{9}(54)$
$x = 30$
So, 86 °F = 30 °C

Figure 5. Results of Written Test for S-R1 Students



English Version Answered S-R2 :

1. ***Solution (Peny):***

   $f(g(x)) = f(x^2) = 2(x^2) + 3 = 2x^2 + 3$

   $g(f(x)) = g\ (2x + 3) = (2x + 3)^2 = 2x^2 + 12x + 9$

   The Difference (Perbedaannya):

   " $(f \circ g)(x)$ is different from $(g \circ f)(x)$. Therefore, function composition is not commutative. The order of functions applied will affects the final result."

2. *a. Coordinate Table for $f(x) = x + 1$:*

x	-2	-1	0	1	2	3
y	-1	0	1	2	3	4
(x, y)	(-2,-1)	(-1,0)	(0,1)	(1,2)	(2,3)	(3,4)

   b. Represented by graph, but there is not inverse functions .

3. *a. Finding $f^{-1}(x)$:*

   $f^{-1}(x) = \frac{x-1}{2}$

   $y = \frac{x-1}{2}$

   $x - 1 = 2y$

   $x = 2y + 1$ *, therefore,* $f^{-1}(x) = 2x + 1$

   b. Proof that $f(f^{-1}(x)) = x$

   $f(f^{1}(x)) = x$

   $f(2x + 1) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$

4. *(Note : The student not answer).*

Figure 6. Results of Written Test for S-R2 Students

Students in the Low Group (S-R1 and S-R2) experienced the most severe communication barriers in the Math Expression aspect, with the lowest average score of 55. The main characteristics of this group were susceptibility to basic algebraic procedural errors (careless errors) and an inability to complete all tasks (incomplete tasks). In the Math Expression & Drawing aspect (Case S-R1), students actually had quite good Math expression skills. However, their cognition was fragile in symbolic manipulation. In Problem No. 1, they made a fatal error in the expansion of (f o g)(x) and (g o f)(x) by simply writing them as functions f(x) and g(x). During the interview, S-R1 expressed his anxiety:

"I was nervous because I had little time left, so I just wrote the function quickly and forgot that the function should be composed."

The student S-R2 completely left out the formula for finding the inverse of a function and drew a graph without a scale (Question No. 2b), and did not allocate it to the Written Text aspect by writing a verbal-analytical description (Question No. 4): Through a clinical interview, S-R2 revealed the reason behind this action:

"I still don't understand how to work on inverse function problems, so I drew a manual drawing that resulted in a deviation because I didn't understand it yet."

Discussion

This descriptive qualitative research successfully uncovered several crucial gaps between conventional learning theories and real cognitive facts in the field when the Discovery Learning model was integrated with the Quizizz gamification media. Through triangulation of macro data (achievement graphs), micro evidence (worksheets), and psychological confirmation (in-depth interviews), two main theoretical gaps were found, which became novel in this research. First, in the discourse of mathematical cognitive development, Jerome Bruner's Representation Theory states that concept mastery progresses in a linear-hierarchical manner from the Enactive (action), Iconic (visual/image), to Symbolic (abstract language/algebra) stages. The majority of literature agrees that image visualization (Drawing) acts as a compulsory bridge that facilitates abstract algebraic understanding (Radford, 2014; Stylianou, 2020). However, the results of this study found a theoretical anomaly in students in the Medium Prior Mathematical Ability (PMA) group, where S-S2 students extremely blanked out the graphic aspect (Drawing) in Question No. 2, but instantly replaced it with a high-level verbal-analytical description: “A straight line with a gradient of 1, intersects the Y-axis at (0,1)”. When confirmed through interviews, S-S2 viewed manual drawings on paper as speculative and inefficient, while text-based arguments and gradients were much more representative of his understanding. This phenomenon sparked a new scientific debate that challenged the rigid nature of Bruner’s stages. S-S2 was shown to have performed Cognitive Bypass, where a student is able to skip the iconic (image) phase and directly operate ideas in the symbolic-verbal realm without losing their conceptual essence.

This is in line with modern criticism from Drijvers et al. (2016), who stated that exposure to technology and digital environments changes the visualization structure of new generation students. Students no longer need physical-manual visualization on paper to understand spatial, but have formed mental images that are analytical-symbolic in their cognition. Mathematical communication skills are referred to as highly abstract operational discourse (Sfard, 2008). This gap shows that high algebraic abilities (Math Expression) and verbal analysis (Written Text) can actually degrade students' motivation to present visual representations (Drawing) if they perceive the manual drawing motor process as consuming the allocated test time.

Second, most international research on gamification (such as Quizizz or Kahoot) within the Discovery Learning framework consistently reports positive impacts on motivation, conceptual understanding, and reduced math anxiety (Zainuddin et al., 2020). The majority of literature claims that instant, automated feedback improves students' overall academic performance. However, this study showed contradictory findings in the Low Group, where their Math Expression score fell tragically to 55.

Through triangulation, S-R1's worksheet showed a fatal careless error in basic algebraic operations by not operating the composition of functions (f o g)(x) and (g o f)(x). The results of in-depth interviews revealed its psychological roots: S-R1 experienced cognitive panic due to time-constrained anxiety running on the classroom whiteboard. Meanwhile, S-R2 experienced an incomplete task cycle where he left the total explanatory text blank at the end of the page because he ran out of time due to taking too long to correct the algebraic calculations in the previous number. This is where the dark side of gamification lies, where the Quizizz game mechanism that relies on a time-attack system (higher scores for faster answers) trains students' cognitive aspects impulsively. According to a study by Attali & Arieli-Attali (2015), constant time pressure in a digital gamification environment can change the cognitive style of lower-group students to be very impulsive. The Quizizz feature successfully helped their short-term memory to understand theoretical definitions in the Written Text aspect (e.g., understanding the meaning of the composition of non-commutative functions. However, the time speed they usually experience in Quizizz destroys their cognitive endurance when forced to perform long, manual algebraic calculations and requires step-by-step algorithmic precision. As a result, low-group students are prone to sudden math anxiety under the time pressure of manual tests, triggering loss of concentration and causing careless errors in basic symbolic expansions.

Conclusions and Suggestions

Based on the results of research and discussion regarding the profile of students' mathematical communication abilities through the Discovery Learning approach assisted by Quizizz in terms of Prior Mathematical Abilities (PMA), three main conclusions can be drawn: The High Prior Mathematic Ability Group (High Group) shows superior and established mastery of inter-representation. The main characteristic of this group is structured metacognitive ability, where they are able to utilize coordinate tables as visual guides to produce precise algebraic calculations (Math Expression) and textual descriptions (Written Text) without being disturbed by time constraints. The Medium Prior Mathematic Ability Group (Medium Group) gives rise to a novel phenomenon in the form of Cognitive Bypass. When faced with limited test time, this group carries out a cognitive trade-off strategy by deliberately reducing or emptying manual visual representations (Drawing) or Math expressions on paper, then jumping directly to encoding geometric properties into verbal-analytical language (Written Text), which they consider more efficient and has theoretical certainty. The Low Prior Mathematic Ability Group (Low Group) experiences the Digital Gamification Paradox. On the one hand, the immediate feedback feature on Quizizz successfully strengthened their short-term memory to understand basic verbal concepts (written text). However, on the other hand, the time-attack mechanism on Quizizz fostered an impulsive cognitive style. As a result, when they switched to manual written tests that required long-term focus, they experienced procedural anxiety, which led to careless errors in basic algebra and incomplete tasks.

Based on the above findings, the researcher proposed several suggestions, namely that teachers are advised to provide more intensive gradual assistance (scaffolding) at the data processing stage for low-ability students, so that they do not fall behind in the concept discovery phase. Teachers are also advised not to impose the same Quizizz time limit on all students. Specifically for the lower group, the time limit for completing each question in the app should be relaxed (or asynchronous/homework mode should be used) to reduce procedural anxiety and train their algorithmic accuracy before the paper-and-pencil written test. Further research can expand the scope of this qualitative analysis by reviewing the mathematical communication profile from the perspective of gender differences or students' learning styles in order to obtain a more comprehensive picture. Given that this study is descriptive-qualitative, it is recommended for future researchers to examine mathematical communication skills in other mathematical materials or use different research approaches, such as quantitative research or mixed methods, in order to obtain a more comprehensive picture of the influence of learning models on students' mathematical communication skills. In addition, students are expected to be more actively involved in the learning process and get used to communicating mathematical ideas in writing and orally so that mathematical communication skills can develop optimally.

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