

Received: 30 August 2025.

Revised: 23 November 2025.

Accepted: 16 December 2025.



# Conceptual Rigor of AI-Generated Mathematical Explanations: The Case of Vector Functions

Enny Listiawati<sup>1\*</sup>, Hendra Kartika<sup>2</sup>, Çiğdem Arslan<sup>3</sup>

<sup>1</sup> Department of Mathematics Education, STKIP PGRI Bangkalan, Indonesia

<sup>2</sup> Department of Mathematics Education, Universitas Singaperbangsa Karawang, Indonesia

<sup>3</sup> Mathematics Education Department, Bursa Uludağ University, Turkey

## Abstract

**Purpose:** The rapid rise of generative artificial intelligence has reshaped discussions in mathematics education, particularly regarding the capacity of advanced systems such as ChatGPT and Gemini to support conceptual rigor. This study aims to investigate how these generative AI tools define and explain vector functions, including the procedures for differentiating and integrating them, in order to evaluate their conceptual rigor of ai-generated mathematical explanations and pedagogical potential. **Methodology:** Employing a qualitative case study design, the research analyzed responses generated by ChatGPT and Gemini to a structured mathematical prompt on vector functions. The explanations were compared with authoritative calculus textbooks using qualitative content analysis and a standardized scoring rubric. **Findings:** Findings reveal that both systems provide broadly accurate introductory descriptions of vector functions, highlighting their component-wise structure. However, notable gaps emerge in mathematical precision, particularly in specifying domains, ranges, and the formal conditions underlying differentiability and integrability. ChatGPT tends to include intuitive geometric interpretations, whereas Gemini provides concise procedural explanations, yet both models lack the rigorous logical framing found in standard mathematical texts. Despite these limitations, the systems demonstrate consistent procedural accuracy in describing differentiation and integration of vector-valued functions. **Significance:** The results underscore the educational potential of generative AI while highlighting the need for teachers to critically evaluate AI-generated mathematical content, particularly when these tools are used to support students' conceptual learning in mathematics. These findings also highlight important implications for AI literacy, instructional design, and future research in mathematics education.

**Keywords:** ChatGPT, Conceptual rigor, Generative artificial intelligence, Gemini, Mathematics education, Qualitative analysis, Vector functions.



© 2025 by the authors. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

\* Corresponding author: Enny Listiawati, [ennylistiawati@stkip PGRI-bkl.ac.id](mailto:ennylistiawati@stkip PGRI-bkl.ac.id)

## Introduction

The mathematical concept of a function is commonly defined as a relationship among two or more variables that can be represented graphically (Del Cerro Velázquez & Méndez, 2021). As a foundational construct, the function serves as a unifying idea both within mathematics and in its connections to real-world phenomena. Extensive scholarship over the past five decades has examined how functions are introduced and learned (Dubinsky & Wilson, 2012), consistently emphasizing that a deep understanding of the function concept is essential for mastering calculus (Bardini et al., 2014).

Calculus occupies a central position in the mathematical sciences due to its rich internal structure and its numerous interdisciplinary applications. Its principles illustrate the versatility and explanatory power of mathematics in fields such as engineering, physics, and economics (Rodríguez-Nieto & Moll, 2025; Yu & Cheng, 2023). Key conceptual connections within calculus include the inverse relationship between differentiation and integration, which is formalized in the Fundamental Theorem of Calculus (García-García & Dolores-Flores, 2019). Furthermore, vector calculus extends single-variable calculus to multivariable contexts, linking vector fields, line and surface integrals, curl, divergence, and curvature (Borji et al., 2024). These connections underscore the importance of strong conceptual foundations for advanced mathematical study.

Despite its centrality, the function concept continues to present challenges for learners at all levels. Prior research has documented persistent difficulties and inconsistencies in students' conceptions of functions, both within their own reasoning and in relation to formal definitions (Harel & Dubinsky, 1992; Carlson, 1998; Oehrtman et al., 2008). Undergraduate students encounter functions repeatedly across courses—from calculus and linear algebra to differential equations and abstract algebra—yet such encounters do not always lead to deeper conceptual understanding (Zandieh et al., 2016). Studies conducted at the upper secondary, tertiary, and teacher-education levels have revealed a range of misconceptions related to functions (Viirman, 2014). Related work also highlights challenges in learning vector-related concepts, including limited conceptual understanding and difficulties with vector and scalar product procedures (Tairab et al., 2020). Nevertheless, research examining conceptions of vector functions generated by generative AI remains extremely limited, even as the use of such tools by students and teachers continues to grow.

The past year has witnessed remarkable advancements in artificial intelligence (AI), accompanied by its unprecedented influence on human creativity, productivity, and knowledge generation (Ali et al., 2023; Badshah et al., 2023). A pivotal milestone in this trajectory was the release of ChatGPT, a Generative Pre-Trained Transformer (GPT) developed by OpenAI, in November 2022, which significantly reshaped global discourse on generative AI (genAI) capabilities and limitations (Tlili et al., 2023). Soon after, Google DeepMind introduced Gemini on 06 December 2023, a state-of-the-art multimodal AI model built on Visual Language Model (VLM) technology and positioned as a direct competitor to ChatGPT, GPT-4, and GPT-4V (Coles, 2023; Perera & Lankathilaka, 2023). Gemini has demonstrated considerable utility in domains such as reinforcement learning, deep learning, and digital education, further expanding the scope of genAI applications (Imran & Almusharraf, 2024).

The integration of AI technologies into education has accelerated substantially over the past two decades, with adaptive learning platforms, automated assessment tools, predictive analytics, and virtual tutoring systems emerging as transformative innovations (Wong, 2025). These AI-driven systems promise to enhance personalization, improve access to high-quality learning experiences, and support greater global collaboration (Luckin & Holmes, 2016). However, their rapid adoption also raises critical ques-

tions regarding their pedagogical, ethical, and epistemological implications, underscoring the need for rigorous inquiry into both their potential benefits and inherent challenges (Lebovits, 2018; Varsik & Vosberg, 2024). As genAI tools continue to gain widespread traction, closer examination of their educational affordances—both within general education and within specific disciplinary contexts—has become increasingly essential (Yoon et al., 2024).

Recent studies indicate that genAI tools can support various educational purposes, including teacher-focused, student-focused, and text-focused tasks (Daher & Anabousy, 2025). The present study focuses specifically on the capacity of genAI systems to assist teachers in the design and construction of lessons, with particular attention to their ability to articulate conceptions of vector functions. To this end, we examine two leading genAI platforms—ChatGPT and Gemini. Although these tools have recently attracted considerable scholarly attention in educational research (Ali et al., 2023; Pan et al., 2023; Ram et al., 2023), limited work has investigated their conceptual competence, particularly in explaining mathematical definitions and conceptual structures. Addressing this gap, the present study adopts an exploratory approach to analyze how ChatGPT and Gemini define, explain, and engage with the concept of vector functions, and to identify the factors shaping the nature and quality of their conceptual output.

Building on the theoretical and empirical gaps highlighted in the Introduction, this study is guided by the following overarching research question: How do generative AI systems—specifically ChatGPT and Gemini—conceptually define vector functions and explain the procedures for differentiating and integrating them, when evaluated against authoritative mathematical references? This inquiry is further elaborated into two sub-questions: (1) What conceptual characteristics appear in AI-generated explanations of vector functions? and (2) How do these systems describe the differentiation and integration of vector-valued functions, and to what extent do these descriptions demonstrate mathematical rigor? These questions provide a coherent analytical frame for examining the depth, accuracy, and pedagogical value of AI-generated mathematical explanations.

## Methods

This study employed a qualitative case study design. A qualitative case study offers a robust methodological framework that facilitates an in-depth, contextually grounded investigation of contemporary phenomena using multiple sources of evidence, thereby enabling analysis from diverse theoretical and analytical perspectives (Baxter & Jack, 2008; Mtisi, 2022). As emphasized by Yin (2018), case studies are particularly suited to addressing *how* and *why* questions, allowing the researcher to generate explanatory and evaluative insights. Thus, this design is well aligned with the aim of the present study, namely to evaluate ChatGPT's and Gemini's conceptualizations of vector functions.

The subjects of this study were ChatGPT and Gemini, selected for their prominence and rapidly expanding influence across educational and professional domains. ChatGPT, in particular, has become a focal point in contemporary educational discourse (Delima et al., 2024). Since its public release on 30 November 2022, interest in leveraging ChatGPT within educational contexts has increased substantially, driven by its demonstrated pedagogical potential (Daher & Gierdien, 2024). Both ChatGPT and Gemini represent state-of-the-art generative AI systems distinguished by extensive user bases and wide-ranging applications across multiple disciplinary fields (Colaco & Antao, 2025).

Data were collected by inputting a prompt into both chatbots. The prompt was adapted from Stewart (2012) and consisted of questions concerning the definition of a vector function and the proce-

dures for determining its derivative and integral. This prompt was selected because it offers a discipline-validated and conceptually grounded instrument for eliciting participants' understanding of vector functions. The two questions are aligned with the "Understand" level of the Revised Bloom's Taxonomy (Anderson & Krathwohl, 2001), as they require participants to *explain*, *interpret*, and *summarize* the definition of a vector function and the procedures for determining its derivative and integral. These items therefore capture essential conceptual and procedural indicators while maintaining methodological focus. The brevity and open-ended design of the prompt promote the articulation of participants' reasoning without constraints, and its grounding in an authoritative calculus source strengthens methodological rigor, transparency, and replicability. The prompt used in this study is presented in Table 1. The responses generated by each chatbot were captured using screenshots and stored as image files.

**Table 1.**

*Question prompt*

Indicator	Question
Understanding of the definition of a vector function	What is a vector function?
Understanding of the methods for finding the derivative and integral of a vector function	How do you find its derivative and its integral?

A qualitative content analysis was conducted to examine the conceptual understanding and completeness of the responses. The outputs were compared against two authoritative calculus textbooks: Stewart (2012) and Rogawski et al. (2019). Conceptual alignment was additionally evaluated using established learning theories and models. To assess response quality and reliability, the study adopted a standardized scoring rubric proposed by Ozdemir and Ozdemir (2025). This rubric evaluates responses on a four-point scale: 1 = entirely incorrect or irrelevant; 2 = partially correct with misleading or inaccurate elements; 3 = generally accurate but insufficient or incomplete; and 4 = fully accurate, comprehensive, and conceptually sound.

To evaluate reproducibility, each prompt was submitted twice from two different computers at different times. Responses identical across devices were classified as "consistent," whereas differing responses were categorized as "inconsistent." In cases of inconsistency, only the first response was scored. Reproducibility rates were analyzed by question category and compared across chatbots.

Ethical approval was not required as the study did not involve human participants. Although broader ethical issues associated with AI—such as privacy, fairness, non-discrimination, and transparency—are frequently highlighted in the literature (Cotton et al., 2023; Mhlanga, 2023), the procedures employed in this study adhered to established ethical standards and did not infringe upon any ethical guidelines (Baytak, 2024).

## Results and Discussions

In this study, the concept of a function is defined as follows. Let  $D$  and  $S$  be two nonempty subsets of the real numbers. A function from  $D$  to  $S$  is a rule that assigns exactly one element of  $S$  to each element of  $D$ . This requirement of assigning a unique output to every input is commonly referred to as

the one-valuedness property of a function. The sets  $D$  and  $S$  are termed the *domain* and *codomain*, respectively (Borke, 2021).

This section addresses the first research question: *What is a vector function?* An initial analysis of responses generated by ChatGPT and Gemini indicates that both systems provide conceptually aligned explanations, particularly in their characterization of vector functions as mappings whose outputs are vector-valued quantities. Although minor differences appear in phrasing and presentation, both responses consistently communicate that a vector function assigns a vector to each value in its domain. The detailed outputs from ChatGPT and Gemini, together with definitions drawn from authoritative mathematics textbooks, are summarized in Table 2.

**Table 2**  
*Definitions of Vector Functions from Multiple Sources*

Source	Explanation
ChatGPT	<p>A <b>vector function</b> is a function whose output is a <b>vector</b>, usually written in terms of a parameter (often <math>t</math>). It describes curves in space (2D or 3D).</p> <p><b>1. What is a vector function?</b></p> <p>A vector function is typically written as:</p> $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ <p>or in 2D:</p> $\mathbf{r}(t) = \langle x(t), y(t) \rangle.$ <p>Each component is a scalar function of <math>t</math>.</p> <p><b>Example:</b></p> $\mathbf{r}(t) = \langle t^2, \sin t, e^t \rangle$
Gemini	<p>◆ A <b>vector function</b> (or <b>vector-valued function</b>) is a mathematical function of a single real variable, often denoted as <math>t</math> (representing time or a parameter), whose output is a <b>vector</b>.</p> <p>In three-dimensional space, a vector function <math>\mathbf{r}(t)</math> can be written in terms of its component functions, <math>f(t)</math>, <math>g(t)</math>, and <math>h(t)</math>:</p> $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ <p>The variable <math>t</math> is the input (a scalar), and the output is a three-dimensional vector. As <math>t</math> changes, the tips of the vectors trace out a <b>space curve</b>.</p>
Stewart (2012)	<p>“A vector function is a function whose domain is a set of real numbers and whose range is a set of vectors.”</p>
Rogawski et al. (2019)	<p>“A vector-valued function is any function <math>\mathbf{r}(t)</math> of the form in <math>\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}</math> whose domain <math>\mathcal{D}</math> is a set of real numbers and whose range is a set of position vectors. The variable <math>t</math> is called a parameter, and the functions <math>x(t)</math>, <math>y(t)</math>, <math>z(t)</math> are called the components or coordinate functions. We usually take as domain the set of all values of <math>t</math> for which <math>\mathbf{r}(t)</math> is defined—that is, all values of <math>t</math> that belong to the domains of all three coordinate func-</p>

tions  $x(t)$ ,  $y(t)$ ,  $z(t)$ .”

Table 2 presents a comparative overview of definitions of vector functions derived from generative AI models (ChatGPT and Gemini) alongside established mathematical references, namely Stewart (2012) and Rogawski et al. (2019). A critical examination reveals a substantial disparity in the level of precision, completeness, and formal rigor across these sources. While the textbook definitions offer comprehensive and mathematically robust descriptions, the AI-generated explanations remain noticeably limited in scope.

The definitions provided by Stewart (2012) and Rogawski et al. (2019) clearly articulate the essential properties of vector-valued functions, emphasizing that such a function maps a set of real numbers (its domain) to a corresponding set of vectors (its range). Rogawski et al. further extend this formalism by specifying the structure of a vector function in terms of its coordinate functions  $x(t)$ ,  $y(t)$ , and  $z(t)$ , the role of the parameter  $t$ , and the requirement that all component functions share a common domain. These features reflect the mathematical rigor expected in standard calculus and analysis literature.

In contrast, the definitions generated by ChatGPT and Gemini are generally accurate at a conceptual level but remain insufficient and incomplete for scholarly use. As previously noted, neither model explicitly states the fundamental mapping relationship from real numbers to vectors, nor do they identify the domain or range with the precision required in mathematical discourse. The omission of these foundational elements results in definitions that lack the structural clarity and formal exactness present in authoritative sources. This indicates that, although AI-driven explanations may serve as accessible introductory descriptions for non-expert audiences, they currently fall short of the definitional standards demanded in academic mathematics.

Table 3 presents the explanations provided by ChatGPT, Gemini, and a standard textbook regarding the derivative and integral of vector functions. This table addresses the second research question—*How do you find its derivative and its integral?*—by comparing how each source describes the component-wise procedures involved in differentiating and integrating vector-valued functions.

**Table 3**

*Conceptions of Derivative and Integral Vector Functions from Multiple Sources*

Source	Explanation
ChatGPT	<p>“To find the derivative, differentiate each component: <math>r'(t) = \langle x'(t), y'(t), z'(t) \rangle</math>. This derivative gives the tangent vector to the curve.”</p> <p>“Integrate each component: <math>\int r(t)dt = \langle \int x(t)dt, \int y(t)dt, \int z(t)dt \rangle + C</math> where <math>C</math> is a constant vector.”</p>
Gemini	<p>“The derivative of a vector function is found by simply taking the derivative of each of its component functions. The derivative, denoted <math>r'(t)</math> or <math>\frac{dr}{dt}</math>, is itself a vector function.”</p>

---

	“The indefinite or definite integral of a continuous vector function is found by simply taking the integral of each of its component functions.”
Stewart (2012)	“To find the derivative or integral, we can differentiate or integrate each component of the vector function.”
Rogawski et al. (2019)	“ $r(t)$ is differentiable if and only if the components are differentiable. In this case, $r'(t)$ is equal to the vector of derivatives $\langle x'(t), y'(t), z'(t) \rangle$ .”
	“The integral exists if each of the components $x(t), y(t), z(t)$ is integrable.”

---

The findings presented in Table 3 provide important insights into how derivative and integral vector functions are conceptualized across different knowledge sources—specifically, generative AI systems (ChatGPT and Gemini) and established calculus textbooks (Stewart, 2012; Rogawski et al., 2019). The central pattern emerging from this comparison is the overall convergence toward a component-wise interpretation of both differentiation and integration. All four sources adhere to the principle that a vector function can be differentiated or integrated by operating independently on each of its scalar components. This points to the durability and widespread acceptance of the component-wise approach in vector calculus across diverse knowledge systems. However, the deeper analysis reveals substantial differences in epistemic framing, linguistic precision, and theoretical rigor among the sources. These variations provide critical insight into the capabilities and limitations of generative AI in mathematical explanation.

ChatGPT’s descriptions demonstrate a pedagogically oriented tendency, mixing formal procedure with intuitive geometric interpretation. By framing the derivative as providing “the tangent vector to the curve,” ChatGPT adds a layer of conceptual visualization not present in the textbook sources. While such contextualization may aid students’ understanding, it also signals a deviation from the formal definitional style of mathematical exposition. Similarly, the mention of a “constant vector” in integration—though correct—lacks formal justification and is not situated within a theorem-driven framework. Gemini, meanwhile, offers concise and technically correct descriptions that closely resemble standard textbook phrasing. Yet, Gemini also refrains from articulating the logical and conditional structure that determines the existence of a derivative or integral. Its explanations are operationally valid but lack the rigorous conditional statements that characterize formal mathematical definitions. This suggests that Gemini, like ChatGPT, predominantly internalizes procedural patterns rather than the deeper logical architecture of calculus.

In contrast, the textbook sources provide definitions grounded in explicit logical structure. Stewart (2012) maintains a pedagogical yet precise emphasis on component-wise computation, reflecting the didactic design of introductory calculus resources. Rogawski et al. (2019), however, articulate a more theoretically mature perspective, framing differentiability and integrability in terms of necessary and sufficient conditions. Their use of biconditional language (“if and only if”) underscores the logical dependency relationships foundational to rigorous mathematical reasoning.

These contrasts reveal a crucial insight into the role of AI in mathematical understanding. While generative AI systems can reproduce correct procedural knowledge, they often lack the capacity to articulate the formal conditions, logical dependencies, and structural coherence that underpin higher-level mathematical concepts. This discrepancy highlights an epistemic boundary: AI-generated mathematical explanations are useful as complementary learning aids but should not be treated as authoritative sub-

stitutes for human-authored, peer-reviewed mathematical texts.

Qadir (2023) cautions that the use of ChatGPT and other generative AI systems may inadvertently perpetuate biases or disseminate misinformation inherent in their training data. Consequently, human users must exercise critical judgment and systematically verify the accuracy of AI-generated responses (Daher & Gierdien, 2024). Furthermore, Cong-Lem et al. (2024) highlight ChatGPT's capacity to generate personalized practice tasks tailored to students' proficiency levels, thereby providing pedagogical support for both teachers and learners, particularly when students encounter difficulties in understanding specific mathematical concepts.

## Conclusions

The comparison further suggests important implications for educational practice, particularly for teachers who increasingly encounter AI-generated explanations in mathematics instruction. As AI-based tools become more prevalent in learning environments, educators must remain attentive to the risk that students may uncritically accept AI-generated content that appears procedurally correct yet lacks conceptual rigor. Such over-reliance may obscure underlying theoretical inaccuracies and foster superficial rather than structural understanding of mathematical ideas. Consequently, teachers need to develop the capacity to critically evaluate AI-produced explanations, identify conceptual omissions or inconsistencies, and guide students toward more coherent and mathematically sound reasoning.

The findings also underscore broader implications for future research in AI literacy and mathematics education. Further investigation is needed to examine how students interpret and evaluate AI-generated mathematical content, how reliably AI systems can produce higher-level mathematical arguments, and what forms of pedagogical scaffolding best support critical engagement with AI outputs. As generative AI capabilities continue to evolve, a nuanced understanding of their epistemic strengths and limitations will be essential for ensuring ethical, effective, and pedagogically robust integration into mathematics teaching, learning, and scholarly practice.

## Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

## References

- Ali, A. H., Alajanbi, M., Yaseen, M. G., & Abed, S. A. (2023). ChatGPT 4, DALL·E, Bard, Claude, BERT: Open Possibilities. *Babylonian Journal of Machine Learning*, 2023, 17–18.
- Anderson, L. W., & Krathwohl, D. R. (2001). *Taxonomy for learning, teaching, and assessing: A revision Bloom's Taxonomy of educational objectives*. Addison Wesley Longman.
- Badshah, A., Rehman, G. U., Farman, H., Ghani, A., Sultan, S., Zubair, M., & Nasralla, M. M. (2023). Transforming educational institutions: Harnessing the power of internet of things, cloud, and fog computing. *Future Internet*, 15(11), 1-20. <https://doi.org/10.3390/fi15110367>
- Bardini, C., Pierce, R., Vincent, J., & King, D. (2014). Undergraduate mathematics students' understanding of the concept of function. *Journal on Mathematics Education*, 5(2). <https://doi.org/10.22342/jme.5.2.1495.85-107>
- Baxter, P., & Jack, S. (2008). Qualitative Case Study Methodology: Study Design and Implementation for Novice Researchers. *The Qualitative Report*, 13(4), 544-559. <https://doi.org/10.46743/2160-3715/2008.1573>

- Baytak, A. (2024). The content analysis of the lesson plans created by ChatGPT and Google Gemini. *Research in Social Sciences and Technology*, 9(1), 329-350. <https://doi.org/10.46303/ressat.2024.19>
- Borji, V., Martínez-Planell, R., & Trigueros, M. (2024). Students' geometric understanding of partial derivatives and the locally linear approach. *Educational Studies in Mathematics*, 115, 69-91. <https://doi.org/10.1007/s10649-023-10242-z>
- Borke, M. (2021). Student teachers' knowledge of students' difficulties with the concept of function. *LUMAT International Journal on Math Science and Technology Education*, 9(1), 670-695. <https://doi.org/10.31129/lumat.9.1.1661>
- Carlson, M. P. (1998). A cross-sectional investigation of the development of the function concept. In A. H. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), *CBMS issues in mathematics education: Research in Collegiate Mathematics Education* (Vol. 3, pp. 114–162). Providence: American Mathematical Society.
- Colaco, D. M., & Antao, D. (2025). Exploring self-learning efficacy: a comparative analysis of ChatGPT and Gemini. *Interactive Technology and Smart Education*, 22(4), 688-698. <https://doi.org/10.1108/itse-12-2023-0251>
- Coles, G. (2023). Google DeepMind Gemini AI release date: The world's new most powerful language model. PCguide.com. Retrieved December 25, 2023, from <https://www.pcguide.com/apps/google-deepmind-gemini-release-date/>
- Cong-Lem, N., Soyoo, A., & Tsering, D. (2024). A Systematic review of the limitations and associated Opportunities of CHATGPT. *International Journal of Human-Computer Interaction*, 1–16. <https://doi.org/10.1080/10447318.2024.2344142>
- Cotton, D. R., Cotton, P. A., & Shipway, J. R. (2023). Chatting and cheating: Ensuring academic integrity in the era of ChatGPT. *Innovations in Education and Teaching International*. <https://doi.org/10.1080/14703297.2023.2190148>
- Daher, W., & Anabousy, A. A. (2025). The didactical knowledge of generative artificial intelligence tools: The case of writing mathematics lessons. *Eurasia Journal of Mathematics Science and Technology Education*, 21(9), em2691. <https://doi.org/10.29333/ejmste/16769>
- Daher, W., & Gierdien, F. (2024). Use of Language By generative AI Tools in Mathematical Problem Solving: The Case of ChatGPT. *African Journal of Research in Mathematics Science and Technology Education*, 28(2), 222–235. <https://doi.org/10.1080/18117295.2024.2384676>
- Davvaz, B. (2023). *Vectors and functions of several variables*. Springer. <https://doi.org/10.1007/978-981-99-2935-1>
- Del Cerro Velázquez, F., & Méndez, G. M. (2021). Application in Augmented Reality for Learning Mathematical Functions: A study for the development of spatial intelligence in secondary education students. *Mathematics*, 9(4), 369. <https://doi.org/10.3390/math9040369>
- Delima, N., Kusuma, D. A., & Paulus, E. (2024). The students' mathematics self-regulated learning and mathematics anxiety based on the use of chat GPT, music, study program, and academic achievement. *Infinity*, 13(2), 349-362. <https://doi.org/10.22460/infinity.v13i2.p349-362>
- Dubinsky, E., & Wilson, R. T. (2012). High school students' understanding of the function concept. *The Journal of Mathematical Behavior*, 32(1), 83–101. <https://doi.org/10.1016/j.jmathb.2012.12.001>
- García-García, J., & Dolores-Flores, C. (2019). Preuniversity students' mathematical connections when sketching the graph of derivative and antiderivative functions. *Mathematics Education Research Journal*, 33, 1-22. <https://doi.org/10.1007/s13394-019-00286-x>
- Harel, G., & Dubinsky, E. (Eds.). (1992). *The concept of function: Aspects of epistemology and pedagogy*. MAA Notes (Vol. 25). Washington, DC: The Mathematical Association of America.
- Imran, M., & Almusharraf, N. (2024). Google Gemini as a next generation AI educational tool: a review of emerging educational technology. *Smart Learning Environments*, 11(1), 1-8. <https://doi.org/10.1186/s40561-024-00310-z>
- Lebovits, H. (2018). Automating Inequality: How High-Tech Tools profile, police, and Punish the poor. *Public Integrity*, 21(4), 448–452. <https://doi.org/10.1080/10999922.2018.1511671>
- Limo, F. A. F., Tiza, D. R. H., Roque, M. M., Herrera, E. E., Murillo, J. P. M., Hualpa, J. J., et al. (2023). Personalized tutoring: ChatGPT as a virtual tutor for personalized learning experiences. *Social Space*, 23(1), 293–312.

- Luckin, R., & Holmes, W. (2016). Intelligence Unleashed: An argument for AI in Education. *Open Research Online (the Open University)*. <https://oro.open.ac.uk/view/person/wh2594.html>.
- Mhlanga, D. (2023). Open AI in education, the responsible and ethical use of ChatGPT towards lifelong learning. *SSRN Electronic Journal*. <https://doi.org/10.2139/SSRN.4354422>
- Mtisi, S. (2022). The Qualitative Case Study Research Strategy as applied on a Rural Enterprise Development Doctoral Research project. *International Journal of Qualitative Methods*, 21. <https://doi.org/10.1177/16094069221145849>
- Oehrtman, M. C., Carlson, M. P., & Thompson, P. W. (2008). Foundational reasoning abilities that promote coherence in students' understandings of function. In M. P. Carlson & C. Rasmussen (Eds.), *Making the connection: Research and practice in undergraduate mathematics* (pp. 150–171). Washington, DC: Mathematical Association of America.
- Ozdemir, M. S., & Ozdemir, Y. E. (2025). Comparison of the performances between ChatGPT and Gemini in answering questions on viral hepatitis. *Scientific Reports*, 15(1), 1712. <https://doi.org/10.1038/s41598-024-83575-1>
- Pan, A., Musheyev, D., Bockelman, D., Loeb, S., & Kabarriti, A. E. (2023). Assessment of artificial intelligence chatbot responses to top searched queries about cancer. *JAMA Oncology*, 9(10), 1437-1440. <https://doi.org/10.1001/jamaoncol.2023.2947>
- Perera, P., & Lankathilaka, M. (2023). Preparing to revolutionize education with the multi-model GenAI tool Google Gemini? A journey towards effective policy making. *Journal of Advances in Education and Philosophy*, 7(8), 246–253.
- Qadir, J. (2023). *Engineering education in the era of ChatGPT: Promise and pitfalls of generative AI for education*. 2023 IEEE Global Engineering Education Conference (EDUCON) (pp. 1–9). Retrieved from <https://doi.org/10.1109/EDUCON54358.2023.10125121>
- Ram, B., & Pratima Verma, P. V. (2023). Artificial intelligence AI-based chatbot study of ChatGPT, Google AI Bard and Baidu AI. *World Journal of Advanced Engineering Technology and Sciences*, 8(1), 258-261. <https://doi.org/10.30574/wjaets.2023.8.1.0045>
- Rodríguez-Nieto, C. A., & Moll, V. F. (2025). Mathematical connections promoted in multivariable calculus' classes and in problems-solving about vectors, partial and directional derivatives, and applications. *Eurasia Journal of Mathematics Science and Technology Education*, 21(4), em2619. <https://doi.org/10.29333/ejmste/16187>
- Rogawski, J., Adams, C., & Franzosa, R. (2019). *Calculus: early transcendentals multivariable*. Macmillan Learning.
- Stewart, J. (2012). *Multivariable Calculus*, Seventh Edition. Cengage Learning.
- Tairab, H., Al Arabi, K., Rabbani, L., & Hamad, S. (2020). Examining grade 11 science students' difficulties in learning about vector operations. *Physics Education*, 55(5), Article 055029. <https://doi.org/10.1088/1361-6552/aba107>
- Tlili, A., Shehata, B., Adarkwah, M. A., Bozkurt, A., Hickey, D. T., Huang, R., & Agyemang, B. (2023). What if the devil is my guardian angel: ChatGPT as a case study of using chatbots in education. *Smart Learning Environments*, 10(1), 1-24. <https://doi.org/10.1186/s40561-023-00237-x>
- Varsik, S., & Vosberg, L. (2024). The potential impact of Artificial Intelligence on equity and inclusion in education. *OECD Artificial Intelligence Papers*, No. 23, OECD Publishing, Paris, <https://doi.org/10.1787/15df715b-en>
- Viirman, O. (2014). *The function concept and university mathematics teaching* (PhD dissertation, Karlstads universitet). Retrieved from <https://urn.kb.se/resolve?urn=urn:nbn:se:kau:diva-31205>
- Wong, J. M. S. (2025). Speculative futures of education: utopian and dystopian scenarios envisioned by ChatGPT, Gemini, and Deepseek. *Discover Education*, 4(1), 1-19. <https://doi.org/10.1007/s44217-025-00692-3>
- Yin, R. K. (2018). *Case Study Research and Applications: Design and Methods* (6th ed.). Sage Publications.
- Yoon, H., Hwang, J., Lee, K., Roh, K. H., & Kwon, O. N. (2024). Students' use of generative artificial intelligence for proving mathematical statements. *ZDM*, 56(7), 1531–1551. <https://doi.org/10.1007/s11858-024-01629-0>
- Yu, C., & Cheng, Y. (2023). Malliavin calculus and its application to robust optimal investment for an insider. *Mathematics*, 11(20), 4378. <https://doi.org/10.3390/math11204378>
- Zandieh, M., Ellis, J., & Rasmussen, C. (2016). A characterization of a unified notion of mathematical function: the case of

high school function and linear transformation. *Educational Studies in Mathematics*, 95(1), 21–38.  
<https://doi.org/10.1007/s10649-016-9737-0>