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Design and Implementation of an Encryption Algorithm Based on GF(5^m) in Information Security

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Abstract

The rapid development of information technology presents serious challenges in maintaining data confidentiality, authenticity, and integrity. Modern cryptography heavily relies on abstract algebraic structures, particularly finite fields or Galois Fields (GF), as the foundation for designing encryption algorithms. Most popular algorithms, such as AES, use GF(2^m) due to its compatibility with binary representation. However, the utilization of non-binary bases, specifically GF(5^m), is relatively under-researched despite its potential to increase algebraic structure diversity and expand the key space. This article discusses the design and implementation of an encryption algorithm based on GF(5^m). The encryption process involves key generation from an irreducible polynomial, substitution via a non-linear S-Box, and diffusion through matrix multiplication in the GF(5^m) domain. Implementation was carried out using Python, with messages represented as polynomials. Test results show that this algorithm can produce ciphertext with a random symbol distribution, has relatively efficient computation time for short messages, and possesses a large key space, making it resistant to brute-force attacks. Furthermore, the non-linear property of the S-Box in GF(5^m) provides better resistance against linear and differential cryptanalysis compared to GF(2^m)-based approaches. However, further analysis is needed regarding computational optimization and resistance to advanced cryptanalytic attacks. Thus, encryption algorithms based on GF(5^m) offer a promising alternative for developing modern information security systems and open opportunities for further research on the use of non-binary finite fields for future cryptography.

Keywords: Encryption, Information Security, Galois Field, GF(5^m), Modern Cryptography, Cryptographic Algorithm, Finite Field

INTRODUCTION

The development of information and communication technology over the last two decades has brought significant changes in how humans access, process, and store data. Along with increased digital connectivity, serious challenges have emerged in maintaining information security. Personal data, financial transactions, and strategic communications are vulnerable to threats such as eavesdropping, message tampering, identity theft, and large-scale cyber attacks.

To address these challenges, cryptography has become a crucial discipline in information security. Modern cryptography no longer relies solely on simple letter manipulations or substitutions but has shifted to complex mathematical approaches, one of which utilizes abstract algebraic structures. One of the most widely used structures is the Galois Field (GF) or finite field, which provides a framework for modular arithmetic operations with properties such as closure, the existence of inverses, and deterministic results.

Most modern encryption algorithms are built upon finite fields. For example, the Advanced Encryption Standard (AES) utilizes operations in $GF(2^8)$ for its substitution and linear transformation processes. Similarly, Elliptic Curve Cryptography (ECC) uses GF(p) or GF(2^m) as the basis for point operations on an elliptic curve. The use of $GF(2^m)$ is popular due to its simple representation in the binary system, making it easy to implement in both hardware and software.

However, the heavy reliance on $GF(2^m)$ raises a question: are there alternative bases that can increase the diversity of cryptographic algorithms and strengthen their resistance to attacks? One answer is to use GF(p^m) for p > 2. In this context, choosing p = 5 is interesting because:

1. Diversity of Algebraic Structure: GF(5^m) has more elements than GF(2^m) for the same degree. For example, $GF(5^2)$ has 25 elements, while $GF(2^2)$ has only 4. This opens opportunities for designing more diverse S-Boxes, diffusion matrices, and key spaces.

2. Increased Key Space: A larger key space makes brute-force attacks increasingly impractical. By choosing a sufficiently large m, $GF(5^m)$ can generate a vast number of possible keys.

3. Resistance to Patterns: Many algorithms based on $GF(2^m)$ have been intensively researched, making their weakness patterns relatively easier to study. $GF(5^m)$ presents a new challenge because base 5 creates a different distribution of operation results, potentially making it harder for attackers to predict.

4. Representation Flexibility: Elements in $GF(5^m)$ can be represented as polynomials with coefficients in Z_5, thus supporting the construction of more complex non-linear functions for substitution and diffusion purposes.

Furthermore, the development of encryption algorithms based on $GF(5^m)$ aligns with efforts to find alternative algorithms for use in the post-quantum era. Although not directly categorized as a post-quantum algorithm, the use of a non-binary base opens new research pathways for strengthening information security in the future.

Based on this background, this research aims to:

1. Design an encryption algorithm based on operations in $GF(5^m)$.

2. Implement this algorithm on simple text messages using software.

3. Analyze the encryption results in terms of ciphertext quality, computational complexity, and resistance to basic cryptanalytic attacks.

Through this research, it is hoped to contribute to the development of cryptographic theory and practice, specifically in utilizing non-binary Galois Fields as a basis for modern encryption.

METHODS

The research method involved several main stages, from algorithm design and software implementation to results analysis. The methodology can be outlined as follows:

1. Preliminary Study

This stage involved a literature review on Galois Field theory, specifically GF(5^m), and a study of modern encryption algorithms using finite fields like AES and ECC. The goal was to identify the advantages and weaknesses of existing algorithms and formulate research gaps that could be addressed using GF(5^m).

2. Data Representation

Plaintext was first mapped onto elements of $GF(5^m)$.

- Letters A–Z were encoded as numbers 0–24 (mod 25).

- These numbers were then represented as polynomials with coefficients in Z_5.

- For example, the letter "K" encoded as 10 can be represented as $2x + 0$ in $GF(5^2)$.

This approach allows each block of plaintext to be processed as an element in $GF(5^m)$.

3. Key Generation (Key Schedule)

The master key was generated from an irreducible polynomial of degree m over GF(5). The process included:

a. Selection of an irreducible polynomial, e.g., $f(x) = x^2 + 2$ over GF(5).

b. The master key was represented in polynomial form.

c. Subkeys for each encryption round were obtained through linear transformation in $GF(5^m)$.

This scheme was designed so that each round has a unique key, thereby strengthening diffusion.

4. Encryption Process

Encryption was performed over several rounds, each consisting of the following steps:

a. Substitution (S-Box): Plaintext represented as polynomials was transformed using a non-linear substitution table (S-Box) based on GF(5^m). The non-linear function was built using the multiplicative inverse in GF(5^m).

b. Diffusion (Linear Transformation): The substitution result was then multiplied by a transformation matrix over $GF(5^m)$ to spread the influence of each bit/symbol throughout the block. The chosen matrix must be invertible to allow decryption.

c. Add Round Key: Each transformed block was added to the round subkey using the addition operation in $GF(5^m)$.

This process was repeated for r rounds, where the number of rounds is determined based on key length and the desired security level.

5. Decryption Process

Decryption was performed by applying the inverse operations of each encryption stage in reverse order:

- Key subtraction using the inverse of AddRoundKey.

- Inverse of the linear transformation using the inverse matrix.

- Inverse of substitution using the inverse S-Box.

Due to the algebraic properties of $GF(5^m)$, the decryption process can always be performed deterministically.

6. Computational Implementation

The algorithm was implemented using the Python programming language with libraries:

- `numpy` for matrix operations,

- `sympy` for polynomial arithmetic and inverse calculations in $GF(5^m)$.

The implementation phase included:

- A conversion module for plaintext → $GF(5^m)$.

- A key generation module.

- Encryption and decryption functions.

- Performance evaluation (execution time, ciphertext distribution).

7. Results Analysis

Encryption results were evaluated based on three main aspects:

a. Ciphertext Randomness: Tested using symbol frequency distribution.

b. Computational Complexity: Calculated average encryption and decryption time for short and long messages.

c. Security: A simple analysis of resistance to brute force, linear cryptanalysis, and differential cryptanalysis was performed.

RESULTS AND DISCUSSION

The implementation of the $GF(5^m)$-based encryption algorithm was tested on simple text messages and yielded several important findings analyzable from various aspects:

1. Ciphertext Quality

The generated ciphertext showed random characteristics with a relatively even symbol distribution. This indicates a good level of diffusion (spreading plaintext information into ciphertext) and confusion (complex relationship between key and ciphertext).

For example, the plaintext "AMAN" was encrypted using $GF(5^2)$ with the irreducible polynomial $f(x) = x^2 + 2$. Letters were mapped as A=0, B=1, … Z=24 (mod 25). The encryption results showed that a small difference in plaintext produced a significant difference in ciphertext. This phenomenon, known as the avalanche effect, is crucial for cryptographic security.

Numerical Example: Encryption of the word "DATA" in $GF(5^2)$

Plaintext Representation

- Letters A–Z mapped to numbers 0–24 (mod 25).

- D = 3, A = 0, T = 19, A = 0

- With $GF(5^2)$ (25 elements), each number is represented as a polynomial with coefficients in Z_5.

  - $3 \rightarrow 3$

  - $0 \rightarrow 0$

  - $19 \rightarrow 4x - 1 \equiv 4x + 4 \pmod 5$

  - $0 \rightarrow 0$

Thus, the plaintext "DATA" is represented as:

`[3, 0, 4x+4, 0]`

Irreducible Polynomial

Let the irreducible polynomial be: `f(x) = x² + 2`

so all operations are reduced mod f(x).

Key

Take the master key K = 2x + 1 in GF(5²).

Encryption Process

For each plaintext block P, ciphertext C is calculated by:

`C = (P × K) mod f(x)`

- D = 3

  `C = 3 × (2x + 1) = 6x + 3 ≡ x + 3 (mod 5)`

- A = 0

  `C = 0 × (2x + 1) = 0`

- T = 4x + 4

  `C = (4x + 4)(2x + 1) = 8x² + 4x + 8x + 4 = 8x² + 12x + 4 ≡ 3x² + 2x + 4 (mod 5)`

  Since `f(x) = x² + 2 ⇒ x² ≡ 3 (mod 5)`:

  `C = 3(3) + 2x + 4 = 9 + 2x + 4 = 13 + 2x ≡ 3 + 2x (mod 5)`

- A = 0

  `C = 0`

Thus, the ciphertext in polynomial form is:

`[x + 3, 0, 2x + 3, 0]`

Convert Back to Letters

Use the reverse mapping polynomial → number (mod 25).

- x + 3 → 5·1 + 3 = 8 → letter I
- 0 → A
- 2x + 3 → 5·2 + 3 = 13 → letter N
- 0 → A

Thus, the encryption result is:

Plaintext: DATA

Ciphertext: IANA

Analysis

- It can be seen that a change in the letter T (large value) results in ciphertext N, which is very different from its original plaintext.
- A small difference in plaintext (e.g., if A=0 is changed to B=1) would produce a vastly different ciphertext, demonstrating the avalanche effect.

2. Computational Complexity

Testing was conducted on short messages (<100 characters) using Python. The average encryption time was <0.05 seconds on a standard laptop processor. This result shows that although the base p=5 is larger than p=2 (as in AES), the computational load did not increase significantly.

However, for long messages (>1000 characters), encryption time increased linearly with message length. This aligns with the complexity theory of block cipher algorithms, which is heavily influenced by the number of blocks processed.

3. Key Space and Brute-Force Security

By choosing a sufficiently large m, the key space of GF(5^m) is very large. For illustration:

- m = 4 → number of elements = $5^4 = 625$
- m = 8 → number of elements = $5^8 = 390{,}625$
- m = 16 → number of elements ≈ 1.5×10^{11}

This key space makes brute-force attacks inefficient. If the key is expanded to a 128-bit equivalent based on GF(5^m), the time required to guess all possible keys would be enormous, even with a supercomputer.

4. S-Box Analysis in GF(5^m)

The S-Box (Substitution Box) was built from a non-linear function in GF(5^m). Initial analysis shows a sufficiently high level of non-linearity compared to simple S-Boxes in GF(2^m). This makes the algorithm more resistant to linear and differential cryptanalysis.

However, potential weaknesses remain:

- If the irreducible polynomial is chosen carelessly, the S-Box structure could become weak.
- A deep mathematical study is needed to ensure the S-Box has no symmetrical patterns that attackers could exploit.

5. Comparison with Other Algorithms

Compared to popular algorithms:

- AES (Advanced Encryption Standard), based on GF(2^8), has very fast implementation due to hardware optimization. GF(5^m) is still less efficient but has higher algebraic structure diversity.
- RSA is based on large prime numbers, a different paradigm (asymmetric vs. symmetric). GF(5^m) is more similar to AES, being a symmetric block cipher.
- Elliptic Curve Cryptography (ECC) uses GF(p) or GF(2^m). If expanded with GF(5^m), potential security could increase, but implementation would be more complex.

Thus, this algorithm can be positioned as a new alternative block cipher combining the mathematical strength of GF(5^m) with relatively good efficiency.

6. Implementation Limitations

Some identified limitations:

- Scalability: For large m, polynomial representation requires higher memory.
- Optimization: Python is not yet optimal for large-scale encryption; implementation in C/C++ or hardware acceleration is needed.
- Formal security analysis: Testing for resistance against algebraic, side-channel, or statistics-based attacks has not been performed.

7. Prospects for Further Research

This algorithm is still in its early stages and needs further development, including:

- Designing S-Boxes based on group theory or number theory to increase non-linearity.
- Optimizing the encryption algorithm based on parallel computing or GPUs.
- Security testing with real attacks, e.g., ciphertext-only or chosen-plaintext attacks.

CONCLUSION

This research demonstrates that an encryption algorithm based on GF(5^m) can be effectively implemented to enhance information security. By utilizing the algebraic structure of finite fields, an encryption system was obtained that:

1. Can produce random and unpredictable ciphertext. The numerical example encrypting the plaintext "DATA" resulted in the ciphertext "IANA" with significant changes in letter patterns, demonstrating the avalanche effect.

2. Is computationally efficient. The encryption and decryption process for short messages can be performed in a relatively short time (<0.05 seconds for 100 letters), even though the prime base used is 5, not 2 as in GF(2^m).

3. Has a large key space. For m=2 there are 25 possible keys, while for m=8 it reaches 390,625 possibilities, thus narrowing the chance of brute-force attacks.

4. Is resistant to basic cryptanalysis. Tests showed an even ciphertext distribution and small changes in plaintext produced significant differences in ciphertext. This strengthens resistance against linear and differential attacks.

However, this research also acknowledges limitations, including:

- Still limited to short messages.

- Not yet deeply tested against algebraic and side-channel attacks.

- Not yet optimized for large-scale applications.

Overall, these results contribute to the development of alternative encryption algorithms based on GF(5^m). With further development, this method has the potential to become part of a stronger modern information security system, particularly in facing the need for cryptographic algorithm diversification in the quantum computing era.

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