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Strengthening Logical Foundation in Mathematics: A Pathway to Enhance Reasoning Skills

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ABSTRACT

Logical thinking serves as the backbone of understanding mathematics which helps learners to connect concepts, justify methods, and construct meaningful explanation. Many students acquire procedural competence to solve a mathematical problem but they remain unable to clearly explain the reasoning that justifies each mathematical step. This gap between performing mathematical problem and explaining solutions undermines conceptual understanding and hinders the development of higher-order reasoning abilities. This research paper addresses this issue by presenting a structured pathway for rebuilding a clear and structured logical framework for mathematics learning. This proposed framework emphasizes on deliberately integration of symbolic logic and formal inference rules into regular mathematical pedagogy. The explicit incorporation of logic within problem-solving practices helps learners to move beyond intuitive thinking toward well-justified reasoning. In support of this objective, this study presents three interrelated approaches: Reasoning Reconstruction Model (RRM), Step–Reason Mapping (SRM) and Logic-Integrated Mathematical Expression (LIME). Together, these approaches support students in structuring their thinking, aligning each solution step to logical rules and rebuilding coherent mathematical arguments.

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INTRODUCTION

Logical reasoning serves the foundation of mathematical understanding as it enables learners to justify their methods, connects concepts and develop valid arguments. Even gaining procedural proficiency, many learners are unable to explain the logical reasoning that justifies each step of the solution. Hanna & de Villers (2012) state that proof is fundamental for fostering learner's explanatory understanding instead of focusing only on procedural skills. Similarly, Stylianides et. al. (2008) emphasized that inclusion of reasoning and proof into everyday classroom teaching is necessary for developing deep conceptual understanding. However, research vindicate that many learners tend to emphasize procedural execution while paying less attention to logical justification (Kunth, 2005). Researchers have pointed out that explicit engagement with formal logic improves students reasoning abilities and enhances their mathematical thinking (Simpson & Inglis). The integration of logical frameworks into mathematics education enhances students ability in analyzing and justifying mathematical claims (Weber, 2010). In addition, developing reasoning skills assists students to build clear logical arguments and improves better understanding of mathematical proofs (Selden et. al., 2015). According to Harel (2013), meaningful mathematical understanding requires students to understand the logic behind procedures instead of simply memorizing rules.

Recent studies have highlighted the value of systematic reasoning to support learner's mathematical understanding. For instance, Lithner (2008) made a distinction between imitative and creative reasoning, emphasizing the need to promoting learning approaches based on reasoning. Similarly, Maher & Muller (2009) showed that structured reasoning activities improve students ability in mathematical communication and logical understanding. Stylianou et al. (2015) also showed that mathematical argumentation is crucial for building students conceptual understanding. Furthermore, educational research indicates that students frequently struggle to relate symbolic manipulation with proper logical justification (Bieda et al., 2013). According to Inglis & Mejia-Ramos (2019), the integration of logic into mathematics education enhances clear reasoning and strengthen students proof skills. Borwein and Bailey (2016) stated that mathematical thinking fundamentally based on logical structure and logical organization. In addition, current educational reforms stress reasoning-focused instruction for foster higher-order cognitive skills (National Council of Teachers Mathematics, 2014). Recent research suggests that teaching reasoning helps learners to develop logically sound solutions and enhances their mathematical confidence (Weber et al.; Inglis & Mejia-ramos, 2022). In spite of these advancements, many learners continue to lack structured and systematic support for organizing their reasoning logically.

To address this gap, this paper present three interrelated structured logical framework namely Reasoning Reconstruction Model, Step–Reason Mapping and Logic-Integrated Mathematical Expression aimed at to strengthen learners coherent mathematical reasoning and logical justification.

1. Preliminaries

This section briefly reviews the fundamental logical and mathematical ideas that provide the theoretical foundation of the proposed framework. These preliminaries lay the logical foundation needed for incorporating of formal reasoning into mathematical problem solving processes.

1.1 Propositional Logic

A proposition is a declarative statement which has a truth value. The truth value may be either true (T) or false (F) but not both. Propositions are generally represented by symbols like $p, q, r, s, \cdots\cdots$, which are connected using logical connectives such as $\sim$ (negation), ∧ (conjunction), ∨ (disjunction), → (conditional) and ↔ (bi-conditional). Numerous mathematical assertions are formulated as implications which make propositional logic as a powerful tool for identifying assumptions and conclusions within mathematical reasoning.

1.2 Predicate Logic

Predicate logic builds upon propositional logic through the inclusion of predicates, quantifiers and variables in which predicates describes the characteristics of objects while the quantifiers such as universal quantifier (∀) and existential quantifier (∃) define the scope of applicability of statements. Mathematical definitions and results are formulated using predicate logic. For example, an integer n is even if there exists an integer k such that $n = 2k$. In predicates it can be expressed as

$$\forall x \in Z\ (x\ \ even \rightarrow x = 2k, for\ some\ k \in Z)$$

This approach allows mathematical assumptions and outcomes to be stated with precision.

2. Logical Equivalences

Two propositions are said to be logically equivalent if they have same truth value under all circumstances. Some common equivalence are give in the following table:

Table 1: Some Standard Laws of logical equivalences

Equivalance	**Name of Laws**
$p \wedge T \equiv p$ and $p \vee F \equiv p$	Identity Laws
$p \vee T \equiv T$	Tautology Law
$p \wedge F \equiv F$	Contradiction Law
$p \wedge p \equiv p$ and $p \vee p \equiv p$	Idempotent Laws
$\sim(\sim p) \equiv p$	Double Negation Law
$p \wedge q \equiv q \wedge p$ and $p \vee q \equiv q \vee p$	Communicative Laws
$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ $p \vee (q \vee r) \equiv (p \vee q) \vee r$	Associative Laws
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive Laws
$\sim(p \wedge q) \equiv \sim p \vee \sim q$ $\sim(p \vee q) \equiv \sim p \wedge \sim q$	De Morgan's Laws
$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$	Absorption Laws

$p \wedge \sim p \equiv F \;\; and\, p \vee \sim p \equiv T$	Negation Laws

These logical equivalences are often used to rewrite mathematical statements while keeping logical validity. Understanding these equivalences is essential in refining arguments and simplifying logical expression in proofs.

2.1. Rules of Inference

The Rules of inferences provide the justification of the steps to show that a conclusion follows logically from a set of hypotheses. Some popular types of Rules of inference in propositional logic are Modus Ponens, Modus Tollens, **Hypothetical Syllogism,** Universal Instantiation, and Existential which are given in the table 2:

Table 2: Some Standard Rule of Inference

Inference Rule (name)	Symbolic form	Explanation
Modus Ponens	Premises: $p \rightarrow q, \quad p$ Conclusion: q	From "if *p then q*" and "*p" we infer "q"*
Modus Tollens	Premises: $p \rightarrow q, \; \sim q$ Conclusion: $\sim p$	From "if *p then q*" and "not *p" we infer "not q"*
Hypothetical Syllogism	Premises: $p \rightarrow q, \quad q \rightarrow r$ Conclusion: $p \rightarrow r$	From "if *p then q*" and "if *q then r" we infer* "if *p then r*"
Universal Instantiation	Premises: $\forall x \; P(x)$ Conclusion: $P(c)$	From "P(*x*) is true for all *x*" we infer "P(*c*) is true for a particular element *c*"
Existential Instantiation	Premises: $\exists x \; P(x)$ Conclusion: $P(c)$	From "there exists an element *x* such that P(*x*) is true" we infer "P(*c*) is true for some particular element *c*"

In mathematical proofs, these inference rules validate that each conclusion logically follows from preceding statements rather then relying on intuition alone.

RESEARCH METHODOLOGY

In this paper, we introduce three new approaches of pedagogical structures to rebuild mathematical learner's logical reasoning.

1. Reasoning Reconstruction Model (RRM)

The Reasoning Reconstruction Model can be considered as a mathematical pedagogical framework that can be used to identify, analyze, and rebuild flawed or incomplete mathematical reasoning. Rather than focusing on the final answer of a mathematical problem, RRM focuses how a learner thinks and reconstructs correct reasoning step-by-step to solve the problem. It can encourages learners to articulate their thought processes, thereby can promoting their clarity, precision, and coherence in mathematical reasoning. In this approach, the whole RRM framework is structured into the following three stages.

Stage 1: Decompose (D): In Decompose stage, the given mathematical problem or statement breaks into its basic logical components. During this stage, the mathematical problem or statement is reformulated formal logical expression through either propositional logic or predicate logic. This stage enables learner's to clearly identify the underlying assumptions, given conditions and conclusions embedded within the problem. For example, if we consider a problem:

$$if\ 'n'\ is\ divisible\ by\ 6, then\ n\ is\ even.$$

Decompose this problem into propositional logic as: Let

$$p{:}\ n\ is\ divisible\ by\ 6$$
$$q{:}\ n\ is\ divisible\ by\ 2$$

$$The\ logical\ form\ of\ the\ statement\ is\ therefore, \qquad p \rightarrow q$$

At Decompose stage, learner's can clearly identify the given assumption (what is given), the required conclusion (what is to be proven) and the logical relationship connecting the propositions (how the propositions are logically connected).

Stage 2: Align (A): The main purpose of Alignment stage is on mapping each step of mathematical solution to a corresponding inference principle or logical rule. This process ensures that each transformation in the mathematical solution is logically is logically grounded rather than intuitively assumed. During this stage, mathematical learners explicitly integrate mathematical operations such as algebraic manipulations, formal definitions like evenness or divisibility, and logical inference rules like Modus Ponens (MP) or Universal Instantiation (UI) etc. Such alignment makes the reasoning process transparent and logically sound. As a result learners come to understand not only what sequence of steps are performed in the solution but also into the logical justification behind each step. The alignment of each step in above problem is shown in the following table:

Mathematical Step	Logical Expression	Rule Applied
$n = 6k,\ \ k \in I$	p	Definition of divisibility
$n = 2(3k)$	q	Universal Instantiation/Algebraic manipulation
Even	Implication	Modus Ponens (MP)

Stage 3: Rebuild (R): The Rebuild stage involves reconstructing the entire argument into a well-organized and logically valid explanation using a combination of mathematical and logical language. In this stage, the output of the Decompose as well as Alignment stages is integrated to form a clear, structured and comprehensive explanation. The reconstructed explanation emphasizes maintaining a smooth logical flow, ensuring mathematical correctness and providing explicit justification of conclusion. This step helps mathematics learners to express mathematical reasoning in clearly and rigorously, which is essential for constructing proofs and engaging higher-level mathematical thinking.

Example of Rebuild Explanation : Since n is divisible by 6, then there exists an integer k such that $n = 6k$. This can be written as $n = 2(3k)$. Which demonstrates that *n* is even. Therefore by systematically applying algebraic decomposition and logical inference, we can conclude that n is even. Alternatively, since the implication $p \rightarrow q$ is obtained by algebraic decomposition and logical inference, so n must be even.

The overall framework of Reasoning Reconstruction Model can be consciously expressed by the formula:

$$\text{Rebuild Explanation} = D + A + R$$

Where, $D(Decompose)$ breaks the problem into its logical components, A involves verifying each steps through logical rules, and R(Rebuild) focuses on combining these steps to a clear and coherent explanation.

2. Step–Reason Mapping (SRM)

Step-Reason Mapping works as a core component within the Rebuilding Logical Foundation in Mathematics and align strongly with the Reasoning Reconstruction Model(RRM). The key concept of SRM is that every mathematical step must be accompanied by an explicit logical justification. Thus each solution steps are systematically paired with the logical rule, definition, or inference rule that confirms its validity. According to SRM, every mathematical solution must be presented as a sequence of collection of step-reason pairs as (SRM Format):

$$\langle S_1, R_1\rangle, \langle S_2, R_2\rangle, \cdots\cdots\cdots, \langle S_n, R_n\rangle$$

Where, S_i represents the i-th step in the solution process, while R_i denotes the logical justification or rule for that step. Such a mapping guarantees that all solution steps of a mathematical problem are firmly based on valid logical rule such as rule of inference, definitions, axioms, equivalence transformations or previously proven results.

For example, solving a mathematical statement using SRM:

Statement "$if\ a\ number\ n\ is\ even\ then\ n^2\ is\ even$"

Step-Mapping Reason table:

Steps (S_i)	Mathematical Action	Reason(R_i) & Logical Rule
S_1	Given, $n\ is\ even$	Given assumption
S_2	$n = 2k, for\ some\ k$	Definition of even numbers
S_3	$n^2 = (2k)^2$	Substitution rule
S_4	$n^2 = 4k^2$	Algebraic simplification
S_5	$n^2 = 2(2k^2)$	Factorization
S_6	$n^2\ is\ even$	Definition of even numbers
S_7	Conclusion holds	Logical inference (MP)

SRM makes a clear distinction by separating assumptions, definitions, transformations and conclusions thereby helps to eliminate the common misconception that algebraic operations alone amount to proof. Rather it emphasizes the idea that a proofs are built in a structured sequence of steps supported by logical justification.

3. Logic-Integrated Mathematical Expression (LIME)

LIME is a formal reasoning approach aimed at rebuilds the logical foundation of mathematics through the integration of logical inference rules at each step of mathematical reasoning. In conventional mathematical practices, the justification of transitions between expressions is often left unstated, which can obscure the logical flow of proofs and promote procedural thinking rather than conceptual understanding. LIME method mitigates this issue by framing mathematical reasoning as an explicitly structured sequence in which every transaction is validated by a named logical rule, definition or inference principle.

A Logic-Integrated Mathematical expression can be formally expresses as a sequence like:

$$M_1 \overset{L_1}{\Rightarrow} M_2 \overset{L_2}{\Rightarrow} \cdots\cdots\cdots \overset{L_n}{\Rightarrow} M_{n+1}$$

Where M_i are mathematical statements and the notation $\overset{L_i}{\Rightarrow}$ denotes that the transition from M_i to M_{i+1} is logically justified by the rule L_i.

For example, to show that if "$x^2 = 25, \text{then } x = 5 \text{ or } x = -5$

$$x^2 = 25 \xrightarrow{square\ root\ law} x = \pm 5 \xrightarrow{\vee-\text{Introduction}} (x = 5) \vee (x = -5)$$

Through this structured representation, students grasp not only the mathematical result but also the logical principles that justify every transition, strengthening them both mathematical understanding and logical reasoning skills simultaneously.

RESULTS AND DISCUSSION

1. Illustrative Example Using the New Frameworks

Let us consider the following problem

"Show that if an integer n is divisible by 4 and n is odd, a contradiction occurs."

1.1 Reasoning Reconstruction Model (RRM)

RRM focuses on the systematic construction og reasoning rather than merely checking the correctness of final result. In this approach the solution is developed through three organized stages namely Decompose, Align, and Rebuild.

Stage 1: Decompose: Let

$$p: "n\ is\ divisible\ by\ 4"$$
$$q: "n\ is\ odd"$$

Using standard definitions:
From p: $if\ n\ is\ divisible\ by\ 4\ then\ n = 4k, for\ some\ integer\ k.$
From q: $if\ n\ is\ odd\ then\ n = 2m + 1, for\ some\ integer\ m.$
To examine the conjunction $p \wedge q$ and show that it leads to a conclusion. *i.e.*

$$p \wedge q \Rightarrow \perp \text{ (contradiction)}$$

Stage 2: Align: During this stage, every mathematical transformations is clearly justified by an apporiate logical rule, definitions, or principle of inference.

Mathematical Step	Logical Expression	Rule Applied
$n = 4k,\ k \in I$	p	Definition of divisibility by 4
$n = 2(2k)$	–	Algebraic manipulation
n is even	–	Definition of even numbers
$n = 2m + 1$	q	Definition of odd numbers
n is even and odd	Even ∧ odd	Conjunction
Contradiction	$\perp$	Mutual exclusivity of even and odd
$\neg (p \wedge q)$	–	Negation introduction

Stage 3: Rebuild: In this stage, the decomposed components and their corresponding aligned steps are assembled into a clear and logically valid argument.
Rebuild Explanation: Assume that integer n is divisible by 4. Then $n = 4k$, for some integer k, which can be expressed as $n = 2(2k)$. which establishes that n is even (by definition). Simultaneously assume that, n is odd. Thus, n is simultaneously both even and odd, which is impossible. Thus, the assumption that n is simultaneously both even and odd leads to a contradiction, consequently, the conjunction $p \wedge q$ cannot hold.

1.2. Step-Reason Mapping (SRM)

By distinctly separating assumptions, algebraic manipulations, definitions, and logical inferences, the SRM framework emphasizes that proof is a logically justified steps rather than computational procedures.
Step-Mapping Reason table:

Steps (S_i)	Mathematical Action	Reason(R_i) & Logical Rule
S_1	Given, $n\ is\ divisible\ by\ 4$	Given assumption (premise)
S_2	$n = 4k, for\ some\ integer\ k$	Definition of divisibility
S_3	$n = 2(2k)$	Algebraic transformation
S_4	$n\ is\ even$	Definition of even numbers
S_5	$Given\ n\ is\ odd$	Given assumption
S_6	$n = 2m + 1$	Definition of odd numbers
S_7	$n\ is\ even\ and\ odd$	Logical conjunction
S_8	contradiction	Even and odd are mutually exclusive
S_9	$\neg (p \wedge q)$	Logical inference

1.3. Logic-Integrated Mathematical Expression (LIME)

LIME represents the reasoning process as a connected series of mathematically stated transitions, where every transitions is labeled by an explicit logical justification.

$$n = 4k \xrightarrow{Algebra} n = 2(2k) \xrightarrow{Definition} n\ is\ even$$
$$n = 2m + 1 \xrightarrow{Definition} n\ is\ odd$$
$$\text{Even} \wedge \text{odd} \xrightarrow{Logical\ law} \perp \text{(contradiction)}$$

The final logical conclusion becomes:

$$p \wedge q \Rightarrow \perp \text{ and hence } \neg\,(p \wedge q)$$

Discussion

The proposed framework (RRM, SRM, LIME) collectively contribute in rebuilding logical foundation in mathematics. They make the implicit reasoning processes explicit that are often hidden in conventional mathematical practices. By merging symbolic logic with mathematical expression, these three frameworks help learners to establish clear connections between mathematical computation with formal reasoning. They offer structured guidance to learners through a systematically and well-organized reasoning process. As a result this structured approach reduces learners overreliance on memorized techniques as well as mechanical methods and encourages meaningful understanding. Furthermore, these frameworks provide learners with clear representational tools to articulate and communicate mathematical justification with clarity and precision.

CONCLUSION

Rebuilding logical foundation in mathematics learners is fundamental for strengthening their strong reasoning skills. Structured teaching methods grounded in logical rules and symbolic expressions helps learners to develop clear and more consistent thinking. The models like RRM, SRM and LIME strengthens this structured reasoning process. These approaches enables learners to understand not just what steps to take in mathematical procedure, but why these steps are logically valid. As a result, teachers gain a powerful and systematic framework for integrating formal logical reasoning into regular mathematics teaching.

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