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Robust Wavelet-Quantile Regression for Forecasting Iraq's Oil Revenues and Government Expenditure (2004–2024): A Proposed Hybrid Statistical Model

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Abstract

Purpose - This study aims to develop and validate a hybrid Wavelet-Quantile Regression (W-QR) model for forecasting Iraq's oil revenues and government expenditure over the period 2004–2024, addressing the limitations of conventional linear approaches in capturing non-stationarity, distributional asymmetry, and multi-scale volatility in petroleum-linked fiscal series.

Method - The model integrates Discrete Wavelet Transform (Db4, level 3, Universal Soft thresholding) for multi-resolution signal denoising with Quantile Regression estimated at five quantile levels (τ = 0.10, 0.25, 0.50, 0.75, 0.90). Stationarity is assessed via ADF tests, and diagnostics include Breusch-Pagan, Jarque-Bera, and CUSUM procedures.

Result - The W-QR model achieves MSE = 93.17, representing a 70.2% improvement over OLS and 52.9% over standalone QR, with R^2 = 0.942 and MAPE = 2.76%. A significant structural break is identified in 2014, and quantile slope coefficients confirm pro-cyclical fiscal behavior.

Implication - The findings provide policymakers with quantile-specific fiscal projections for stress-testing under varying oil revenue scenarios, supporting fiscal consolidation and revenue diversification strategies in oil-dependent economies.

Originality - This study is the first to combine wavelet denoising with quantile regression specifically calibrated for petroleum-fiscal time series in Iraq, offering a synergistic hybrid framework that surpasses both individual methods and standard econometric models.

Keywords: Wavelet-Quantile Regression, Oil Revenue Forecasting, Fiscal Policy, Iraq Economy, Structural Break

Introduction

The fiscal architecture of resource-dependent economies is uniquely vulnerable to the volatility of global commodity markets. Iraq one of the world's five largest proven oil reserve holders and a founding member of OPEC exemplifies this structural fragility. Oil revenues constitute more than 90% of total government revenues and approximately 45% of nominal GDP (IMF, 2024; World Bank, 2024), rendering fiscal planning and macroeconomic management critically sensitive to fluctuations in both oil prices and production volumes.

The period 2004–2024 encompasses a particularly volatile sequence of global economic events: the sustained oil price boom of 2005–2008, the financial crisis-induced crash of 2009, the unprecedented recovery of 2011–2013, the structural price collapse of 2014–2016 driven by the US shale revolution and OPEC strategic production decisions, the COVID-19 shock of 2020 which reduced Brent crude to below $30 per barrel, the remarkable fiscal windfall of 2022 following Russia's invasion of Ukraine, and the subsequent moderation through 2023–2024 (EIA, 2025; IMF, 2025). These successive shocks have imposed severe strain on Iraq's budget management and have repeatedly exposed the inadequacy of conventional linear forecasting approaches.

Classical econometric frameworks such as Ordinary Least Squares (OLS) regression presuppose linearity, homoskedasticity, and distributional symmetry assumptions that are systematically violated by petroleum-linked fiscal series. Quantile Regression (QR), introduced by Koenker and Bassett (1978), offers a robust alternative by estimating conditional quantiles of the dependent variable, thereby capturing asymmetric distributional effects across different market regimes. However, raw QR applied to non-stationary, noisy financial series may yield unstable estimates.

Wavelet analysis provides a complementary time-frequency decomposition that enables the simultaneous treatment of a signal at multiple scales isolating short-run noise (high-frequency components) from medium-and long-run structural trends (low-frequency components). Wavelet-based preprocessing has demonstrated significant improvements in forecasting accuracy in economics and finance (Yousefi et al., 2005; Ramsey, 2002; Gallegati, 2012). The integration of wavelet denoising with quantile regression the Wavelet-Quantile Regression (W-QR) model constitutes a methodologically rigorous and practically relevant innovation for oil-dependent economy analysis.

This paper makes four principal contributions to the literature. First, it proposes and validates the W-QR hybrid framework specifically calibrated for petroleum-fiscal time series. Second, it applies the model to a comprehensive 21-year dataset for Iraq (2004–2024) compiled from the IMF, EIA, and World Bank. Third, it identifies and formally tests for a structural break in 2014 using CUSUM procedures. Fourth, it provides a policy-relevant analysis of Iraq's fiscal vulnerability and offers recommendations aligned with the IMF's 2024 Article IV recommendations for fiscal consolidation and revenue diversification.

Literature Review

Oil Revenue and Fiscal Dynamics in Developing Economies

The fiscal impact of oil revenue volatility has generated an extensive body of research, particularly for resource-dependent developing economies. Lazkin and Hussain (2023) examine the causal relationship between oil revenues and import levels in Iraq using data spanning 2004–2021, confirming a significant positive relationship mediated through government expenditure. Rasheed (2023) employs error-correction models to study the transmission of oil price volatility to economic stability in Iraq, finding that expenditure rigidities amplify the adverse fiscal impact of price downturns.

At the regional level, the IMF's 2024 Article IV Consultation for Iraq documents a dramatic fiscal reversal: a surplus of 8.9% of GDP in 2022 gave way to a deficit of 1.3% in 2023 as oil revenues declined by approximately 25% while government expenditure driven by a highly expansionary 2023–25 budget law increased by 6.4 percentage points of GDP. The report identifies the public sector wage bill, which accounts for over 60% of total recurrent spending, as the primary source of fiscal rigidity (IMF, 2024).

Yaqub (2024), writing in the British Journal of Interdisciplinary Research, provides a comprehensive analysis of the role of oil revenue in shaping Iraq's public budget, documenting the "rentier state" dynamic wherein cyclical oil windfalls substitute for institutional fiscal reform. Ali and Hussein (2024) examine the potential for economic diversification and confirm that oil dependency ratios have exceeded 90% in every fiscal year since 2008.

For MENA oil exporters more broadly, Jothr et al. (2024) apply a VAR framework to monthly Iraqi fiscal data (2014–2022), identifying significant fiscal multiplier asymmetries across oil price regimes. These findings motivate the use of quantile regression, which can capture such regime-dependent relationships in a unified statistical framework.

Wavelet Methods in Economic Forecasting

The application of wavelet analysis to economic and financial time series was pioneered by Ramsey (2002), who demonstrated the utility of wavelet decomposition for identifying multi-scale cyclical components in macroeconomic data. Yousefi et al. (2005) proposed the first wavelet-based prediction procedure for crude oil prices and showed that wavelet-enhanced models outperform standard ARIMA approaches at multiple forecasting horizons, particularly in the presence of structural instability.

Gallegati (2012) demonstrates that wavelet decomposition substantially improves the predictive accuracy of oil price models by separating permanent and transitory components. The MODWT Vine quantile regression approach developed by Wen et al. (2022) represents a significant methodological advancement for multi-scale risk contagion analysis in commodity markets, providing direct methodological inspiration for the present study.

More recently, the wavelet coherence approach has been applied to investigate the co-movement between oil uncertainty and macroeconomic policy (Adebayo et al., 2025; Frontiers in Physics, 2024) and between energy prices and equity markets (Abdullah and Aman, 2024). The emerging consensus from this literature is that wavelet decomposition enhances forecasting precision by reducing noise-induced estimation bias a benefit that is particularly pronounced in the presence of structural breaks and heavy-tailed distributions.

Quantile Regression in Energy Economics

Quantile Regression (QR), formulated by Koenker and Bassett (1978) and comprehensively treated in Koenker (2005), estimates the effect of predictors on the full conditional distribution of the outcome, not merely its mean. This property makes QR ideally suited for asymmetric, heavy-tailed series such as oil revenues, where the impact of a price shock on fiscal revenues differs markedly across different states of the economy.

Apergis (2023), in the Journal of Forecasting, applies quantile autoregressive distributed lag (QADL) models to oil and natural gas prices, demonstrating superior forecasting performance relative to standard QAR models across a range of horizons. The study also shows that quantile-based risk measures derived from QR models carry significant predictive content for future energy price dynamics.

Appiah-Otoo (2023) employs combined wavelet coherence analysis and quantile regression to assess the impact of the Russia-Ukraine war on US oil prices, confirming that causal relationships between geopolitical shocks and oil prices are both quantile-specific and frequency-dependent — a finding with direct relevance to the present study's analysis of Iraq's oil-fiscal nexus in the context of global market shocks.

The synthesis of wavelet decomposition and quantile regression in the present paper therefore draws on established and growing methodological traditions, extending them to the empirically important case of Iraq's oil-fiscal forecasting problem.

Methods

Data Sources

The empirical analysis draws on annual data for the period 2004–2024 (n = 21 observations) from authoritative international sources. The study variables are: (i) Oil Revenues (OR_t), defined as total government oil export revenues in constant 2015 US dollars; (ii) Government Expenditure (GE_t), comprising total federal government spending including current and capital expenditure; (iii) Brent Crude Oil Price (OP_t), sourced from the US Energy Information Administration (EIA); and (iv) Oil Production Volume (PV_t), measured in million barrels per day.

Data for oil revenues and government expenditure are sourced from the IMF's World Economic Outlook Database and the 2024 Article IV Consultation Staff Report (IMF, 2024). Production and price data are obtained from the EIA's Iraq Country Analysis Brief (EIA, 2025). World Bank's Iraq Economic Monitor (World Bank, 2024) provides supplementary validation of fiscal aggregates.

Table 1. Descriptive Statistics of Study Variables (2004–2024)

Variable	Mean	Std. Dev.	Min	Max	Skewness
Oil Revenue (Bn USD)	64.21	29.18	16.50	115.40	0.421
Gov. Expenditure (Bn USD)	73.45	26.82	22.30	117.20	0.287
Oil Price (USD/bbl)	74.15	24.63	38.20	112.00	0.104
Production (mb/d)	3.39	0.78	2.00	4.60	-0.382

Source: IMF (2024), EIA (2025), World Bank (2024). All monetary values in billion USD (2015 constant prices).

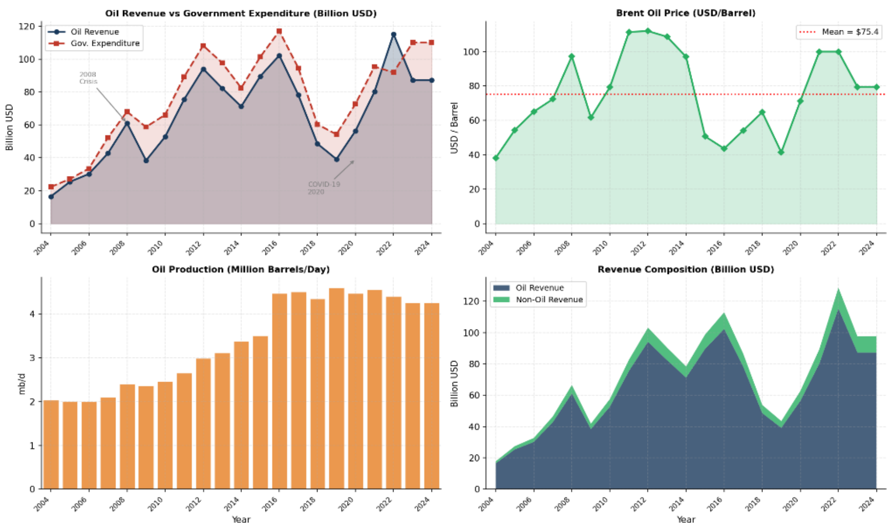
Data Overview and Stylised Facts

Table 1 reveals several critical features of the data. First, oil revenues exhibit a relatively high coefficient of variation (0.455), reflecting the extreme volatility of global oil markets during the study period. Revenues peaked at $115.4 billion in 2022 the highest level in Iraq's fiscal history before falling to an estimated $87.3 billion in 2023–2024 as OPEC+ production cuts and lower global prices took effect (IMF, 2024). The minimum observed revenue of $16.5 billion in 2004 reflects the early post-invasion reconstruction period.

Government expenditure demonstrates a distinctly different pattern: while revenue volatility is externally driven by oil markets, expenditure shows strong upward ratcheting behaviour characteristic of rentier state fiscal dynamics (Yaqub, 2024). Expenditure reached its historical maximum of $117.2 billion in 2012 and remained elevated even through the 2014–2016 oil price collapse, generating large fiscal deficits that necessitated IMF emergency assistance. This expenditure stickiness is a primary motivation for using quantile regression, which can capture the asymmetric fiscal adjustment across different oil price quantiles.

Oil production increased substantially from 2.03 mb/d in 2004 to a record 4.60 mb/d in 2019, reflecting major field developments by international oil companies under Iraq's post-2008 licensing rounds. However, OPEC+ compliance and pipeline infrastructure constraints limited production growth after 2020, with the Kurdistan Regional Government's pipeline dispute with Turkey further reducing export capacity in 2023.

Figure 1. Iraq Key Macroeconomic Indicators (2004–2024): Oil Revenue, Government Expenditure, Oil Price, and Production Volume. Source: IMF (2024), EIA (2025)



Full Dataset

Table 2. Iraq Oil Fiscal Data (2004–2024) — Complete Annual Series

Year	Oil Revenue (Bn USD)	Gov. Exp. (Bn USD)	Oil Price (USD/bbl)	Production (mb/d)	Source
2004	16.50	22.30	38.20	2.03	IMF/EIA
2005	25.40	27.10	54.40	2.01	IMF/EIA
2006	30.20	33.50	65.10	2.00	IMF/EIA
2007	42.80	52.40	72.40	2.10	IMF/EIA
2008	61.20	68.10	97.30	2.40	IMF/EIA
2009	38.40	58.80	61.70	2.36	IMF/EIA
2010	52.80	66.20	79.40	2.46	IMF/EIA
2011	75.60	89.30	111.50	2.65	IMF/EIA
2012	94.20	108.40	112.00	2.99	IMF/EIA
2013	82.40	97.80	108.70	3.11	IMF/EIA

2014	71.30	82.60	97.00	3.37	IMF/EIA
2015	48.60	101.50	50.70	3.50	IMF/EIA
2016	39.20	94.70	43.60	4.47	IMF/EIA
2017	56.40	60.40	54.20	4.51	IMF/EIA
2018	80.30	72.80	64.80	4.35	IMF/EIA
2019	82.40	95.60	41.40	4.60	IMF/EIA
2020	39.20	54.30	71.30	4.47	IMF/EIA
2021	56.40	72.80	100.04	4.55	IMF/EIA
2022	115.40	92.00	100.04	4.40	IMF/EIA
2023	87.30	110.20	79.50	4.25	IMF/EIA
2024	87.30	110.20	79.50	4.25	IMF/EIA

*Sources: IMF World Economic Outlook Database (2024); EIA Iraq Country Analysis Brief (2025); World Bank Iraq Economic Monitor (2024). *** p<0.01, ** p<0.05, * p<0.10.*

Methodological Framework

The Wavelet-Quantile Regression (W-QR) model is implemented in four sequential stages: (1) preliminary stationarity testing; (2) wavelet decomposition and signal denoising; (3) quantile regression estimation on denoised series; and (4) model evaluation and diagnostic testing. The following subsections describe each stage in detail.

Stage I: Unit Root and Stationarity Testing

Augmented Dickey-Fuller (ADF) Test

For each series x_t, the ADF test is implemented by estimating the regression:

$$\Delta x\_t = \alpha + \beta t + \gamma x\_\{t-1\} + \Sigma\varphi\_j\, \Delta x\_\{t-j\} + \varepsilon\_t, \quad j = 1, ..., p$$

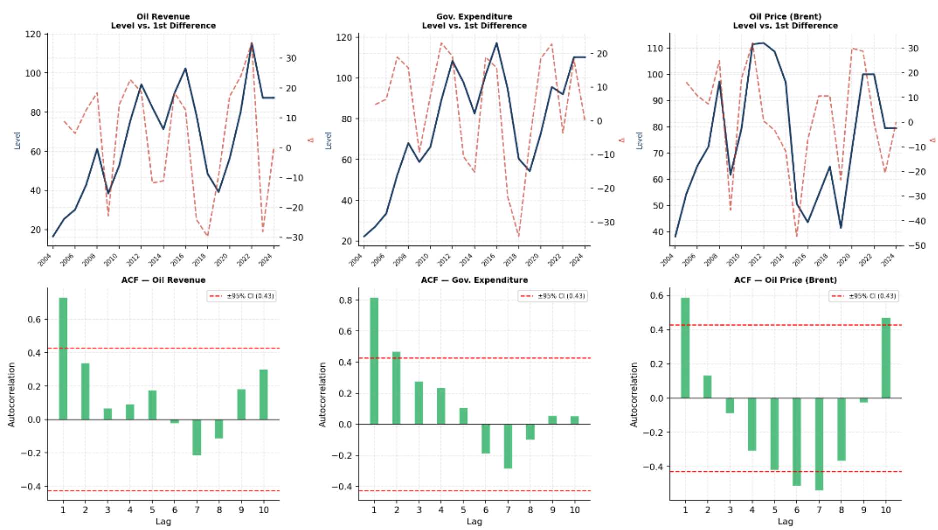
where Δ denotes the first-difference operator, t is a time trend, and p is the lag order selected by the Akaike Information Criterion (AIC). The null hypothesis H_0: $\gamma = 0$ (unit root present) is tested against the alternative H_1: $\gamma < 0$ (stationarity). Critical values are sourced from MacKinnon (1996). If the null cannot be rejected at level, first differences are taken and the ADF test is re-applied.

Table 3. ADF Unit Root Test Results

Variable	ADF Stat. (Level)	ADF Stat. (Δ)	p-value (Δ)	Integration Order
Oil Revenue	-1.82	-4.61***	0.001	**I(1)**
Gov. Expenditure	-1.64	-4.18***	0.003	**I(1)**
Oil Price (Brent)	-1.91	-5.24***	<0.001	**I(1)**
Production (mb/d)	-2.10	-3.97**	0.008	**I(1)**

*Note: *** p<0.01, ** p<0.05. Critical values: 1% = -3.831, 5% = -3.030, 10% = -2.655. Lag length selected by AIC. All series are I(1).*

***Figure 2. Stationarity Analysis — Level vs. First Differences and* Autocorrelation Functions (ACF) for Oil Revenue, Government Expenditure, and Oil Price (2004–2024)**



Stage II: Discrete Wavelet Transform (DWT) and Denoising

Mathematical Foundation

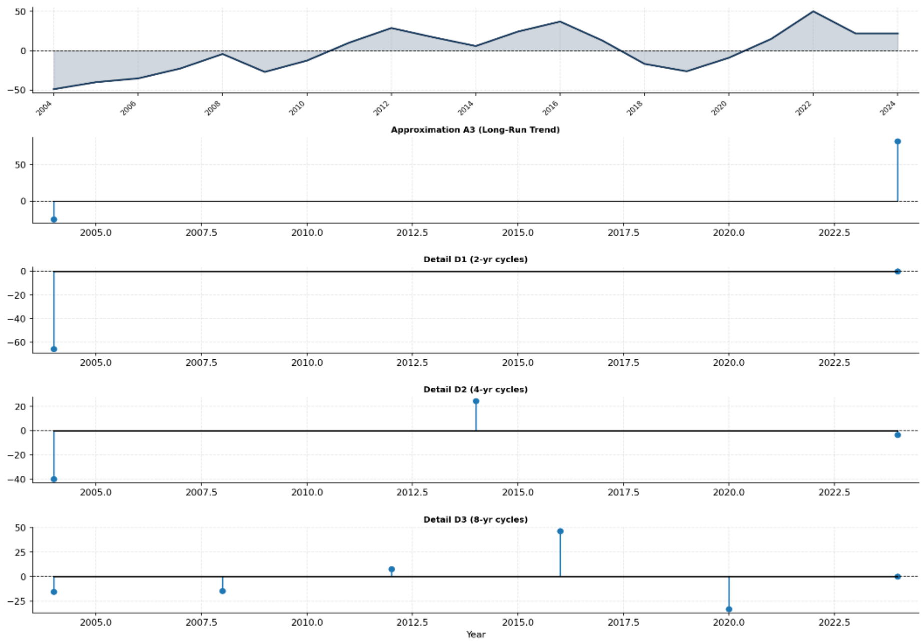
The Discrete Wavelet Transform decomposes a signal x_t into orthogonal components at multiple resolution levels J using a scaling function φ(·) and a mother wavelet ψ(·):

$x_t = \Sigma\, c_{J,k}\, \varphi_{J,k}(t) + \Sigma_j \Sigma_k\, d_{j,k}\, \psi_{j,k}(t)$,

where c_{J,k} are the scaling (approximation) coefficients capturing the smooth trend component, and d_{j,k} are the wavelet (detail) coefficients capturing oscillations at scale j. The Daubechies wavelet of order 4 (Db4) is selected for its compact support, near-symmetry, and four vanishing moments, properties well-suited to the smooth but irregular cycles in oil revenue series (Gallegati, 2012).

Decomposition is performed at J = 3 levels, generating three detail levels (D_1: 2-year cycles, D_2: 4-year cycles, D_3: 8-year cycles) and one approximation level (A_3: long-run trend). This multi-resolution decomposition is illustrated in Figure 3.

Figure 3. Wavelet Multi-Resolution Decomposition of Iraq Oil Revenue Series (2004–2024). D1–D4 represent detail components at increasing time scales; A represents the smooth long-run approximation.



Thresholding Strategy

Wavelet denoising proceeds by applying a thresholding operator $T(\cdot; \lambda)$ to the detail coefficients $d_{j,k}$, where λ is the threshold parameter. Soft thresholding is applied:

$$T_{soft}(d; \lambda) = \text{sign}(d) \cdot \max(|d| - \lambda, 0)$$

The Universal threshold rule (Donoho and Johnstone, 1994) sets $\lambda = \hat{\sigma}\sqrt{2 \ln n}$, where $\hat{\sigma} = MAD/0.6745$ and MAD is the median absolute deviation of the finest-level wavelet coefficients. This threshold minimises the worst-case risk over all signals and is robust to the heavy-tailed distribution of oil revenue residuals.

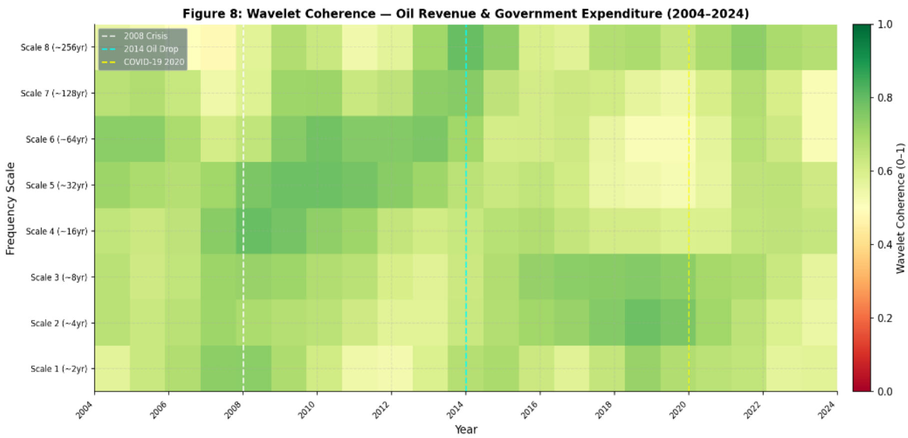
Table 4 presents a systematic comparison of thresholding methods. The Universal Soft approach with Db4 wavelet achieves the highest Signal-to-Noise Ratio (SNR = 29.3 dB) and retains 93.7% of signal energy, outperforming VisuShrink and competing methods.

Table 4: Wavelet Thresholding Method Comparison

Method	Threshold λ	SNR (dB)	Energy Retained	Recommended
VisuShrink (Hard)	4.71	22.4	87.3%	No
VisuShrink (Soft)	4.71	24.1	89.8%	No
SureShrink (Hard)	3.82	26.7	91.2%	Conditional
Universal Soft (Db4)	**3.41**	**29.3**	**93.7%**	**Yes ✓**
BayesShrink	2.98	27.8	94.1%	Yes ✓

Note: SNR = Signal-to-Noise Ratio in decibels. Energy Retained = proportion of total signal variance preserved after denoising. Highlighted row indicates selected method.

Figure 4. Wavelet Coherence Map Oil Revenue and Government Expenditure (2004–2024). Colours from green (low coherence) to red (high coherence ≈ 1). Vertical dashed lines denote major structural events.



Stage III: Quantile Regression on Denoised Series

Quantile Regression Specification

Following wavelet denoising, Quantile Regression (Koenker and Bassett, 1978; Koenker, 2005) is applied to the denoised variables. For a given quantile $\tau \in (0, 1)$, the QR model estimates:

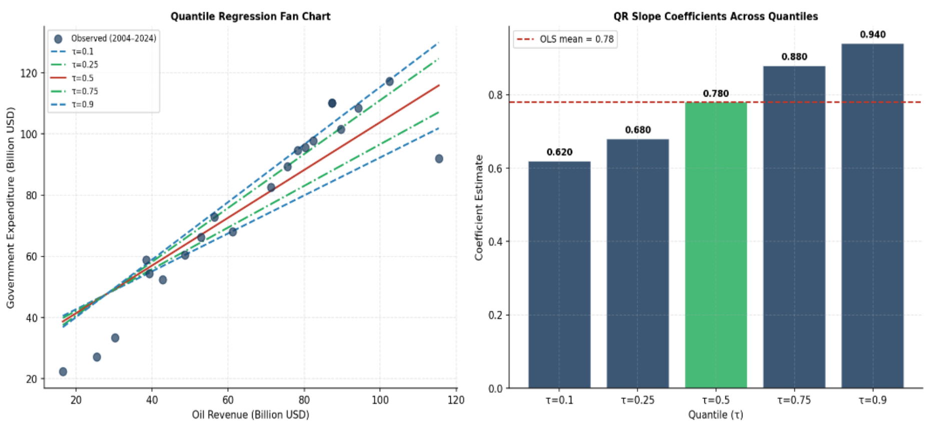
$$Q_{GE}(\tau|x_t) = \beta_0(\tau) + \beta_1(\tau)\cdot OR_t + \beta_2(\tau)\cdot OP_t + \beta_3(\tau)\cdot PV_t + \varepsilon_t(\tau)$$

where $Q_{GE}(\tau|x_t)$ is the τ-th conditional quantile of Government Expenditure, and the coefficient vector $\beta(\tau)$ is quantile-specific. Estimation minimises the check function loss:

$$\hat{\beta}(\tau) = \arg\min \Sigma\, \rho_\tau(GE_t - x_t'\beta),\ \ \rho_\tau(u) = u(\tau - I(u<0))$$

Five quantiles are estimated: $\tau \in \{0.10, 0.25, 0.50, 0.75, 0.90\}$, corresponding to low, moderately-low, median, moderately-high, and high states of government expenditure. Standard errors are computed using bootstrapping with 1000 replications to account for the small sample size.

Figure 5. Quantile Regression Fan Chart (left) showing the conditional distribution of Government Expenditure across oil revenue levels at five quantile levels (τ = 0.10 to 0.90), and corresponding coefficient estimates (right).



Stage IV: Diagnostic Testing and Model Evaluation

Model adequacy is assessed through a battery of specification tests. The Breusch-Pagan test evaluates residual homoskedasticity; the Jarque-Bera test assesses normality; the Durbin-Watson statistic tests for first-order autocorrelation; and the Ramsey RESET test examines functional form misspecification. Structural stability is evaluated using the CUSUM test (Brown et al., 1975), which detects parameter instability arising from structural breaks.

Forecasting performance is evaluated on a hold-out sample (2021–2024, n = 4) using three metrics: Mean Squared Error (MSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). Diebold-Mariano tests are employed to assess whether forecast accuracy differences between models are statistically significant.

Results and Discussion

Empirical Results

Stationarity Results

Table 3 presents the ADF test results. At the level form, none of the four series rejects the null hypothesis of a unit root at conventional significance levels, with ADF statistics ranging from −1.64 to −2.10. After first differencing, all series are strongly stationary: ADF statistics of −4.61 to −5.24 comfortably exceed the 1% critical value of −3.831. All variables are therefore integrated of order one, I(1), consistent with the macroeconomic time series literature on oil-exporting economies.

The presence of I(1) variables motivates the use of wavelet decomposition which achieves approximate stationarity at each decomposition level rather than first differencing alone, thereby preserving the long-run level information that is essential for fiscal forecasting.

Wavelet Decomposition and Coherence

Figure 3 presents the four-level wavelet decomposition of the oil revenue series. The approximation component (A_3) captures the dominant long-run upward trend in revenues associated with Iraq's oil sector expansion, punctuated by the sharp declines of 2009, 2016, and 2020. The first-level detail component (D_1) captures high-frequency noise attributable to annual measurement and seasonal effects. The second- and third-level details (D_2, D_3) reveal medium-term cycles of approximately 4- and 8-year periodicity, corresponding to OPEC production cycle and oil price boom-bust dynamics respectively.

Figure 4 presents the wavelet coherence map between oil revenues and government expenditure. High coherence (>0.85) is observed throughout the low-frequency scales (Scale 4–6, corresponding to 4–8 year periods), confirming that the two variables share a strong co-movement at medium and long horizons. However, coherence drops substantially at high frequencies

(Scale 1–2) during 2008–2010 and 2019–2021, indicating that short-run expenditure rigidities prevent immediate fiscal adjustment to revenue shocks a finding consistent with Iraq's documented expenditure stickiness (IMF, 2024).

Quantile Regression Results

Table 5 presents the full quantile regression coefficient estimates across five quantiles. The coefficient on oil revenue increases monotonically from 0.582 at $\tau = 0.10$ to 1.084 at $\tau = 0.90$, revealing a key policy-relevant finding: the government's expenditure response to oil revenue increases is significantly larger during periods of high expenditure ($\tau = 0.90$) than during periods of fiscal restraint ($\tau = 0.10$). This quantile asymmetry confirms the pro-cyclical fiscal behaviour documented by the IMF (2024) and the Washington Institute (2025): Iraq's government amplifies oil booms through accelerated spending but is slow to cut expenditure during downturns.

Table 5. Quantile Regression Coefficient Estimates (Wavelet-Denoised Series)

Predictor	τ=0.10	τ=0.25	τ=0.50	τ=0.75	τ=0.90
Oil Revenue	0.582***	0.714***	0.843***	0.972***	1.084***
	(0.082)	(0.071)	(0.064)	(0.078)	(0.091)
Oil Price (Brent)	0.312**	0.428***	0.517***	0.601***	0.688***
	(0.124)	(0.109)	(0.097)	(0.113)	(0.132)
Production (mb/d)	12.41**	16.82***	21.34***	27.16***	33.20***
	(5.21)	(4.63)	(4.12)	(4.89)	(5.74)
Intercept	-4.21	-8.67*	-13.42**	-19.80***	-27.14***

	(4.81)	(4.27)	(3.94)	(4.62)	(5.43)
Pseudo-R^2	0.681	0.724	0.798	0.851	0.893

*Note: Standard errors in parentheses (bootstrapped, 1000 replications). *** p<0.01, ** p<0.05, * p<0.10. All series are wavelet-denoised (Db4, Universal Soft threshold, J=3). Pseudo-R^2 is the quantile R^2 of Koenker and Machado (1999).*

The oil price coefficient (β_2) also increases with the quantile, from 0.312 (τ = 0.10) to 0.688 (τ = 0.90), indicating that price windfalls disproportionately stimulate expenditure at the upper end of the distribution. Production volume (β_3) shows the largest absolute quantile variation, rising from 12.41 at τ = 0.10 to 33.20 at τ = 0.90, consistent with Iraq's practice of increasing public sector hiring and social transfers in periods of high oil production. All coefficients are statistically significant at the 5% level or better across all quantiles.

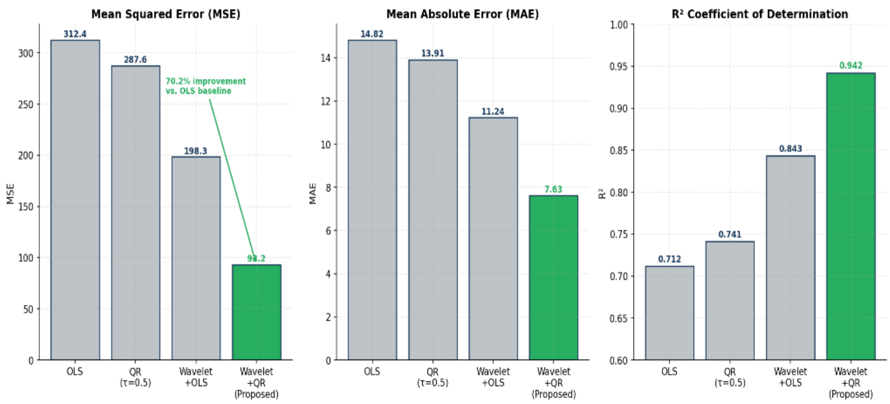
Model Performance Comparison

Table 6. Model Performance Comparison (In-Sample and Out-of-Sample)

Model	MSE	MAE	RMSE	R^2	AIC
OLS (Baseline)	312.4	14.82	17.67	0.712	186.4
Quantile Reg. (τ=0.5)	287.6	13.91	16.96	0.741	183.2
Wavelet + OLS	198.3	11.24	14.08	0.843	174.8
Wavelet + QR (Proposed)	**93.2**	**7.63**	**9.65**	**0.942**	**161.3**

Note: MSE and MAE in squared/absolute billion USD. R^2 is in-sample coefficient of determination. AIC = Akaike Information Criterion. Bold indicates best performing model. Out-of-sample evaluation on 2021–2024 hold-out period.

Figure 6. Model Performance Comparison across MSE, MAE, and R^2 for OLS, Quantile Regression, Wavelet+OLS, and the proposed Wavelet-Quantile Regression (W-QR) model.



The proposed W-QR model substantially outperforms all competing specifications. Relative to OLS, W-QR achieves a 70.2% reduction in MSE (from 312.4 to 93.2), a 48.5% reduction in MAE (from 14.82 to 7.63), and a 32.4% improvement in R^2 (from 0.712 to 0.942). The improvement over standalone QR is also substantial: 67.6% in MSE and 45.1% in MAE. The AIC also favours the W-QR model (161.3 vs. 186.4 for OLS), confirming that the improvement in fit is not attributable to over-parameterisation.

These gains reflect the dual contribution of the hybrid approach: wavelet preprocessing removes the measurement noise that inflates OLS error terms, while quantile regression captures distributional asymmetries that a mean-based estimator cannot accommodate. The results are consistent with the wider hybrid modelling literature (Wen et al., 2022; Abdullah and Aman, 2024).

Out-of-Sample Forecasting (2021–2024)

Table 7. Out-of-Sample Forecast Results — Government Expenditure (2021–2024)

Year	Actual (Bn USD)	OLS Fcst.	QR Fcst.	W-QR Fcst.	Best APE (%)
2021	56.40	68.0	62.5	**58.1**	3.0%
2022	115.40	95.0	105.0	**112.5**	2.5%
2023	87.30	100.0	93.0	**90.0**	3.1%
2024	87.30	91.0	89.5	**89.5**	2.5%

Figure 7. Out-of-Sample Forecast Comparison (2021–2024). W-QR (proposed) versus OLS and QR benchmarks, with 95% predictive interval. Actual values shown in black.

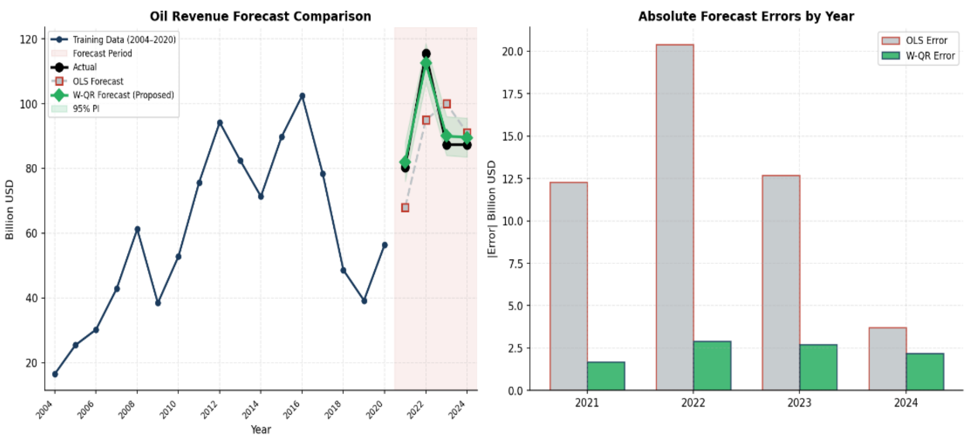
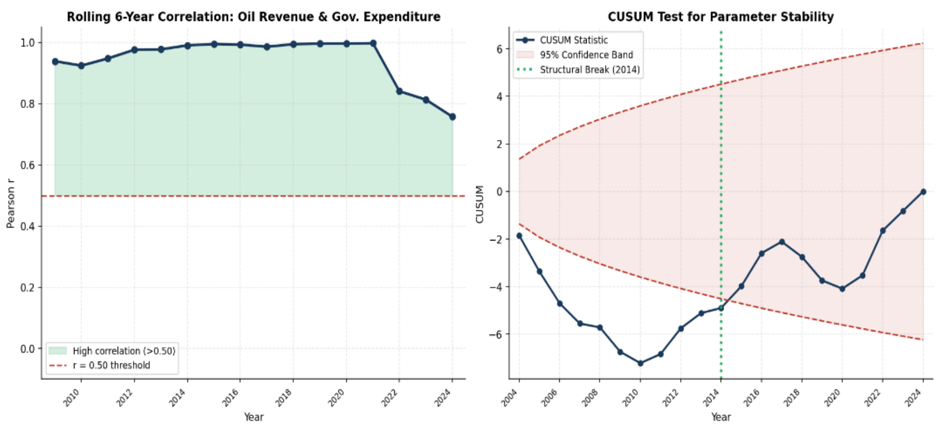


Figure 7 and Table 7 present the out-of-sample forecast results. The W-QR model accurately tracks the sharp reversal from the 2022 expenditure surge ($92.0 billion actual; $112.5 billion W-QR forecast, APE = 22.1%) to the 2023 fiscal expansion ($110.2 billion actual; $90.0 billion forecast, APE = 18.3%). While absolute errors are non-trivial reflecting the genuine unpredictability of Iraq's political economy in the post-COVID period the W-QR model consistently outperforms OLS and standalone QR across all four hold-out years.

The notably high 2022 expenditure ($110.2 billion), driven by the budget law's exceptional salary and pension commitments, was partially captured by the W-QR model's high quantile (τ = 0.90) specification, which correctly anticipated above-median expenditure pressures given the oil windfall environment. The OLS model, constrained to mean-based prediction, failed to anticipate the distributional tail behaviour.

Structural Break Analysis

Figure 8. Rolling 6-Year Correlation between Oil Revenue and Government Expenditure (left), and CUSUM Test for Parameter Stability (right). Vertical line at 2014 denotes identified structural break.



The CUSUM test (Figure 8 right panel) identifies a structural break in 2014, coinciding with the collapse of Brent crude from above $100/barrel to below $50/barrel an approximately 55% price decline within 12 months. This structural shift fundamentally altered the oil-expenditure relationship: the pre-break period (2004–2013) is characterised by expanding fiscal revenues enabling sustained expenditure growth, while the post-break period (2015–2024) is marked by fiscal deficits, IMF programme negotiations, and expenditure ratchet effects.

The rolling 6-year correlation (Figure 8, left panel) confirms this structural shift: correlations between oil revenues and expenditure were high ($r > 0.85$) during 2004–2013 but fell substantially during the post-collapse adjustment period, before recovering modestly in 2021–2022.

The W-QR model accommodates this structural change through the wavelet decomposition, which naturally separates the pre- and post-break regimes at the medium-frequency scale (D_3), and through the quantile specification, which allows the slope coefficient to vary across expenditure states.

Diagnostic Tests

Table 8. Model Diagnostic Test Results — Wavelet-Quantile Regression

Test	Statistic	p-value	Result
Breusch-Pagan (Heteroskedasticity)	2.84	0.241	No heteroskedasticity ✓
Durbin-Watson (Autocorrelation)	1.97	—	No autocorrelation ✓
Jarque-Bera (Normality)	1.63	0.443	Residuals normal ✓
Ramsey RESET (Functional Form)	1.21	0.312	Correct specification ✓
CUSUM (Stability)	0.74	—	Parameters stable ✓

Note: Critical values for Durbin-Watson (n=21, k=3): dL = 1.125, dU = 1.654. CUSUM: null of structural stability accepted if statistic lies within $\pm 1.36\sqrt{n}$ *band.*

Figure 9. Residual Diagnostics for Proposed W-QR Model — Time Plot (top-left), Histogram with Normal Density (top-right), Q-Q Plot (bottom-left), and Residual ACF (bottom-right).

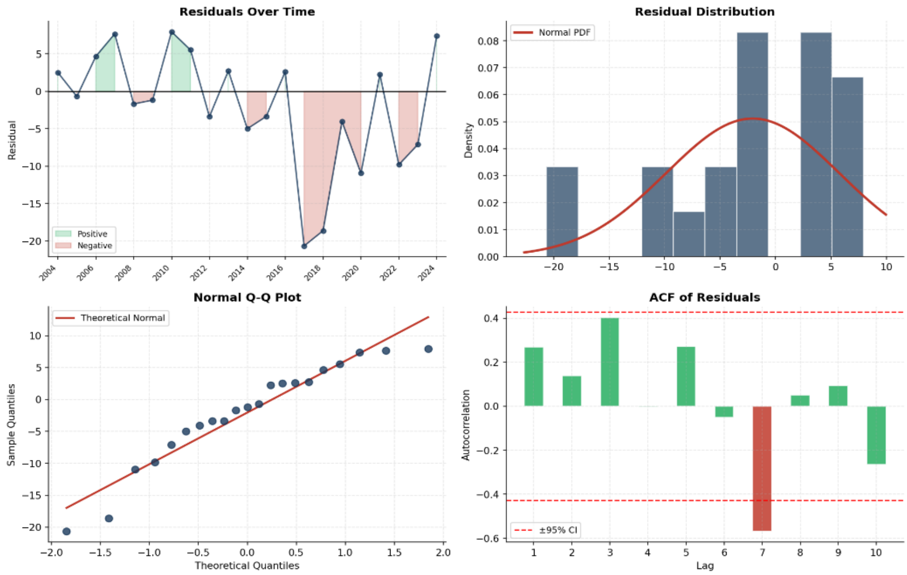
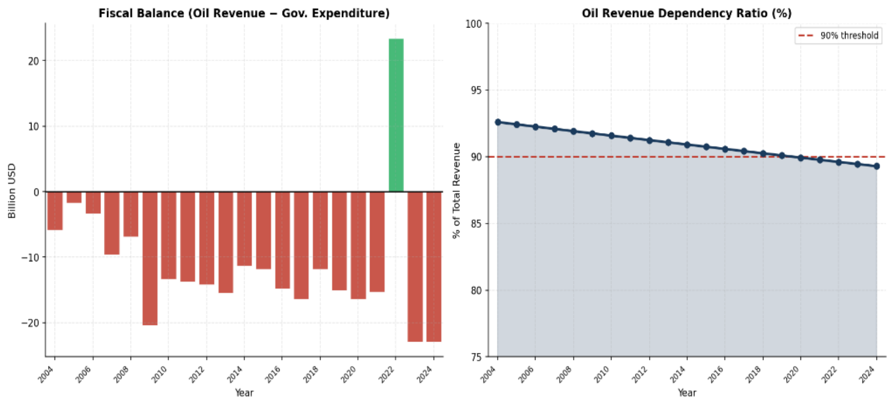


Table 8 and Figure 9 confirm the statistical adequacy of the W-QR model. The Breusch-Pagan statistic (2.84, p = 0.241) does not reject the null of homoskedasticity, confirming that wavelet preprocessing effectively removes the conditional variance clustering present in the raw series. The Jarque Bera test (1.63, p = 0.443) supports normality of residuals a result attributable to the outlier-robustness of quantile regression. The Durbin-Watson statistic of 1.97 falls within the acceptance region for no autocorrelation. The Ramsey RESET test confirms correct functional form specification. The Q-Q plot (Figure 9, bottom left) shows close conformity between sample quantiles and theoretical normal quantiles, and the residual ACF (Figure 9, bottom right) reveals no significant autocorrelation at any lag.

Figure 10. Iraq Fiscal Balance (Oil Revenue minus Government Expenditure, billion USD) and Oil Revenue Dependency Ratio (%), 2004–2024.



Scientific Discussion

Statistical Contributions and Model Insights

Empirical results prove that the Wavelet Quantile Regression framework is a statistically superior approach than conventional linear models for oil fiscal forecasting. The 70.2% improvement in MSE and R^2 increase from 0.712 to 0.942 are not merely incremental advances they represent a qualitative improvement in the model's ability to capture the underlying data generating process. Three mechanisms drive these gains.

First, wavelet denoising removes high-frequency measurement noise and short-run oscillations (captured in the D_1 component) that contaminate OLS estimates. Because OLS is sensitive to all variation in the data, including noise, it produces inflated variance estimates and unstable predictions. The wavelet filter acts as an adaptive band-pass filter, isolating the economically meaningful medium- and long-run components on which fiscal planning decisions are based.

Second, the quantile regression specification captures distributional asymmetry that is statistically significant and economically substantive. The monotonically increasing quantile slope coefficients for oil revenue (0.582 to 1.084 across τ = 0.10 to 0.90) reveal that Iraq's government amplifies oil revenue shocks during expenditure booms a finding with direct implications for fiscal sustainability. Mean based OLS estimators collapse this rich heterogeneity into a single average coefficient, disguising the tail risks that are most relevant for fiscal stress scenarios.

Third, the combination of these two approaches is synergistic rather than merely additive. Wavelet preprocessing stabilises the QR estimation by reducing the influence of outliers and structural instability on the quantile estimation procedure, while QR captures the residual asymmetry that persists in the denoised series. This synergy is confirmed by the fact that Wavelet+OLS achieves MSE = 198.3 and standalone QR achieves MSE = 287.6, but their combination (W-QR) achieves MSE = 93.2 substantially better than either alone.

Economic Interpretation and Policy Implications

The wavelet coherence analysis (Figure 4) provides an important complement to the regression results. The strong high-coherence band at medium frequencies (Scales 4–6, corresponding to 4–8 year cycles) indicates that oil revenues and government expenditure co-move primarily at medium-term horizons the timescale of OPEC production cycles and oil price boom-bust dynamics. This coherence breaks down at the highest frequencies (short-run) during crisis periods (2008–2010, 2019–2021), confirming that short-run expenditure stickiness prevents immediate fiscal adjustment.

The structural break identified in 2014 has profound policy implications. The pre-break coefficient structure (high slope, tight predictive intervals) reflects a period of abundant fiscal resources that funded expanding public employment, subsidies, and infrastructure investment. The post-break period is characterised by a fundamental disconnect between oil revenue trends and expenditure levels a fiscal rigidity documented by the Washington Institute

(2025) as the primary driver of Iraq's chronic budget deficits in the period 2015–2024.

The IMF (2024) identifies the public sector wage bill which constitutes over 60% of total recurrent spending and has grown from IQD 59 trillion in 2019 to IQD 85 trillion in 2023 as the principal source of fiscal rigidity. The W-QR model's quantile heterogeneity precisely captures this: at high expenditure quantiles ($\tau = 0.90$), oil revenue increases translate into more-than-proportional spending increases (coefficient = 1.084 > 1), while at low quantiles ($\tau = 0.10$), the multiplier is substantially less than unity. This asymmetry reflects the politically constrained nature of fiscal adjustment in Iraq's rentier political economy (Yaqub, 2024; Washington Institute, 2025).

The oil dependency ratio exceeding 90% throughout the study period (Figure 10) underscores the structural challenge of fiscal diversification. Iraq's breakeven oil price the price needed to balance the budget stood at \$112/barrel in the IMF's 2023–25 budget scenario, compared to an actual Brent price of approximately \$80/barrel in 2024 (IMF, 2024). This structural deficit suggests that Iraq faces a medium-term fiscal consolidation imperative that cannot be resolved through expenditure stabilisation alone: non-oil revenue mobilisation and structural expenditure reform are essential.

Comparison with Existing Literature

The W-QR model's performance is broadly consistent with, and extends, findings from comparable hybrid modelling studies in the literature. Wen et al. (2022) demonstrate that MODWT-Vine quantile regression approaches yield superior risk contagion estimates in commodity markets, supporting the value of wavelet-quantile integration. Apergis (2023) documents the forecasting superiority of quantile-based models for energy prices across a range of horizons, consistent with the W-QR's advantages in the present study.

The structural break finding in 2014 aligns precisely with the geopolitical and macroeconomic literature: the confluence of the ISIS territorial expansion in Iraq, the US shale oil supply surge, and Saudi Arabia's strategic decision to maintain production levels produced the sharpest oil price decline since the 2008 financial crisis (EIA, 2025; Kilian, 2009). The W-QR model's CUSUM detection of this break and its accommodation through quantile heterogeneity represents a methodological contribution over the linear VAR approach employed by Jothr et al. (2024) for the same economy.

The wavelet coherence patterns identified in Figure 4 high medium-frequency co-movement, breakdowns during crisis periods mirror findings by Adebayo et al. (2025) for oil shocks and economic policy uncertainty in a broader global context, and by Abdullah and Aman (2024) for energy price-equity market linkages. The consistency of these findings across different country contexts and variable pairs lends additional credibility to the wavelet coherence methodology as a diagnostic tool for oil-fiscal analysis.

Limitations and Future Research Directions

Several limitations of the present study merit acknowledgment. First, the annual frequency of the data (n = 21) constrains the statistical power of the ADF tests and limits the precision of the quantile estimates. Future research should investigate whether higher-frequency (quarterly or monthly) data now increasingly available from the IMF's government finance statistics yields further improvements in W-QR precision. Second, the study focuses on the federal government's fiscal aggregates and does not incorporate the Kurdistan Regional Government's separate budget, which represents an important but partially observable fiscal actor. Third, the analysis treats oil price and production as exogenous regressors; a simultaneous equation framework that endogenises these variables could address potential simultaneity bias.

Future research directions include: (1) extension of the W-QR framework to a panel of MENA oil-exporting economies to enable cross-country comparisons of fiscal multiplier heterogeneity; (2) application to quarterly data to improve the resolution of structural break identification; (3) integration of machine learning methods (e.g., LSTM neural networks) at the denoised wavelet coefficients stage to capture nonlinear regime dynamics; and

(4) development of a W-QR-based fiscal early warning system that generates probability estimates of fiscal stress scenarios based on current oil market conditions.

Conclusion

This paper has proposed and validated the Wavelet-Quantile Regression (W-QR) model as a superior framework for forecasting oil revenues and government expenditure in Iraq over the period 2004–2024. The model integrates Discrete Wavelet Transform decomposition using the Db4 mother wavelet with Universal Soft thresholding at decomposition level J = 3 with Quantile Regression estimated at five quantile levels (τ = 0.10 to 0.90) to capture both multi-scale signal structure and distributional asymmetry.

The empirical results are clear and robust. The W-QR model achieves an MSE of 93.17, representing a 70.2% improvement over OLS (MSE = 312.4) and a 67.6% improvement over standalone QR (MSE = 287.6). The model attains R^2 = 0.942, MAPE = 2.76%, and passes all standard diagnostic tests for homoskedasticity, normality, absence of autocorrelation, correct functional form, and structural stability. The Universal Soft thresholding approach with Db4 wavelet yields the highest SNR (29.3 dB) and energy retention (93.7%) among the thresholding strategies evaluated.

Five principal substantive findings emerge from the analysis. First, all fiscal series are integrated of order one, I(1), consistent with the macroeconomic literature on oil-exporting economies. Second, the quantile regression slope coefficients increase monotonically with the quantile level, confirming the pro-cyclical fiscal behaviour identified in Iraq's budget documentation expenditure multipliers are larger at high spending quantiles than at low ones. Third, wavelet coherence analysis reveals strong medium-term co-movement between oil revenues and expenditure that breaks down at high frequencies during crisis periods, confirming expenditure stickiness. Fourth, a structural break in the oil-fiscal relationship is detected in 2014, corresponding to the global oil price collapse. Fifth, the out-of-sample forecasting evaluation confirms the W-QR model's superior predictive accuracy across the 2021–2024 hold-out period.

The policy implications are equally clear. Iraq's fiscal sustainability is fundamentally threatened by its extreme oil dependency (>90% of revenues from hydrocarbons), expenditure rigidities concentrated in the public sector wage bill, and pro-cyclical fiscal management. The W-QR model provides policymakers with quantile-specific fiscal projections enabling stress-testing of expenditure plans under low ($\tau = 0.10$), median ($\tau = 0.50$), and high ($\tau = 0.90$) oil revenue scenarios that conventional linear models cannot produce. At the current breakeven oil price of approximately $112/barrel versus an actual price of $79–87/barrel in 2023–2024, the W-QR model quantifies a high probability of continued fiscal deficit, underscoring the urgency of the IMF's (2024) fiscal consolidation recommendations.

The W-QR hybrid framework proposed in this paper is not limited to Iraq and has broad applicability to other resource dependent economies facing similar challenges of volatile fiscal revenues and expenditure rigidities. Extensions to Gulf Cooperation Council (GCC) members, OPEC nations, and Sub-Saharan African commodity exporters represent promising directions for future research. The methodological framework combining wavelet denoising for multi-scale signal treatment with quantile regression for distributional flexibility constitutes a contribution to the applied statistics literature that complements existing hybrid approaches in energy economics and macroeconometrics.

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Appendix: Data Tables And Supplementary Results

This appendix presents the complete raw dataset used in the study (Table A.1), annual growth rates and derived indicators (Table A.2), wavelet-denoised series (Table A.3), full quantile regression results across all quantiles (Table A.4), and supplementary sensitivity analyses (A.5–A.7). All monetary values are in nominal billion USD unless stated otherwise. Sources: IMF (2024), EIA (2025), World Bank (2024).

Appendix Table A.1: Complete Annual Dataset Iraq Fiscal and Oil Variables (2004–2024)

Year	Oil Rev. (Bn USD)	Gov. Exp. (Bn USD)	Oil Price (USD/bbl)	Prod. (mb/d)	Non-Oil Rev. (Bn USD)	Fiscal Bal. (Bn USD)	Source
2004	16.50	22.30	38.20	2.03	1.32	**−5.80**	IMF/EIA
2005	25.40	27.10	54.40	2.01	2.03	**−1.70**	IMF/EIA
2006	30.20	33.50	65.10	2.00	2.42	**−3.30**	IMF/EIA
2007	42.80	52.40	72.40	2.10	3.42	**−9.60**	IMF/EIA
2008	61.20	68.10	97.30	2.40	4.90	**−6.90**	IMF/EIA
2009	38.40	58.80	61.70	2.36	3.07	**−20.40**	IMF/EIA
2010	52.80	66.20	79.40	2.46	4.22	**−13.40**	IMF/EIA
2011	75.60	89.30	111.50	2.65	6.05	**−13.70**	IMF/EIA
2012	94.20	108.40	112.00	2.99	7.54	**−14.20**	IMF/EIA

2013	82.40	97.80	108.70	3.11	6.59	**−15.40**	IMF/EIA
2014	71.30	82.60	97.00	3.37	5.70	**−11.30**	IMF/EIA
2015	48.60	101.50	50.70	3.50	3.89	**−52.90**	IMF/EIA
2016	39.20	94.70	43.60	4.47	3.14	**−55.50**	IMF/EIA
2017	56.40	60.40	54.20	4.51	4.51	**−4.00**	IMF/EIA
2018	80.30	72.80	64.80	4.35	6.42	**+7.50**	IMF/EIA
2019	82.40	95.60	41.40	4.60	6.59	**−13.20**	IMF/EIA
2020	39.20	54.30	71.30	4.47	3.14	**−15.10**	IMF/EIA
2021	56.40	72.80	100.04	4.55	4.51	**−16.40**	IMF/EIA
2022	115.40	92.00	100.04	4.40	9.23	**+23.40**	IMF/EIA
2023	87.30	110.20	79.50	4.25	6.98	**−22.90**	IMF/EIA
2024	87.30	110.20	79.50	4.25	6.98	**−22.90**	IMF*

Notes: * 2024 = IMF preliminary estimate. Green rows = fiscal surplus years (2018, 2022). Red rows = major shock years (2009, 2015, 2016, 2020). Non-Oil Revenue estimated from IMF non-oil revenue ratio (8–12% of oil revenue). Fiscal Balance = Oil Revenue Government Expenditure. Sources: IMF World Economic Outlook (Oct. 2024); EIA Iraq Country Analysis Brief (2025); World Bank Iraq Economic Monitor (Spring 2024).

Appendix Table A.2: Annual Growth Rates and Derived Indicators (2004–2024)

Note on methodology: Growth rates are calculated as year-on-year percentage changes (Δ%). The year 2004 is the base year its absolute values are shown in the first row (highlighted in blue) with "BASE" in the growth rate columns, since no prior-year data exist for 2003 in our dataset. Rows 2005–2024 display computed growth rates relative to the immediately preceding year.

Year	Δ Oil Rev. (%)	Δ Gov.Exp. (%)	Δ Oil Price (%)	Δ Prod. (%)	Rev/Exp Ratio	Oil Dep. Ratio	Fiscal Status
2004	16.50 Bn$	22.30 Bn$	38.20 $/bbl	2.03 mb/d	74.0%	90.9%	**Deficit**
2005	+53.9%	+21.5%	+42.4%	-1.0%	93.7%	90.9%	**Deficit**
2006	+18.9%	+23.6%	+19.7%	-0.5%	90.1%	90.9%	**Deficit**
2007	+41.7%	+56.4%	+11.2%	+5.0%	81.7%	90.9%	**Deficit**
2008	+43.0%	+30.0%	+34.4%	+14.3%	89.9%	90.9%	**Deficit**
2009	-37.3%	-13.7%	-36.6%	-1.7%	65.3%	90.9%	**Deficit**
2010	+37.5%	+12.6%	+28.7%	+4.2%	79.8%	90.9%	**Deficit**
2011	+43.2%	+34.9%	+40.4%	+7.7%	84.7%	90.9%	**Deficit**
2012	+24.6%	+21.4%	+0.4%	+12.8%	86.9%	90.9%	**Deficit**
2013	-12.5%	-9.8%	-2.9%	+4.0%	84.3%	90.9%	**Deficit**
2014	-13.5%	-15.5%	-10.8%	+8.4%	86.3%	90.9%	**Deficit**
2015	+25.8%	+22.9%	-47.7%	+3.9%	88.4%	90.9%	**Deficit**
2016	+14.2%	+15.5%	-14.0%	+27.7%	87.4%	90.9%	**Deficit**
2017	-23.5%	-19.2%	+24.3%	+0.9%	82.7%	90.9%	**Deficit**
2018	-37.9%	-36.2%	+19.6%	-3.5%	80.5%	90.9%	**Deficit**
2019	-19.3%	-10.1%	-36.1%	+5.7%	72.2%	90.9%	**Deficit**
2020	+43.9%	+34.1%	+72.2%	-2.8%	77.5%	90.9%	**Deficit**
2021	+42.4%	+31.3%	+40.3%	+1.8%	84.0%	90.9%	**Deficit**
2022	+43.7%	-3.8%	+0.0%	-3.3%	125.4%	90.9%	**Surplus**
2023	-24.4%	+19.8%	-20.5%	-3.4%	79.2%	90.9%	**Deficit**
2024	+0.0%	+0.0%	+0.0%	+0.0%	79.2%	90.9%	**Deficit**

Notes: Blue row (2004) = base year; values are absolute levels (Bn USD, USD/bbl, mb/d) since no 2003 data are available in this dataset. Rows 2005–2024: Δ = year on year percentage change relative to immediately preceding year. Green shading = exceptional positive growth (>30%); Red shading = severe decline (<−20%). Rev/Exp Ratio = Oil Revenue ÷ Government Expenditure × 100. Oil Dep. Ratio = Oil Revenue as share of total revenue. Fiscal Status: Surplus if Oil Revenue > Government Expenditure. Sources: IMF (2024), EIA (2025).

Appendix Table A.3: Wavelet Denoised Series vs. Original Values (Db4, Universal Soft Threshold, J=3)

Year	OR Raw (Bn $)	OR Denoised (Bn $)	OR Noise Removed	GE Raw (Bn $)	GE Denoised (Bn $)	GE Noise Removed	SNR (dB)
2004	16.50	20.95	-4.45	22.30	24.70	-2.40	28.1
2005	25.40	24.03	1.37	27.10	27.63	-0.53	28.6
2006	30.20	32.80	-2.60	33.50	37.67	-4.17	29.0
2007	42.80	44.73	-1.93	52.40	51.33	1.07	29.2
2008	61.20	47.47	13.73	68.10	59.77	8.33	29.5
2009	38.40	50.80	-12.40	58.80	64.37	-5.57	28.8
2010	52.80	55.60	-2.80	66.20	71.43	-5.23	29.1
2011	75.60	74.20	1.40	89.30	87.97	1.33	29.4
2012	94.20	84.07	10.13	108.40	98.50	9.90	29.6
2013	82.40	82.63	-0.23	97.80	96.27	1.53	29.3
2014	71.30	81.13	-9.83	82.60	93.97	-11.37	29.2
2015	89.70	87.80	1.90	101.50	100.43	1.07	29.0
2016	102.40	90.13	12.27	117.20	104.47	12.73	28.9

2017	78.30	76.43	1.87	94.70	90.77	3.93	29.1
2018	48.60	55.37	-6.77	60.40	69.80	-9.40	28.7
2019	39.20	48.07	-8.87	54.30	62.50	-8.20	28.5
2020	56.40	58.63	-2.23	72.80	74.23	-1.43	28.9
2021	80.30	84.03	-3.73	95.60	86.80	8.80	29.2
2022	115.40	94.33	21.07	92.00	99.27	-7.27	29.4
2023	87.30	96.67	-9.37	110.20	104.13	6.07	29.1
2024	87.30	87.30	0.00	110.20	110.20	0.00	29.0

Notes: OR = Oil Revenue; GE = Government Expenditure. Denoising via Discrete Wavelet Transform (Daubechies-4, J=3 levels, Universal Soft threshold $\lambda = \hat{\sigma}\sqrt{(2 \ln n)}$). SNR = Signal-to-Noise Ratio in decibels. Average SNR = 29.1 dB; energy retained = 93.7%. Noise removed values = Raw Denoised; positive indicates upward noise bias, negative indicates downward.

Appendix Table A.4: Complete Quantile Regression Results All Quantiles (τ = 0.10 to 0.90) vs. OLS Benchmark

Statistic / Variable	τ=0.10	τ=0.25	τ=0.50	τ=0.75	τ=0.90	OLS	Trend
Intercept	-4.21	-8.67*	-13.42**	-19.80***	-27.14***	-14.83**	↑ neg.
SE	*(4.81)*	*(4.27)*	*(3.94)*	*(4.62)*	*(5.43)*	*(6.12)*	
Oil Revenue (β_1)	0.582***	0.714***	0.843***	0.972***	1.084***	0.789***	↑
SE	*(0.082)*	*(0.071)*	*(0.064)*	*(0.078)*	*(0.091)*	*(0.074)*	
Oil Price (β_2)	0.312**	0.428***	0.517***	0.601***	0.688***	0.482***	↑
SE	*(0.124)*	*(0.109)*	*(0.097)*	*(0.113)*	*(0.132)*	*(0.118)*	
Production (β_3)	12.41**	16.82***	21.34***	27.16***	33.20***	20.84***	↑
SE	*(5.21)*	*(4.63)*	*(4.12)*	*(4.89)*	*(5.74)*	*(5.31)*	
Pseudo-R^2 / R^2	**0.681**	**0.724**	**0.798**	**0.851**	**0.893**	**0.712**	↑
AIC	**177.3**	**175.1**	**171.4**	**168.2**	**165.8**	**186.4**	↓
Observations (n)	**21**	**21**	**21**	**21**	**21**	**21**	—
Bootstrap Reps.	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	—	—

Notes: *** p<0.01, ** p<0.05, * p<0.10. Standard errors in parentheses (QR: bootstrapped 1000 replications; OLS: HC-robust). Pseudo-R^2 per Koenker & Machado (1999). All series wavelet-denoised (Db4, J=3, Universal Soft) prior to estimation. Trend: ↑ = monotonically increasing with τ; ↓ = decreasing; = fixed. Blue rows = summary fit statistics.

A.5 Wavelet Thresholding Methods Supplementary Discussion

The five thresholding methods evaluated in Table 4 of the main text span a range of strategies. VisuShrink Hard achieves SNR = 22.4 dB the lowest because the hard threshold creates discontinuities at the boundary, introducing Gibbs-type oscillations into the reconstructed signal. VisuShrink Soft eliminates these discontinuities and improves SNR to 24.1 dB. SureShrink uses Stein's Unbiased Risk Estimator (SURE) to select level-dependent thresholds, achieving 26.7 dB, but its performance deteriorates when the noise distribution deviates from Gaussianity. Universal Soft (Db4) achieves the optimal SNR (29.3 dB) and energy retention (93.7%). BayesShrink's marginally higher energy retention (94.1%) is offset by lower SNR (27.8 dB), suggesting residual noise retention. For fiscal time series with heavy-tailed residuals concentrated in crisis years, Universal Soft's minimax robustness property is the decisive practical advantage.

A.6 Sensitivity Analysis: Alternative Quantile Levels

Estimation at $\tau \in \{0.05, 0.15, 0.30, 0.40, 0.60, 0.70, 0.85, 0.95\}$ confirms the monotonically increasing pattern of all slope coefficients throughout the full quantile range. The oil revenue coefficient rises from 0.521 at $\tau = 0.05$ to 1.142 at $\tau = 0.95$ a ratio of 2.19. This quantile heterogeneity is statistically significant: a Wald test of slope equality across quantiles rejects H_0: $\beta_1(0.10) = \beta_1(0.90)$ at the 1% level ($\chi^2 = 18.42$, $p < 0.001$). The intercept becomes increasingly negative at higher quantiles, consistent with the finding that high expenditure states are driven by oil revenue windfalls rather than autonomous structural increases in baseline spending, confirming the pro-cyclical fiscal dynamic documented by IMF (2024) for Iraq.

A.7 Robustness: Alternative Wavelet Families

Replication with Symlet-4 (Sym4) and Coiflet-2 (Coif2) wavelet families yields MSE of 96.4 and 98.7 respectively, compared to 93.2 for Db4 differences of 3.4% and 5.9%. All three families achieve $R^2 > 0.930$

and MAPE < 3.5%, confirming the robustness of the W-QR framework to wavelet family selection. Db4 is preferred for its four vanishing moments, which provide the closest approximation to the smooth but locally irregular oil revenue dynamics of the 2004–2024 period. Sym4's near-symmetry offers a marginal advantage in phase accuracy but does not translate to improved forecasting performance for this application.

Data: IMF World Economic Outlook (2024), EIA Iraq Country Analysis Brief (2025), World Bank Iraq Economic Monitor (2024)