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Premium Sufficiency Reserve on Joint Life Insurance With Laplace Distribution

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Abstract

Each insurance participant pays a premium to the insurance company during the coverage period. In paying the sum insured to insurance participants, insurance companies need to prepare reserve costs. This reserve fee is used to pay for the needs of insurance companies and insurance participants. This research explains the calculation of premium sufficiency reserve for joint life insurance for life insurance participants aged x years and y years with Laplace distribution. The parameters of the Laplace distribution are estimated using the method of momen and the method of maximum likelihood. The solution of the problem is obtained by determining the initial life annuity term, single premium, and annual premium so that the premium sufficiency reserve formula based on Laplace distribution is obtained. The results of the calculation of premium sufficiency reserves of joint life insurance using Laplace distribution are more less the same as the prospective reserves of joint life insurance using Laplace distribution.

Keywords: Premium Sufficiency reserve, endownment joint life insurance, method of moment, method of maximum likelihood, Laplace distribution

1. Introduction

Nowadays, all aspects that are important to human life will not always be in a good and safe condition. In real life, these important aspects will always be surrounded by various things that might threaten security, safety and cause financial losses. One of the efforts to overcome this is by transferring the financial loss to another party which then led to the existence of insurance, one type of insurance is endowment life insurance. Endowment life insurance is a combination of pure endowment life insurance and term life insurance where the insured must pay the sum insured during or at the end of the policy coverage period, either death or survival (Futami, 1993).

Multi life insurance id divided into two parts according to the payment pe- riod, namely joint life and last survivor. Joint life insurance is joint life insurance.

where the premium is paid until the first death of the participants. last survivor life insurance is life insurance with premium payments until the last death of the participants (Bowers et al., 1997).

Based on theory, the amount of money available to the company during the coverage period is called reserves. Based on the method of colculation, reserves are divided into two types, namely retrospective reserves and prospective reserves. In prospective reserves there are several reserve modifications, one of which is premium sufficiency. Premium sufficiency is the calculation of insurance premium reserves based on gross premium assumptions. Gross premium is an annuity, but gross premium has a greater value than net premium (Futami, 1994).

Laplace distribution is one of the continuous probability distributions named after Pierre Simon Laplace. Laplace distribution is also called double exponential distribution which has very wide applications (Al-noor & Rasheed, 2012).

In this article, premium reserves are determined for dual-use joint life insur- ance with premium sufficiency reserves using the Laplace distribution function. In this case, the author limits to include only two insurance participants aged x years and y years in one policy.

2. Literature Review

2.1. Survival Function And Laplace Distribution

The survival function is obtained from the distribution function using a probability density function. The probability of x is denoted by $p(x) = P(X = x)$. The symbol X represents the name of the random variable, while x is the value of the random variable with the probability density function symbolized by $f(x)$. By therefore, the probability density function and distribution function are explained first. Definition 1 (Walpole et al., 2012) Cumulative distribution function $F(x)$ of a continuous random variable X with probability density function $f(x)$ is.

$$F(x) = (X \leq x) = \int_{-\infty}^{x} f(t)\, dt \; for - \infty < x < \infty$$

The survival function or life function denoted by $S(x)$ is used to express the chance of a person living to the age of x years where $x \geq 0$. If X is a continuous random variable that expresses the age of the insurance participant to survive, then the survival function of the insurance participant described (Bowers et al., 1997), namely

$$S(x)$$

The relationship between the survival function and the cumulative distribution function is as follows:

$$S(x) = 1 - P(X \leq x) \tag{1}$$

Based on Definisi 1 and equation (1), the survival function of an insurance participant aged x years is obtained, namely

$$S(x) \tag{2}$$

The cumulative distribution function of a continuous random variable $T(x)$ for $t \geq 0$ denoted by $F_{T(x)}(t)$ can be expressed as follows (Dickson et al., 2009):

$$F_{T(x)}(t) = P(T(x) \leq t)),\, t \geq 0 \tag{3}$$

The probability of a person aged x in the next t years is expressed as follows (Bowers et al., 1997):

$${}_tq_x = P(T(x) \leq t), t \geq 0 \tag{4}$$

Based on equation (3) and equation (4) the relationship between the cumulative distribution function and the probability of death is obtained, namely

$$F_{T(x)}(t) = \quad {}_tq_x \tag{5}$$

The cumulative distribution function of a continuous random variable $T(x)$ can also be expressed as follow (Walpole et al., 2012):

$$F_{T(x)}(t) = \frac{F(x+t) - F(x)}{S(x)} \tag{6}$$

The survival function of a continuous random variable T (x) is denoted by $S_{T(x)}(t)$ which is expressed as follows:

$$S_{T(x)}(t) = 1 - F_{T(x)}(t) \tag{7}$$

Furthermore equation (5) is substituted into equation (7) so that it can be writen into

$$S_{T(x)}(t) = 1 - \quad {}_tq_x \tag{8}$$

The function $S_{T(x)}(t)$ expresses the chance that a person aged x years can survive up to t years which is expressed as follows:

$$S_{T(x)}(t) = 1 - \quad {}_tp_x \tag{9}$$

Based on equation (8) and equation (9) the relationship between chance and probability of death is obtained, namely

$${}_tp_x = 1 - \quad {}_tq_x \tag{10}$$

Joint life insurance is an insurance program that provides protection to two or more people who have a family relationship (Futami, 1994). In this article the marger is limited to two insurance participants aged x years and y years respectively which is then expressed as a joint life.

The probability of life insurance participants in the joint life status is expressed in a combined form. This is because the chances of each participant are independent of each other, so that the life chances for the combined status of joint life are described in (Bowers et al., 1997), and (Promislow, 2011) as follow:

$$ {}_tp_{xy} = \quad {}_tp_{x\,t}p_y \tag{11}$$

In this article, the life expectancy and death expectancy of insurance participants are determined using the Laplace distribution. The probability density function of the Laplace distribution is expressed as follows (Ogunsanya & Job, 2023):

$$ f(x;\theta,\beta) = \frac{1}{2\beta}e^{\left(\frac{-|x-\theta|}{\beta}\right)}, -\infty < x < \infty,\ \ \theta > 0, \qquad \beta > 0. \tag{12}$$

where θ is a shape parameter, and β is a scalar parameter.

Furthermore, from equation (12) the cumulative distribution function of the Laplace distribution is obtained, namely

$$ F(x) = 1 - \frac{1}{2}e^{\left(-\frac{x-\theta}{\beta}\right)}. \tag{13}$$

Then by substituting equation (13) into equation (2) we obtain the survival function $S(x)$ of Laplace distribution, namely

$$ S(x) = \left(\frac{1}{2}e^{\left(-\frac{x-\theta}{\beta}\right)}\right). \tag{14}$$

Based on equation (13), the cumulative distribution function for $(x + t)$ years of Laplace distribution, namely

$$ F(x+t) = \left(1 - \frac{1}{2}e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}\right). \tag{15}$$

The cumulative distribution function for an insurance participant aged x years dying at an interval of t years is obtained by substituting equation (13), equation (14), and equation (15) into equation (6) as follows:

$$ F_{T(x)}(t) = \left(\frac{e^{\left(-\frac{x-\theta}{\beta}\right)} - e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}}\right). \tag{16}$$

Based on equation (5) and equation (16), the probability that a person aged x years will die in the next t years with Laplace distribution is obtained, namely

$$ {}_tq_x = \left(\frac{e^{\left(-\frac{x-\theta}{\beta}\right)} - e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}}\right). \tag{17}$$

Life chances of a person aged x years living up to t years then using the Laplace distribution is obtained by equation (17) is substituted into equation (10), namely

$$ {}_tp_x = \left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}}\right). \tag{18}$$

Based on the equation (18) life chances of a life insurance participant aged y years can survive until t years later with a Laplace distribution, namely

$$ {}_tp_y = \left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}}\right). \tag{19}$$

Life chances the joint life status of an insured person aged x years and y years with Laplace distribution is obtained by substituting equation (18) and equation (19) into equation (11), namely

$$ {}_tp_{xy} = \left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}}\right)\left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}}\right). \tag{20}$$

3. Materials and Methods

3.1. Parameter Estimation on Laplace Distribution

In Laplace distribution there are parameters θ and β whose values are unknown. Therefore, the parameters θ and β are estimated using the method of moments and the maximum likelihood method.

Determining the estimated value of the parameter θ using the method of moments, the population moment of the Laplace distribution is obtained, namely

$$\begin{aligned} E(x) &= \int_{-\infty}^{0} x \cdot \frac{1}{2\beta} e^{\frac{-x-\theta}{\beta}} dx + \int_{0}^{\infty} x \cdot \frac{1}{2\beta} e^{\frac{x-\theta}{\beta}} dx \\ &= -\frac{\beta}{2} + \frac{\theta}{2} + \frac{\beta}{2} + \frac{\theta}{2} \\ &= \frac{\theta}{2} + \frac{\theta}{2}, \\ E(x) &= \theta. \end{aligned} \tag{21}$$

Furthermore by equating the population moments and sample moments the esti- mated value of the parameter θ is obtained which is denoted by $\hat{\theta}_{MME}$, namely

$$\hat{\theta}_{MME} = \bar{x}. \tag{22}$$

Next determine the estimated parameter β by using the maximum likelihood method of the Laplace distribution, namely

$$L(\beta) = (2\beta)^{-n^*} e^{\left(-\frac{1}{\beta}\sum_{i=1}^{n^*} |x_i - \theta|\right)}. \tag{23}$$

Equation (23) is expressed in natural logarithm (ln) transformation, namely

$$L(\beta) = -n^* In(2) - n^* In(\beta) - \frac{1}{\beta}\sum_{i=1}^{n^*} |x_i - \theta|. \tag{24}$$

Then determine the first derivative of β to obtain:

$$\frac{\partial}{\partial \beta}[\mathrm{In}L(\beta)] = \frac{\partial}{\partial \beta}\left[-n^* In(2) - n^* In(\beta) - \frac{1}{\beta}\sum_{i=1}^{n^*} |x_i - \theta|\right],$$

$$\frac{\partial}{\partial \beta}[\mathrm{In}L(\beta)] = -\frac{n^*}{\beta} + \frac{\sum_{i=1}^{n^*} |x_i - \theta|}{\beta^2}. \tag{25}$$

Then equalized to zero obtained

$$0 = -n^*\beta + \sum_{i=1}^{n^*} |x_i - \theta|. \tag{26}$$

Furthermore by using equation (26) and substituting equation (22) obtained the estimated value of the parameter β denoted by $\hat{\beta}$, namely

$$\hat{\beta} = \frac{1}{n^*}\sum_{i=1}^{n^*} |x_i - \hat{\theta}_{MME}|. \tag{27}$$

3.2. Innitial Joint Life Annuity Term Life Insurance With Laplace Distribution

Life annuity is an annuity whose payment is influenced by the chance of life and the chance of death of insurance participants over a certain period of time. A life annuity whose payments are made at the beginning of each year for a certain period of time is called a term early life annuity (Futami, 1993). The value of the initial life annuity is influenced by the interest rate denoted by i. The interest rate used in the study is the compound interest rate, which is a way of calculating interest where the principal amount of the next investment period is the previous principal amount plus the amount of interest earned. In compound interest there is a discount factor denoted by v, which is the present value of a payment of 1 unit of payment made one year later (Futami, 1993), namely

$$v = \frac{1}{1+i}. \tag{28}$$

The discount rate which is the amount of interest lost if the payment is made one year earlier denoted by d (Futami, 1993) can be expressed as follows:

$$d = 1 - v \tag{29}$$

The cash value of the initial joint life annuity for an insured person aged x years and y years with a payout period of n years, namely

$$\ddot{a}_{xy:\overline{n}|} = \sum_{i=1}^{n^*} v^t \; {}_tp_{xy}. \tag{30}$$

The cash value of the initial annuity of x years old and y years old with a coverage period of n years with Laplace distribution is obtained by substituting equation (20) into equation (30), namely

$$\ddot{a}_{xy:\overline{n}|} = \sum_{t=0}^{n-1} v^t \left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}} \right) \left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}} \right). \tag{31}$$

Furthermore the cash value of the initial life annuity of $(x + t)$ years and $(y + t)$ years old insurance participants and the coverage period of $(n - t)$ years based on equation (31) with Laplace distribution, namely

$$\ddot{a}_{x+t,y+t:\overline{n-t}|} = \sum_{k=0}^{(n-1)-1} v^k \left(\frac{e^{\left(-\frac{(x+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}} \right) \left(\frac{e^{\left(-\frac{(y+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}} \right). \tag{32}$$

The cash value of the initial joint life annuity of x years old and y years old insureds with a payout period of m years based on equation (29) with Laplace distribution, namely

$$\ddot{a}_{xy:\overline{m}|} = \sum_{t=0}^{m-1} v^t \left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}} \right) \left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}} \right). \tag{33}$$

Furthermore based on equation (32), the cash value of the initial joint life annuity of participants aged $(x + t)$ years and $(y + t)$ years with a payment coverage period of $(m - t)$ years with Laplace distribution, namely

$$\ddot{a}_{x+t,y+t:\overline{m-t}|} = \sum_{k=0}^{(m-1)-1} v^k \left(\frac{e^{\left(-\frac{(x+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}} \right) \left(\frac{e^{\left(-\frac{(y+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}} \right). \tag{34}$$

3.3. Premium Of Joint Life Insurance With Laplace Distribution

Before calculating the annual premium, fisrt calculate the single premium of joint life insurance. The single premium of joint life insurance is denoted by $A_{xy:\overline{n}|}$ and can be expressed as follows:

$$A_{xy:\overline{n}|} = 1 - d\ddot{a}_{xy:\overline{n}|}.. \tag{35}$$

Furthermore, by substituting equation (31) into equation (35) the single premium of joint life insurance is obtained, namely

$$A_{xy:\overline{n}|} = 1 - d \sum_{t=0}^{n-1} v^t \left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}} \right) \left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}} \right). \tag{36}$$

Based on the equation (36) the single premium for insurance participants aged $(x + t)$ years and $(y + t)$ years with a coverage period of $(n - t)$ years with Laplace distribution, namely

$$A_{x+t,y+t:\overline{n-t}|} = 1 - d \sum_{k=0}^{(n-1)-1} v^k \left(\frac{e^{\left(-\frac{(x+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}} \right) \left(\frac{e^{\left(-\frac{(y+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}} \right). \tag{37}$$

The annual premium is the premium paid at the beginning of each year, which can be the same or change every year (Futami, 1993). Annual premium of joint life insurance with Laplace distribution paid for m years where $m < n$ by insurance participants aged x years and y years denoted by ${}_{m}p_{xy:\overline{n|}}$ can be expessed as follows:

$$ {}_{m}p_{xy:\overline{n|}} = \frac{1 - d\sum_{t=0}^{n-1} v^t \left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}} \right) \left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}} \right)}{\sum_{t=0}^{m-1} v^t \left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}} \right) \left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}} \right)} \tag{38} $$

3.4. Premium Sufficiency Reserve On Joint Life Insurance With Laplace Distribution

Premium sufficiency reserve is the calculation of insurance premium reserves based on gross premium assumptions. Gross premium or gross premium is the premium paid by life insurance participants to the insurance company. Gross premium is also an annuity, but gross premium has a greater value than net premium. In the gross premium, the detemination is influenced by management fees. The gross premium of endowment life insurance can be expressed as follows (Futami, 1994):

$$ {}_{m}p^{*}_{xy:\overline{n|}} = \frac{1}{1-\beta}\left({}_{m}p_{xy:\overline{n|}} + \frac{\alpha'}{\ddot{a}_{xy:\overline{m|}}} + \gamma + \gamma' \frac{\ddot{a}_{xy:\overline{n|}} - \ddot{a}_{xy:\overline{m|}}}{\ddot{a}_{xy:\overline{m|}}} \right). \tag{39} $$

Premium sufficiency reserve is a modification of prospective reserve. In prospec- tive reserve, the calculation of reserves is obtained from the difference between the present value of future payments, denoted by A, and the present value of future revenues denoted by P , namely (Futami, 1994)

$$ {}^{m}_{t}V_{xy:\overline{n|}} = A_{x+t,y+t:\overline{n-t|}} - {}_{m}p_{xy:\overline{n|}}\, \ddot{a}_{x+t,y+t:\overline{m-t|}}\,. \tag{40} $$

In prospective reserves, the calculation of reserve can be obtained from the difference between the present value of future payments denoted by A, and the present value of future receipts denoted by P a. thus equation (40) of prospective reserve can be expressed as follows:

$$ {}^{m}_{t}V_{xy:\overline{n|}} = A - Pa. \tag{41} $$

The calculation of premium sufficiency reserve is carried out on the basis of future expenditures added to the insurance company's management costs in the form of agent commission fees for each premium collection, premium maintenance costs during the payment period, and premium maintenance costs after the payment period until the end of the coverage period. So that the present value of future payments based on prospective reserves changes to (Futami, 1994):

$$ \begin{aligned} A = {} & A_{x+t,y+t:\overline{n-t|}} + \beta\, {}_{m}p^{*}_{xy:\overline{n|}}\, \ddot{a}_{x+t,y+t:\overline{m-t|}} + \gamma\, \ddot{a}_{x+t,y+t:\overline{m-t|}} \\ & + \gamma'\left(\ddot{a}_{x+t,y+t:\overline{n-t|}} - \ddot{a}_{x+t,y+t:\overline{m-t|}}\right). \end{aligned} \tag{42} $$

Meanwhile the present value of future revenue use gross premium instead of net premium. Therefore the present value of future revenue in equation (40) based on prospective reserves changes (Futami, 1994):

$$ Pa = {}_{m}p^{*}_{xy:\overline{n|}}\, \ddot{a}_{x+t,y+t:\overline{m-t|}}\,. \tag{43} $$

Premium sufficiency reserve denoted by ${}^{m}_{t}V^{(ps)}_{xy:\overline{n|}}$, with a premium payment period on *m* years and a reserve calculation time of *t* years is obtain by substituting equation (42) and equation (43) into the equation (41) obtained as follows (Futami, 1994):

$$ {}^{m}_{t}V^{(ps)}_{xy:\overline{n|}} = A_{x+t,y+t:\overline{n-t|}} - (\beta - 1)\, {}_{m}p^{*}_{xy:\overline{n|}}\, \ddot{a}_{x+t,y+t:\overline{m-t|}} + \gamma\, \ddot{a}_{x+t,y+t:\overline{m-t|}}. \tag{44} $$

Furthermore premium sufficiency reserve using the gross premium obtained by substituting the equation (39) into the equation (44) is expressed as follows:

$$ {}^{m}_{t}V^{(ps)}_{xy:\overline{n|}} = A_{x+t,y+t:\overline{n-t|}} - \left({}_{m}p_{xy:\overline{n|}} + \frac{\alpha'}{\ddot{a}_{xy:\overline{m|}}} \right) \ddot{a}_{x+t,y+t:\overline{m-t|}} $$

$$+\gamma'\left(\ddot{a}_{x+t,y+t:\overline{n-t}|}-\frac{\ddot{a}_{xy:\overline{n}|}}{\ddot{a}_{xy:\overline{m}|}}\right)\ddot{a}_{x+t,y+t:\overline{m-t}|}. \tag{45}$$

Premium sufficiency reserve for joint life insurance life n years and insured aged x years and y years with annual premium for m years paid in advance in t years using Laplace distribution is obtained from substituting equation (31), equation (32), equation (33), equation (34), equation (36), equation (38) into the equation (45), namely

$$
\begin{aligned}
{}_{t}^{m}V_{xy:\overline{n}|}^{(ps)} &= R\left(1-d\sum_{k=0}^{(n-1)-1} v^k\left(\frac{e^{\left(-\frac{(x+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}\right)\left(\frac{e^{\left(-\frac{(y+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}\right)\right)\\
&-\left[\frac{R\left(1-d\sum_{t=0}^{n-1} v^t\left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}}\right)\left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}}\right)\right)}{\sum_{t=0}^{m-1} v^t\left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}}\right)\left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}}\right)}+\frac{\alpha'}{\sum_{t=0}^{m-1} v^t\left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}}\right)\left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}}\right)}\right]\\
&\sum_{k=0}^{(m-1)-1} v^k\left(\frac{e^{\left(-\frac{(x+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}\right)\left(\frac{e^{\left(-\frac{(y+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}\right)\\
&+\gamma'\left(\sum_{k=0}^{(n-1)-1} v^k\left(\frac{e^{\left(-\frac{(x+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}\right)\left(\frac{e^{\left(-\frac{(y+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}\right)\right.\\
&\left.-\frac{\sum_{t=0}^{n-1} v^t\left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}}\right)\left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}}\right)}{\sum_{t=0}^{m-1} v^t\left(\frac{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{x-\theta}{\beta}\right)}}\right)\left(\frac{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}{e^{\left(-\frac{y-\theta}{\beta}\right)}}\right)}\sum_{k=0}^{(m-1)-1} v^k\left(\frac{e^{\left(-\frac{(x+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(x+t)-\theta}{\beta}\right)}}\right)\left(\frac{e^{\left(-\frac{(y+t+k)-\theta}{\beta}\right)}}{e^{\left(-\frac{(y+t)-\theta}{\beta}\right)}}\right)\right)
\end{aligned} \tag{46}
$$

4. Results and Discussion

A couple aged 47 years and 40 years respectively joined a joint life insurance program with a term of 20 years and a premium payment period of 18 years. If the sum insured received by the heirs is IDR 100,000,000 and the prevailing interest rate is 6% and the new policy coverage fee (α′) is 0, 5% and the insurance company management fee (γ′) is 5% of the sum insured. Determine:

(i) Prospective reserve of *joint life* insurance using Laplace distribution.

(ii) *Premium sufficiency* reserve of *joint life* insurance using Laplace distribution.

Based on the problem, it is known that $x = 47, y = 40, n = 20, m = 18, i = 0.06,$ $\alpha = 0.005, \gamma = 0.5 \, and \, R = 100,000,000$. Based on equation (28), the discount factor is obtained, namely

$$v=\frac{1}{1+0.06}=0.943396264150.$$

Discount rate based on equation (29), namely

$$d = 1 - 0.943396264150 = 0.0566037358.$$

The calculation of reserves using Laplace distribution is done with Maple 13. Before calculating the reserves first determine the estimated value of the parameters of the Laplace distribution, with the help of *software* Maple 13 obtained the value of $\theta_x = 74.03000000, \theta_y = 67.82000000, \beta_x = 14.01060000$ and $\beta_y = 15.19080000$.

Furthermore the calculation at $t = 1$ for each case using *software* Maple 13 and obtained the following results:

(i) Prospective reserve of *joint life* insurance using Laplace distribution. The prospective reserve for the end of the first year of a married couple who are members of a joint life insurance with Laplace distribution, using the equation (40) obtained
$${}^{18}_{1}V_{47:\overline{20}|} = IDR2{,}670{,}301.70$$

(ii) *Premium sufficiency* reserve of *joint life* insurance using Laplace distribution.
The *premium sufficiency* reserve of endowment life insurance with Laplace distribution for the couple at the end of the first year based on the equation (46) is obtained
$${}^{18}_{1}V^{(ps)}_{47,40:\overline{20}|} = IDR2{,}670{,}301.71$$

The complete calculation of prospective reserves and premium sufficiency of Laplace distribution joint life insurance for a couple aged 47 years and 40 years respectively in year t for the above two cases is presented in Table 1 and illustrated in figure 1.

Table 1: Prospective reserve and premium sufficiency reserve with Laplace distribution

Years(t)	Prospective reserve (IDR) ${}^{18}_{1}V_{47:\overline{20}|}$	Premium Sufficiency reserve (IDR) ${}^{18}_{1}V^{(ps)}_{47,40:\overline{20}|}$
1	2,670,301.70	2,670,301.71
2	4,159,123.56	4,159,123.59
3	5,854,031.69	5,854,031.72
4	7,783,552.93	7,783,552.98
5	9,980,163.06	9,980,163.14
6	12,480,833.12	12,480,833.23
7	15,327,651.80	15,327,651.95
8	18,568,533.83	18,568,534.00
9	22,258,026.29	22,258,026.51
10	26,458,227.04	26,458,227.30

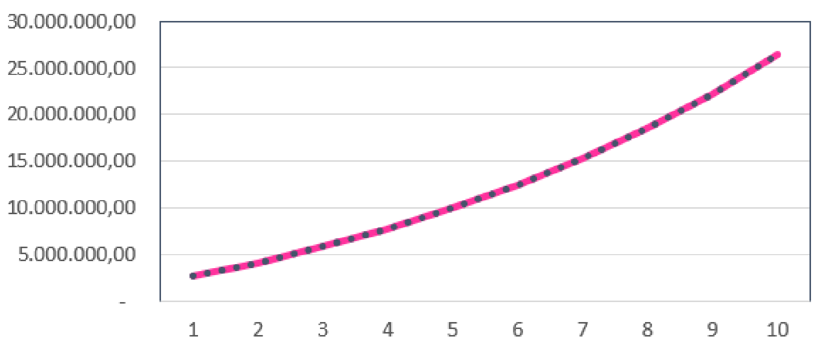

Figure 1: Prospective reserve and *premium sufficiency* reserve *joint life* insurance with Laplace distribution

Based on Table 1 and the illustration of figure 1 it is found that the size of the premium sufficiency reserve of joint life insurance using Laplace distribution are more less the same as the prospective reserve of joint life insurance using Laplace distribution.

5. Conclussion

Based on the discussion, it is known that the premium amount depends on the coverage period, payment period, interest rate, annuity cash value, and the age of the insured. This thesis uses a compound interest rate that depends on the discount factor v and the discount rate d. the calculation premium sufficiency reserve in joint life insurance is affected by management costs namely the cost of covering new policies and maintenance costs.

Premium sufficiency reserve using Laplace distribution are more less the same as the prospective reserve. Premium sufficiency reserve there are insurance main- tenance costs in the calculation of reserves. Prospective reserve and premium suffi- ciency reserve with Laplace distribution increases every year. The single premium with Laplace distribution charged to insurance participants is getting bigger with a smaller initial life annuity.

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