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Unveiling students' cognitive patterns in word problem solving: The case of single-variable linear equations

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Abstract

Students often struggle to transfer their procedural knowledge of linear equations into meaningful solutions when faced with word problems, revealing a persistent gap in integrating conceptual understanding, strategic reasoning, and metacognitive evaluation. This study investigates students' cognitive profiles in solving mathematical word problems involving single-variable linear equations. Grounded in a synthesized framework from Polya's heuristic model and the NCTM problem-solving process, the research focuses on five cognitive stages: understanding, analyzing, strategizing, executing, and evaluating. Using a qualitative descriptive method, data were collected from 30 eighth-grade students through written tasks and semi-structured interviews. The findings indicate strong performance in problem comprehension; however, there was a notable decline in evaluation and reflection stages. Interview data revealed that low-performing students often lacked conceptual understanding and demonstrated limited metacognitive awareness, whereas high performers integrated conceptual, procedural, and reflective thinking. This study highlights the gap between procedural fluency and strategic reasoning across performance levels, emphasizing the need for instructional approaches that integrate metacognitive scaffolding to enhance problem-solving competence. A diagnostic framework is proposed to support teachers in identifying and addressing students' cognitive needs.

Keywords: cognitive profile; linear equation; mathematical thinking; metacognition; problem solving; word problem

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Introduction

Problem-solving has been widely acknowledged as a core competency in 21st-century mathematics education (Albay, 2020; Fayakuun & Agoestanto, 2023; Fitriati & Marlaini, 2020; Olivares et al., 2021; Szabo et al., 2020). It plays a central role in equipping students with essential skills to deal with real-life situations by applying mathematical reasoning. Beyond demonstrating mastery of mathematical content, the ability to solve problems also reflects students' critical, logical, and reflective thinking capacities (Harahap et al., 2024; Widyawati & Rahayu, 2020). These competencies are essential in fostering students' readiness for future academic and professional challenges. As such, cultivating problem-solving proficiency is not only a pedagogical goal but also a strategic educational priority.

Mathematical word problems—defined as verbal descriptions of mathematical tasks that require students to extract relevant information, model the situation mathematically, and solve for unknowns—are an integral component of mathematics instruction (Agusfianuddin et al., 2024; Prasetya et al., 2019; Teo Lian Wan & Abdullah, 2023; Verschaffel et al., 2020). These problems serve as meaningful contexts that challenge students to translate everyday or academic scenarios into formal mathematical representations. This process is cognitively demanding, as it involves understanding the problem scenario, formulating appropriate mathematical models, selecting and executing relevant strategies, and interpreting results in context. Success in solving word problems, therefore, depends on the coordination of multiple cognitive domains. Furthermore, mathematical word problems provide opportunities to foster higher-order thinking skills such as reasoning, communication, and the ability to connect abstract mathematical ideas to real-world applications (Ida et al., 2021; Irmayanti et al., 2020). These features make them a powerful pedagogical tool for promoting deep mathematical understanding.

Despite their instructional value, many students still struggle significantly with word problem solving. The Programme for International Student Assessment (PISA) has consistently reported that Indonesian students perform poorly in mathematical literacy, which encompasses the capacity to solve real-world problems using mathematical knowledge. According to the 2018 PISA report, over 70% of Indonesian students failed to attain Level 2, the minimum benchmark indicating the ability to interpret and apply simple mathematical models in everyday contexts (Ismail et al., 2018; Nurutami et al., 2018). This alarming statistic underscores the pervasive gap in students' cognitive readiness to engage with problem-solving tasks. It suggests that many students lack the ability to move from verbal descriptions to symbolic reasoning, which is a critical component of mathematical problem solving (Ruamba et al., 2024).

Additional studies corroborate this finding by showing that students face difficulties in several key aspects of problem solving, including identifying relevant information, converting text into mathematical symbols, choosing suitable operations, and analyzing problem structure. These challenges suggest a disconnect between procedural knowledge and conceptual understanding. Addressing such issues requires not only improved instruction but also a deeper insight into students' cognitive processing during problem solving. A promising avenue for tackling this challenge lies in mapping students' cognitive profiles to better understand their thinking processes. Cognitive profiling enables educators to pinpoint where and why students encounter obstacles and how instruction can be tailored accordingly (Irianti, 2020; Pradestya et al., 2020).

To that end, the current study adopts a cognitive diagnostic approach by integrating two prominent frameworks in mathematical problem solving: Polya's problem-solving steps and the process standards proposed by the National Council of Teachers of Mathematics (NCTM). Both frameworks consist of key stages—understanding the problem, devising a plan, executing the plan, and reviewing the solution—yet they have rarely been synthesized into a unified, operational framework. Such integration is necessary to develop a more comprehensive and measurable model of student cognition. By aligning these stages with observable indicators, educators can more precisely assess students' problem-solving processes. This also creates opportunities for more targeted assessment tools and instructional designs.

This study synthesizes the two models into five cognitive stages: (1) understanding and formulating the problem, (2) analyzing and diagnosing the problem, (3) designing a solution strategy, (4) executing the strategy and solving the problem, and (5) evaluating and reflecting. Each stage is accompanied by specific indicators to support cognitive analysis. For example, in the first stage, students are expected to identify given and required information, while in later stages, they must be able to apply accurate procedures and verify their results. These indicators serve not only as diagnostic tools but also as formative assessment instruments that can inform instructional improvement. The integration of Polya and NCTM's approaches within this framework provides both theoretical depth and practical utility.

Recent studies have highlighted persistent challenges students face in solving linear equations in one variable, particularly when presented in contextualized or word problem formats (Mondal et al., 2025; Sanders et al., 2025; Schreiber, 2025; Sigus & Mädamürk, 2025). These difficulties often stem from students' limited ability to construct mathematical representations from verbal descriptions and a lack of understanding of the inverse relationship between operations (Kania et al., 2023; Kania & Juandi, 2023; Tao et al., 2025). Word problems involving linear equations have been shown to require not only procedural fluency but also conceptual understanding and strategic competence (Jupri & Drijvers, 2016; Kellman et al., 2010). However, few studies have systematically classified such problems based on cognitive demands and difficulty levels. The current study aims to fill this gap by developing and analyzing word problems involving linear equations according to defined cognitive stages defined in this study.

The significance of this study lies in its contribution to both theory and practice in mathematics education. The proposed framework supports the development of differentiated and responsive pedagogical strategies that address students' individual cognitive needs (Jimenez et al., 2024). It also provides empirical grounding for the design of process-based assessments that go beyond evaluating final answers to capturing the quality of students' thinking. In doing so, this research responds to the urgent need for instructional practices that are aligned with the goals of 21st-century education. Moreover, by focusing on a foundational topic—single-variable linear equations—this study ensures high applicability and relevance to secondary mathematics instruction (Rohimah et al., 2022). The cognitive profiles generated through this framework can serve as a reference for future studies and educational interventions.

Existing studies have predominantly focused on procedural fluency or strategy use in isolation, leaving limited insights into how students cognitively navigate the entire problem-solving process. Addressing this gap, the present study explores students' cognitive profiles in solving mathematical word problems involving single-variable linear equations. Based on the synthesized problem-solving framework combining Polya's and NCTM's models, at which stages are students able to solve word problems involving single-variable linear equations, and where do they experience cognitive difficulties?

Methods

This study employed a qualitative descriptive approach to explore students' mathematical problem-solving abilities in depth. The participants were 30 eighth-grade students from a *Madrasah Tsanawiyah* (Islamic junior high school) located in Majalengka Regency, West Java, Indonesia. At the time of the study, students had previously been introduced to linear equations in one variable as part of the national mathematics curriculum for Grade 8 in the previous semester, but had not yet received formal reinforcement or problem-based instruction related to real-life applications. Preliminary classroom observations and discussions with the mathematics teacher indicated that while most students demonstrated basic procedural fluency, their conceptual understanding and ability to apply mathematical reasoning in contextual problems remained limited. The sample included 18 female and 12 male students, with an average age of 14.1 years. The school is located in a peri-urban area with students coming from varied socioeconomic backgrounds.

Data were collected during the second semester of the 2024/2025 academic year through a written mathematical problem-solving test. The test was administered over 80 minutes (2 × 40-minute sessions) under controlled conditions. Students were not permitted to use notes, calculators, or reference materials, and the administration was closely supervised to ensure the authenticity of individual responses. This setting was designed to elicit students' natural cognitive strategies and provide valid insight into their mathematical thinking processes.

The instrument consisted of a structured set of three open-ended word problems, each representing a distinct level of difficulty—low, moderate, and high. The instrument consisted of a structured set of three open-ended word problems, each representing a distinct level of difficulty—low, moderate, and high. To ensure content validity, the instrument was reviewed by three experts in mathematics education: a professor, an associate professor, and a senior secondary school mathematics teacher with decades of teaching experience. Their evaluations confirmed that the problems were appropriate, clear, and aligned with the cognitive demands targeted in this study. Reliability was addressed by piloting the instrument with a group of students not involved in the main study to ensure clarity of wording and consistency of student interpretation.

The items were carefully constructed to assess students' ability to solve contextual problems involving single-variable linear equations. Each task was systematically aligned with five cognitive stages adapted from a synthesis of Polya's four-step problem-solving model and the National Council of Teachers of Mathematics (NCTM) process standards: (1) understanding and formulating the problem, (2) analyzing and diagnosing the problem, (3) designing a solution strategy, (4) executing the strategy and solving the problem, and (5) evaluating and reflecting on the result. For each stage, multiple performance indicators were developed to enable a fine-grained analysis of students' cognitive responses. The three test items were as follows:

1) Item 1 (Low difficulty): A basic relational age problem requiring students to apply simple addition to determine an individual's age.
2) Item 2 (Moderate difficulty): A transactional problem involving the purchase of pencils, requiring students to use subtraction and division based on price and change received.
3) Item 3 (High difficulty): A proportional distribution problem that required students to construct and solve single-variable linear equations based on the comparative amounts received by two individuals.

Each item was scored using a rubric that awarded points based on the five cognitive stages, with increasing complexity reflected in the maximum possible scores: 25 for low, 35 for moderate, and 40 for high difficulty. This rubric allowed evaluators to assess not only whether students arrived at the correct answer, but also how they reasoned through the problem.

Table 1. Structure and scoring of word problems by cognitive stage and difficulty

No.	Difficulty Level	Problem Statement	Total Score	Indicators
1	Low	Adi is currently 10 years older than his younger sibling. If the sibling is 8 years old, how old is Adi?	25	1. Understanding and Formulating the Problem • Identify known information (2) • Determine what is asked (2) 2. Analyzing and Diagnosing the Problem • Recognize the relationship in age (2) • Understand that only addition is needed (2) 3. Designing a Solution Strategy • Choose addition as the main operation (2) • Outline calculation steps (2) 4. Executing the Strategy and Solving the Problem • Perform addition correctly (4) • Ensure no calculation error (3) 5. Evaluation and Reflection • Recheck the result against given information (3) • Confirm answer fits the relationship (3)
2	Moderate	A store sells pencils for Rp2,000 each. If a customer pays Rp18,000 and receives Rp2,000 in change, how many pencils were bought?	35	1. Understanding and Formulating the Problem • Identify known information (2) • Determine what is asked (2) 2. Analyzing and Diagnosing the Problem • Link to relevant concepts/theories (3) • Identify relationships in the problem (3) 3. Designing a Solution Strategy • Choose the division to find the quantity (3) • Plan calculation steps accurately (3)

No.	Difficulty Level	Problem Statement	Total Score	Indicators
				4. Executing the Strategy and Solving the Problem • Perform division correctly (5) • Ensure consistency with problem data (4) 5. Evaluation and Reflection • Recheck results by recalculating (4) • Confirm consistency with the question scenario (4)
3	High	A father divides money between his two children. The first child receives three times as much as the second. If the total amount is Rp48,000, how much does each child receive?	40	1. Understanding and Formulating the Problem • Identify known information (3) • Determine what is asked (3) 2. Analyzing and Diagnosing the Problem • Recognize proportional relationship (3) • Identify pattern in distribution (3) 3. Designing a Solution Strategy • Choose to solve equation via addition and division (3) • Plan steps to find required value (4) 4. Executing the Strategy and Solving the Problem • Solve the equation (7) • Ensure no calculation errors (5) 5. Evaluation and Reflection • Verify result by recalculating (5) • Confirm answer matches the scenario (4)

The data analysis followed three major steps: (1) reviewing and interpreting students' written responses individually, (2) categorizing the data according to the cognitive indicators for each problem-solving stage, and (3) drawing conclusions based on emerging patterns and tendencies. These steps were carried out manually and collaboratively to enhance reliability and ensure consistent coding. The goal of the analysis was to identify the specific cognitive stages that students were able to complete and to diagnose the stages where cognitive breakdowns occurred. This structured approach facilitated a nuanced understanding of students' problem-solving behavior in relation to the synthesized theoretical model. Findings were validated through peer debriefing and triangulation across scorers.

In the Indonesian education system, the Minimum Mastery Criterion (*Kriteria Ketuntasan Minimal*, KKM) serves as a benchmark to determine whether students have achieved the expected level of competence in a given subject area. In this study, the KKM for mathematics in the participating school was set at 75, in accordance with national curriculum standards. This threshold was used as a reference point to interpret students' performance on the problem-solving test. Responses that demonstrated understanding and accurate application of concepts were compared against the KKM to determine whether students had met the minimum expected proficiency. The KKM also informed the classification of students' overall mastery levels and guided the discussion on pedagogical implications, particularly in identifying areas where instructional reinforcement may be needed.

In addition to the written test, semi-structured interviews were conducted to gain deeper insights into students' cognitive strategies and reasoning processes during problem solving. A purposive sample of six students was selected based on the diversity of their written test responses—representing low, moderate, and high levels of performance. The interviews were conducted individually within one week after the test administration, each lasting approximately 20–30 minutes. Guided by a flexible interview protocol, students were asked to explain their thought processes, justify their answers, and reflect on the difficulties they encountered. All interviews were audio-recorded with participants' consent and transcribed verbatim for qualitative analysis. The data were then coded inductively to identify emerging patterns related to problem comprehension, strategy formulation, and error tendencies.

Subsequently, interviews were conducted with nine students selected as samples representing low, medium, and high performance levels based on their scores on problem-solving tasks categorized as easy, moderate, and difficult. This selection aimed to explore variations in students' cognitive processes across different levels of problem-solving ability. The interviews provided deeper insights into students' reasoning and thought patterns, complementing the written test data.

Results

Achievement of indicators based on problem-solving stages

The analysis was conducted by categorizing student achievement data into five stages of problem-solving, synthesized from Polya's model and the framework of the National Council of Teachers of Mathematics (NCTM). These stages include: (1) Understanding and Formulating the Problem, (2) Analyzing and Diagnosing the Problem, (3) Designing a Solution Strategy, (4) Executing the Strategy and Solving the Problem, and (5) Evaluation and Reflection. The quantitative data are summarized in Table 2.

Table 2. Average student achievement at each problem-solving stage

Problem-Solving Stage	Average Score (%)	Description
Understanding and Formulating the Problem	93.5%	Students were able to identify key information and understand the question
Analyzing and Diagnosing the Problem	65.3%	Students struggled to link information with relevant concepts or patterns
Designing a Solution Strategy	56.0%	Students used memorized methods without contextual adaptation
Executing the Strategy and Solving	55.3%	High frequency of procedural and calculation errors
Evaluation and Reflection	32.3%	Most students did not check or reflect on the accuracy of their answers

The highest achievement was found in the first stage—understanding and formulating the problem—with an average score of 93.5%. This result indicates that most students were proficient in identifying important information and grasping the literal meaning of the question.

However, performance significantly declined in subsequent stages: only 65.3% in analyzing and diagnosing, 56.0% in designing a strategy, 55.3% in executing it, and merely 32.3% in the final evaluation and reflection stage. These findings suggest that while students could decode explicit information in the text, they struggled with abstract reasoning, planning, and verifying their solutions. As reflected in students' cognitive behavior, a lack of metacognitive awareness emerged as a critical barrier—highlighting that students not only failed to deeply understand the problem but were also unable to monitor and evaluate their own thinking processes.

Interview findings based on task level and student performance

Additional insights were obtained from interviews with students of low, moderate, and high performance across the three difficulty levels. Thematic open coding revealed five dominant themes: understanding the problem, difficulty recalling formulas, basic arithmetic ability, limitations in procedural planning, and weak reflection on results. These are summarized in Table 3.

Table 3. Summary of interviews by question difficulty and student performance

Task Level	Student Category	Main Difficulty	Recognized Strength
Easy	Low	Cannot perform calculations	Understands the problem
	Moderate	Forgot the formula	Can perform calculations
	High	-	Understands and applies the formula
Moderate	Low	Hard to recall the formula	Understands the problem
	Moderate	Does not know the formula	Understands and can calculate
	High	-	The problem is easy to understand
Difficult	Low	Does not understand the problem	Can apply the formula
	Moderate	Cannot understand or recall formula	Can perform calculations
	High	-	Understands the problem thoroughly

Comprehension emerged as a consistent predictor of high performance across all levels. In contrast, low-performing students frequently cited confusion or forgetfulness regarding formulas as their main challenge. Some students with strong calculation skills still showed difficulty in planning and articulating systematic solution steps. Moreover, very few students demonstrated metacognitive behaviors such as checking or validating their solutions—further reinforcing the weakness of the reflective dimension in their thinking. The presence or absence of reflection may therefore be a distinguishing factor between surface-level understanding and deeper cognitive engagement.

Analysis of student responses based on problem-solving stage

Students' written responses were analyzed using a rubric aligned with five cognitive stages adapted from Polya's and NCTM's problem-solving frameworks: (1) understanding and formulating the problem, (2) analyzing and diagnosing the problem, (3) designing a solution strategy, (4) executing the strategy and solving the problem, and (5) evaluating and reflecting. Each stage contained specific performance indicators, and responses were scored accordingly to generate a profile of each student's problem-solving ability across tasks of varying difficulty. The analysis process involved both descriptive coding and interpretative categorization to identify key patterns, misconceptions, and strategic behaviors. Based on these profiles, three students were purposively selected to represent distinct levels of performance—low, moderate, and high—which are illustrated respectively in Figure 1, Figure 2, and Figure 3. The selection was grounded on the total score distribution and the consistency of students' cognitive performance across tasks, ensuring that each figure reflects a prototypical example of the corresponding ability level.

A store sells pencils for Rp2,000 each. If a customer pays Rp18,000 and receives Rp2,000 in change, how many pencils were bought?
Solution:
Known:
- Price per pencil = Rp2,000
- Amount paid by the customer = Rp18,000
- Change received = Rp2,000

Asked:
How many pencils did the customer buy?
Answer:
Amount used to buy pencils = Amount paid – Change
= 18,000 – 2,000
= 16,000
= 16,000 ÷ Price per pencil
= 16,000 ÷ 2,000
= 8 pencils
Therefore, the number of pencils bought is 8 pencils.

Figure 1. High-performing student response

The high-performing student demonstrated complete problem comprehension and accurate data representation. Key variables such as unit price, payment amount, and change were clearly identified and logically processed. The student correctly subtracted the change from the total payment to obtain the cost of the pencils, and then divided by the unit price. Importantly, the student ended their solution with a clear and contextually relevant conclusion. This indicates strong integration of conceptual understanding, procedural execution, and reflective thinking—key attributes of high-level problem-solving per the NCTM framework.

Interview excerpt: "I first checked what the question was asking, then I subtracted the change and divided by the price. I checked again to make sure it made sense."

A store sells pencils for Rp2,000 each. If a customer pays Rp18,000 and receives Rp2,000 in change, how many pencils were bought?
Solution:
Known:
- Price of pencil = Rp2,000 each
- Money = Rp18,000
- Change = Rp2,000

Asked:
How many pencils were bought?
Answer:
18,000 ÷ 2 = 9,000 – 2,000 = 7
So, the number of pencils bought is 7 pencils.

Figure 2. Moderate-performing student response

The moderate-performing student displayed an initial understanding of the problem but lacked strategic clarity. Although essential information was noted, the representation of relationships among variables was unclear. The calculation steps appeared intuitive rather than systematically derived. Mathematical symbols such as fractions and values were present, but their relevance to the problem context was not explicitly explained. This reflects a partial strategic approach and low metacognitive control, as the solution lacked documentation of reasoning and internal validation.

Interview excerpt*:* "I remembered the numbers but wasn't sure what to do. I just tried something that seemed to work."

A store sells pencils for Rp2,000 each. If a customer pays Rp18,000 and receives Rp2,000 in change, how many pencils were bought?
Solution:
Known:
- The price of a pencil is Rp2,000 each
- A customer pays Rp18,000
- Receives Rp2,000 in change

Asked:
How many pencils were bought?
Answer:
Rp18,000 – Rp2,000 = Rp16,000
8 x 2 = 16
So, the number of pencils bought is 8.

Figure 3. Low-performing student response

The low-performing student exhibited conceptual misunderstanding from the outset. Although some given data were transcribed, the student proceeded to divide the payment by the unit price without accounting for the change. This error revealed a failure to connect the numeric data to the transaction context. No strategy was evident prior to calculations, and the process lacked analysis or verification. The thinking was procedural and mechanical, without reflective judgment, indicative of a concrete operational cognitive stage.

Interview excerpt: "I just divided the money by the price, like we usually do. I didn't think about the change."

The interview excerpt "I just divided the money by the price, like we usually do. I didn't think about the change" reflects a procedural but non-strategic approach to problem solving, where the student relies on habitual methods rather than contextual reasoning. This response indicates the presence of surface-level procedural knowledge—the student applies a memorized formula or operation (division) without assessing its relevance to the problem's context. The failure to account for the "change" suggests a lack of conceptual understanding, particularly in representing and manipulating real-world quantities within a mathematical model.

Table 4 presents a synthesized comparison of student thinking characteristics, which refer to the observable patterns of cognitive engagement demonstrated in students' written problem-solving responses. These characteristics encompass the degree of conceptual understanding, the strategic quality of their solution approach, and the extent of reflective or metacognitive behavior (e.g., checking results, justifying steps).

The classification into "high," "moderate," and "low" levels was based on a holistic analysis of students' responses across the three word problems, using the five-stage cognitive framework described earlier (understanding, analyzing, strategizing, executing, and evaluating). Each student's thinking profile was assessed through qualitative coding of their written work, considering aspects such as clarity of reasoning, coherence of mathematical models, use of appropriate strategies, and signs of error-checking or reflection. Representative students were selected to exemplify each level, ensuring consistency in the cognitive patterns observed.

Table 4. Synthesized comparison of student thinking based on responses

Student Level	Thinking Characteristics	Main Error	Strength
High	Strategic, conceptual, reflective	Almost none	Clear reasoning sequence
Moderate	Partially strategic, intuitive	Unclear steps and inconsistent strategy	General idea of solution path
Low	Procedural, lacks context	Misinterprets the problem and omits key variables	Some basic arithmetic ability

The interview process in this study was conducted in two distinct stages, each serving a complementary purpose. The first stage was a diagnostic interview, conducted shortly after the written test, and aimed at clarifying students' initial reasoning, thought processes, and any ambiguous responses in their written work. This stage helped validate interpretations of student strategies and errors, ensuring alignment between observed responses and student intent. Interviews were semi-structured and guided by students' actual test papers, allowing them to elaborate on their solution paths or explain skipped steps.

The second stage was a reflective interview, conducted approximately one week later, focusing on students' broader perceptions of problem-solving, their self-identified strengths and challenges, and metacognitive reflections such as how they checked or revised their answers. This stage provided richer insights into the underlying cognitive and affective factors influencing performance and formed the basis for thematic analysis, as summarized in Table 5. While both stages used semi-structured protocols, the first stage was tightly bound to specific responses, whereas the second stage allowed for more open-ended reflection and cross-item discussion

Table 5. Interview themes and descriptions

Theme	Description
Problem Comprehension	Key indicator of high performance across all problem levels
Formula Difficulty	Reported mainly in moderate and difficult tasks by low-performing students
Arithmetic Ability	Strength recognized by high- and moderate-performing students
Procedural Limitation	Difficulty in outliInterviewning solution steps despite understanding the problem
Weak Reflection	Rarely expressed awareness of checking or evaluating final answers

Discussion

The findings from this study indicate that students' mathematical problem-solving processes do not yet fully demonstrate optimal performance across all cognitive stages, particularly in the metacognitive dimension. The highest achievement occurred at the stage of understanding and formulating the problem (93.5%), suggesting that most students could identify explicit information such as numbers and keywords. This aligns with previous research which emphasized that literal reading ability in mathematical contexts does not always translate into comprehensive problem-solving skills (Díaz, 2022). However, performance declined sharply in the more advanced stages, especially evaluation and reflection, which scored only 32.3%. This suggests a significant weakness in metacognitive regulation, including the ability to verify, revise, and evaluate one's own strategies and solutions (Teng, 2020).

When considered alongside the written test analysis, the interview findings presented in Table 3 provide further depth and validation to the cognitive profiles synthesized in Table 4. Notably, students categorized as high-performing consistently demonstrated strong problem comprehension and minimal difficulty across all task levels, aligning with the "strategic, conceptual, and reflective" thinking profile observed in their written responses (Kania et al., 2023; Kholid, Swastika, et al., 2022; Susilo et al., 2023). Similarly, moderate-performing students often exhibited partial understanding and reliance on intuition, as reflected in both their ability to perform calculations and their lack of systematic planning—corroborating the characterization of "partially strategic" thinking with inconsistent execution (Angraini et al., 2023; Nufus et al., 2024; Shi & Qu, 2022).

In contrast, low-performing students frequently reported difficulty recalling formulas or interpreting the problem context (Geraci et al., 2023; Park & Cheon, 2025; Tschisgale et al., 2025). This mirrors the procedural, surface-level engagement identified in their written work, characterized by omitted variables and misinterpretations (XX). Across all performance levels, the lack of metacognitive awareness during interviews—particularly in terms of checking or justifying answers—reinforces the observed weakness in the "evaluation and reflection" stage of problem solving (Alias et al., 2024; Kania et al., 2024; Prabandari et al., 2024). Thus, the combined analysis of Table 3 and Table 4 offers a coherent portrait of how students' observable behaviors and verbalized reasoning converge to shape their mathematical thinking profiles.

Qualitative interview findings reinforce this pattern. Low-performing students often cited misunderstanding the problem or forgetting formulas as their primary difficulties. While some students managed to perform calculations, they lacked strategic reasoning or reflection regarding the contextual relevance of their answers. This points to a disconnect between procedural competence and conceptual understanding. The findings support Schoenfeld's assertion that problem-solving involves a complex interaction of conceptual knowledge, strategic behavior, and metacognitive control (Braithwaite & Sprague, 2021; Levin, 2018; Novianti & Aini, 2023).

Further analysis of students' written work highlights the role of cognitive quality in influencing performance. High-performing students displayed strategic and reflective thinking, processing information systematically and concluding with a contextualized validation. This is indicative of active metacognition—an essential trait in NCTM's problem-solving framework and Polya's model (Hancock & Karakok, 2021; Kholid, Sa'Dijah, et al., 2022). In contrast, students with moderate ability demonstrated partially structured strategies. Their reasoning was often intuitive, inconsistent, and lacked explicit explanation, reflecting a transitional stage from procedural to conceptual thinking. Within Bloom's taxonomy, these students operate mostly at the application level, falling short of analysis or evaluation stages (Shaikh et al., 2021).

More concerning are the findings from low-performing students, who largely operated within a mechanical procedural thinking framework. These students recorded data but failed to interpret it contextually, often ignoring key variables like change. This indicates a conceptual misconception in which relationships among information are not perceived as part of a unified problem structure. As Hiebert and Lefevre, highlighted, conceptual understanding enables flexible connections between mathematical ideas; its absence inhibits knowledge transfer (Kim, 2020; Mutawah et al., 2019).

Interview themes revealed five primary barriers: lack of problem comprehension, formula recall difficulty, weak procedural planning, low reflective capacity, and insufficient strategic awareness. Of these, problem comprehension consistently distinguished high- from low-performing students, underlining its foundational importance in the problem-solving process. These findings reinforce the need for instructional practices that develop both cognitive and metacognitive competencies.

The interview excerpts illustrate dual-layered student difficulties in both cognitive and metacognitive aspects. Students faced obstacles not only in generating strategies but also in evaluating their thinking processes. This underscores the importance of instruction that goes beyond formulaic mastery, promoting strategy design, cognitive flexibility, and self-regulated reflection (García-Pérez et al., 2021; Hartelt & Martens, 2024). From a metacognitive perspective, the student demonstrates minimal self-monitoring and evaluation, as evidenced by the admission of not reflecting on the logical fit between the operation performed and the problem scenario. This aligns with previous findings that students often default to routine procedures in the absence of metacognitive control mechanisms (Clark et al., 2024; Wang et al., 2023). The reliance on "what we usually do" suggests cognitive rigidity and a limited repertoire of problem-solving strategies, which impedes flexible adaptation to novel or slightly altered contexts (Kim, 2020).

These findings emphasize the value of mathematics instruction that cultivates not only procedural skills but also strategic and reflective thinking. Educators should incorporate strategy-based problem-solving models supported by metacognitive scaffolding such as guiding questions, self-reflection, and classroom discussions on strategies. Such approaches deepen understanding and empower students to monitor and regulate their own thinking processes—an essential competency for 21st-century learners (Radovan, 2019; Tachie, 2019).

Conclusion

This study revealed that students' mathematical problem-solving performance is heavily concentrated in the early stages of the cognitive process—particularly in identifying and interpreting explicit information—yet declines markedly in later stages requiring strategic planning, execution, and metacognitive reflection. The quantitative findings show that while most students are capable of understanding the surface structure of problems, fewer demonstrate the ability to connect mathematical concepts, construct solution strategies, or evaluate the appropriateness of their answers. These patterns were further confirmed by qualitative interviews, which underscored the gap between procedural fluency and conceptual understanding, particularly among low- and moderate-performing students.

The integration of Polya's and NCTM's frameworks proved effective in mapping the diverse cognitive profiles exhibited by students, offering a practical tool for diagnosing thinking patterns at each stage of problem solving. High-performing students consistently exhibited metacognitive awareness, including planning, self-monitoring, and reflective judgment—traits notably absent in low-performing peers. These results reinforce the importance of explicitly teaching not only mathematical procedures but also cognitive and metacognitive strategies as core components of problem-solving instruction.

From a pedagogical perspective, the findings advocate for the adoption of instructional approaches that combine conceptual scaffolding with structured opportunities for metacognitive engagement, such as guided self-questioning, collaborative reflection, and strategic modeling. Future research should extend this inquiry by incorporating longitudinal data and intervention-based designs to assess how cognitive and metacognitive training impacts student achievement over time. Ultimately, the development of reflective, strategic problem solvers requires a shift from content delivery to the cultivation of independent and adaptive mathematical thinkers—an imperative for 21st-century education.

Despite its contributions, this study has several limitations that should be acknowledged. The participant sample was confined to 30 eighth-grade students from a single Islamic junior high school in West Java, thereby restricting the generalizability of the findings across different educational and cultural contexts. Additionally, the scope of the inquiry was limited to linear equations in one variable, and may not capture cognitive behaviors across other mathematical domains. The absence of classroom observations also limits contextual interpretation of students' reasoning processes. Future research should involve larger, more diverse student samples and incorporate multimodal data sources—such as classroom discourse, eye-tracking, or think-aloud protocols—to gain deeper insight into the real-time dynamics of mathematical problem solving. Longitudinal and intervention-based designs are also needed to evaluate how cognitive and metacognitive training can systematically improve student outcomes over time.

Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript

Author Contributions

Zaenal Arifin: Conceptualization, writing - original draft, editing, and visualization; **Al Jupri:** Formal analysis, methodology, validation and supervision.

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