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ON H-IRREGULARITY STRENGTHS OF SQUARE CHAIN GRAPHS

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ABSTRACT

This paper investigates three types of graph labeling schemes, namely H-irregular vertex, edge, and total labeling. These frameworks are examined for square chain graphs SC_n, where $n \geq 1$. Furthermore, by establishing matching lower and upper bounds, the exact values of the vertex, edge, and total SC_m-irregularity strengths of SC_n are determined for all m with $1 \leq m \leq n$.

Keywords : **$\boldsymbol{H}$-Irregular Vertex Labeling; $\boldsymbol{H}$-Irregular Edge Labeling; $\boldsymbol{H}$-Irregular Total Labeling; Square Chain Graph**

I. INTRODUCTION

Graph theory provides a rigorous framework for modeling complex structures across various scientific and engineering domains by representing objects as vertices and their interactions as edges. Owing to this expressive capability, this mathematical approach has been widely applied in numerous areas, including financial fraud detection [7], the study of large-scale interconnected structures [10], and stock market modeling and forecasting [13].

A prominent branch of this field is graph labeling, which concerns the assignment of integers to the components of a graph such as vertices, edges, or both, according to specified rules [8]. Depending on which components receive labels, graph labeling can be classified into vertex, edge, or total labeling.

Among the many graph labeling schemes studied in the literature, irregular labeling constitutes one of the earliest and most fundamental approaches. This concept was introduced by Chartrand et al. [6] and is based on assigning integers to the edges of a graph in such a way that the induced vertex weights are pairwise distinct.

Inspired by the notion of irregular labeling, Ashraf et al. [1,17] proposed three generalized labeling schemes, namely the H-irregular vertex labeling, the H-irregular edge labeling, and the H-irregular total labeling. In contrast to classical irregular labeling, which assigns labels to edges so that the resulting vertex weights are pairwise distinct, the H-irregular approach shifts the focus to subgraph structures. More precisely, it requires that all subgraphs of a given graph that are isomorphic to a fixed graph H possess distinct weights, thereby providing a broader and more flexible framework for studying irregularity phenomena in graphs.

Let G be a finite, simple, undirected graph, and let H be a subgraph of G. We say that G admits an *H-covering* if every edge of G is contained in at least one subgraph of G that is isomorphic to H.

Definition 1 ([1]). Let G be a graph that admits an H-covering. An *H-irregular vertex k-labeling* of G is a function $\alpha: V(G) \to \{1,2,\dots,k\}$ such that the induced weights of all subgraphs of G that are isomorphic to H are pairwise distinct. The *vertex H-weight* of a subgraph H under α is defined by

$$wt_{\alpha}(H) = \sum_{v \in V(H)} \alpha(v).$$

The smallest positive integer k for which such a labeling exists is called the *vertex H-irregularity strength* of G and is denoted by $vhs(G,H)$.

Definition 2 ([1]). Let G be a graph admitting an H-covering. An *H-irregular edge k-labeling* of G is a mapping $\beta: E(G) \to \{1,2,\dots,k\}$ such that the edge weights induced on all subgraphs of G isomorphic to H are mutually distinct. The *edge H-weight* of a subgraph H is given by

$$wt_{\beta}(H) = \sum_{e \in E(H)} \beta(e).$$

The minimum integer k for which G admits such a labeling is referred to as the *edge H-irregularity strength* of G, denoted by $ehs(G,H)$.

Definition 3 ([2]). Let G be a graph admitting an H-covering. An *H-irregular total k-labeling* of G is a function $\gamma: V(G) \cup E(G) \to \{1,2,\dots,k\}$ such that the total weights associated with all subgraphs of G isomorphic to H are pairwise distinct. The *total H-weight* of a subgraph H is defined as

$$wt_\gamma(H) = \sum_{v\in V(H)} \gamma(v) + \sum_{e\in E(H)} \gamma(e).$$

The smallest positive integer k for which G admits an H-irregular total k-labeling is called the *total H-irregularity strength* of G and is denoted by $ths(G,H)$.

The study of vertex, edge, and total H-irregularity strength for different families of graphs has attracted considerable attention and has been widely explored in the literature. Initial results in this direction were obtained by Ashraf et al. [1], who determined the vertex H-irregularity strength for several basic graph classes such as paths, ladders, and fan graphs. Subsequent work by Labane et al. [11] extended these investigations by establishing exact values for the vertex H-irregularity strength of diamond graphs. Moreover, Ashraf et al. [4] examined H-irregular vertex labelings in the context of graphs formed through $\mathcal{G}$-amalgamation. More recently, Bača et al. [5] analyzed join product graphs and determined their vertex H-irregularity strength.

Parallel developments have also been reported for edge H-irregularity strength. Ashraf et al. [3] investigated this parameter for ladders and fan graphs. Subsequently, Naeem et al. [12] extended the analysis to several other graph families, including prisms, antiprisms, triangular ladders, diagonal ladders, wheels, and gear graphs. In addition, Ibrahim et al. [9] focused on grid-based structures and obtained corresponding results for hexagonal and octagonal grid graphs.
Research on total H-irregularity strength has similarly progressed. Ashraf et al. [2] determined the total H-irregularity strength for paths, ladders, and fan graphs, laying the groundwork for further studies. Later contributions by Shulhany et al. [14] addressed butterfly and eclipse graphs, while Wahyujati and Susanti [15] investigated comb product graphs and established exact values for their total H-irregularity strength.

Despite these extensive developments, the family of square chain graphs has not yet been examined within the framework of H-irregular labeling. Motivated by this gap, the present work investigates vertex, edge, and total H-irregular labeling schemes for square chain graphs. In particular, we determine the exact values of the corresponding vertex, edge, and total H-irregularity strengths for this class of graphs.

II. METHODS

This section presents the methodology employed to determine the vertex SC_m-irregularity strength, the edge SC_m-irregularity strength, and the total SC_m-irregularity strength of the square chain graph SC_n. Our analysis aims to establish the exact values of these parameters for $1 \le m \le n$.

The exact values are determined by establishing tight lower and upper bounds. The lower bounds follow from previously established results due to [1] and [2], as summarized in the following theorems.

Theorem 1 ([1]). Let G be a graph that admits an H-covering consisting of s subgraphs each isomorphic to H. Then the vertex H-irregularity strength of G satisfies

$$vhs(G,H) \ge \left\lceil \frac{s-1}{|V(H)|} \right\rceil + 1.$$

Theorem 2 ([1]). Suppose that a graph G admits an H-covering formed by s subgraphs isomorphic to H. Then the edge H-irregularity strength of G is bounded below by

$$ehs(G,H) \ge \left\lceil \frac{s-1}{|E(H)|} \right\rceil + 1.$$

Theorem 3 ([2]). Let G be a graph admitting an H-covering with s subgraphs isomorphic to H. Then the total H-irregularity strength of G satisfies the inequality

$$ths(G,H) \ge \left\lceil \frac{s-1}{|V(H)| + |E(H)|} \right\rceil + 1.$$

In contrast, the upper bounds are established constructively by explicitly demonstrating the existence of suitable SC_m-irregular vertex labeling, SC_m-irregular edge labeling, and SC_m-irregular total labeling of SC_n. The combination of these approaches yields the exact values of the irregularity strength parameters for the class of square chain graphs.

III. RESULTS AND DISCUSSION

In this section, we examine the vertex, edge, and total SC_m-irregularity strengths of the square chain graph SC_n for integers m and n satisfying $1 \le m \le n$. Let SC_n, with $n \ge 1$, denote a square chain graph whose structure is described as follows. The vertex set of SC_n is defined by

$$V(SC_n) = \{a_i : 1 \le i \le n+1\} \cup \{b_i, c_i : 1 \le i \le n\},$$

while its edge set is given by

$$E(SC_n) = \{a_i b_i, a_i c_i, a_{i+1} b_i, a_{i+1} c_i : 1 \le i \le n\}.$$

Clearly, for any integers m and n with $1 \le m \le n$, the graph SC_n admits an SC_m-covering and contains exactly $n - m + 1$ subgraphs that are isomorphic to SC_m.

For each integer j with $1 \le j \le n - m + 1$, let us denote by SC_m^j the jth subgraph of SC_n that is isomorphic to SC_m. The vertex set of this subgraph is

$$V(SC_m^j) = \{a_i : j \le i \le j+m\} \cup \{b_i, c_i : j \le i \le j+m-1\},$$

and its edge set is

$$E(SC_m^j) = \{a_i b_i, a_i c_i, a_{i+1} b_i, a_{i+1} c_i : j \le i \le j+m-1\}.$$

The square chain graph SC_5 is shown in Figure 1.

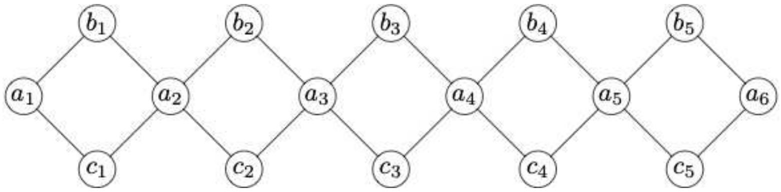

Figure 1: The square chain graph SC_5.

The first main result of this paper concerns the vertex SC_m-irregularity strength of square chain graphs and is presented in the following theorem.

Theorem 4. Let m, n be integers with $1 \leq m \leq n$. Then

$$vhs(SC_n, SC_m) = \left\lceil \frac{2m+n+1}{3m+1} \right\rceil.$$

Proof. Let $1 \leq m \leq n$. Since SC_n has $n-m+1$ subgraphs isomorphic to SC_m, and since SC_m has $3m+1$ vertices, it follows from Theorem 1 that

$$vhs(SC_n, SC_m) \geq \left\lceil \frac{n-m+1-1}{3m+1} \right\rceil + 1 = \left\lceil \frac{2m+n+1}{3m+1} \right\rceil.$$

To prove the equality, we provide a suitable SC_m-irregular vertex $\left\lceil \frac{2m+n+1}{3m+1} \right\rceil$-labeling of SC_n. To do this, we define a vertex labeling α of SC_n in the following way.

$$\alpha(a_i) = \left\lceil \frac{2m+i}{3m+1} \right\rceil \quad \text{for } 1 \leq i \leq n+1,$$
$$\alpha(b_i) = \left\lceil \frac{m+i}{3m+1} \right\rceil \quad \text{for } 1 \leq i \leq n,$$
$$\alpha(c_i) = \left\lceil \frac{i}{3m+1} \right\rceil \quad \text{for } 1 \leq i \leq n.$$

Clearly, the vertex labels used in the labeling α are at most $\left\lceil \frac{2m+n+1}{3m+1} \right\rceil$.

Now, we show that all the vertex SC_m-weights are distinct. Note that it is enough to show that $wt_\alpha(SC_m^j) < wt_\alpha(SC_m^{j+1})$ for $1 \leq j \leq n-m$. We have that

$$wt_\alpha(SC_m^j) = \sum_{i=j}^{j+m} \alpha(a_i) + \sum_{i=j}^{j+m-1} \big(\alpha(b_i) + \alpha(c_i)\big)$$
$$= \alpha(a_j) + \alpha(b_j) + \alpha(c_j) + \sum_{i=j+1}^{j+m} \alpha(a_i) + \sum_{i=j+1}^{j+m-1} \big(\alpha(b_i) + \alpha(c_i)\big)$$

and

$$wt_\alpha(SC_m^{j+1}) = \sum_{i=j+1}^{j+m+1} \alpha(a_i) + \sum_{i=j+1}^{j+m} \big(\alpha(b_i) + \alpha(c_i)\big)$$
$$= \alpha(a_{j+m+1}) + \alpha(b_{j+m}) + \alpha(c_{j+m}) + \sum_{i=j+1}^{j+m} \alpha(a_i) + \sum_{i=j+1}^{j+m-1} \big(\alpha(b_i) + \alpha(c_i)\big)$$
$$= 1 + \left\lceil \frac{2m+j}{3m+1} \right\rceil + \left\lceil \frac{m+j}{3m+1} \right\rceil + \left\lceil \frac{j}{3m+1} \right\rceil + \sum_{i=j+1}^{j+m} \alpha(a_i) + \sum_{i=j+1}^{j+m-1} \big(\alpha(b_i) + \alpha(c_i)\big)$$
$$= 1 + \alpha(a_j) + \alpha(b_j) + \alpha(c_j) + \sum_{i=j+1}^{j+m} \alpha(a_i) + \sum_{i=j+1}^{j+m-1} \big(\alpha(b_i) + \alpha(c_i)\big)$$
$$= 1 + wt_\alpha(SC_m^j),$$

which means that $wt_\alpha(SC_m^j) < wt_\alpha(SC_m^{j+1})$ for $1 \le j \le n-m$. Therefore, α is an SC_m-irregular vertex $\left\lceil\frac{2m+n+1}{3m+1}\right\rceil$-labeling of SC_n, and so $vhs(SC_n, SC_m) = \left\lceil\frac{2m+n+1}{3m+1}\right\rceil$. This completes the proof. ■

As an example, Figure 2 provides an SC_2-irregular vertex 2-labeling of the square chain graph SC_5.

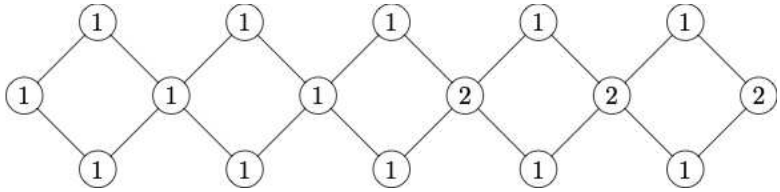

Figure 2: An SC_2-irregular vertex 2-labeling of SC_5.

The next theorem provides the edge SC_m-irregularity strength for the square chain graphs.

Theorem 5. Let m, n be integers with $1 \le m \le n$. Then

$$ehs(SC_n, SC_m) = \left\lceil\frac{3m+n}{4m}\right\rceil.$$

Proof. Let $1 \le m \le n$. Because SC_n has $n-m+1$ subgraphs isomorphic to \$SC_m\$, and because SC_m has $4m$ edges, Theorem 2 yields

$$ehs(SC_n, SC_m) \ge \left\lceil\frac{n-m+1-1}{4m}\right\rceil + 1 = \left\lceil\frac{3m+n}{4m}\right\rceil.$$

Let us define an edge labeling β of SC_n in the following way.

$$\beta(a_ib_i) = \left\lceil\frac{3m+i}{4m}\right\rceil \quad \text{for } 1 \le i \le n,$$
$$\beta(a_ic_i) = \left\lceil\frac{2m+i}{4m}\right\rceil \quad \text{for } 1 \le i \le n,$$
$$\beta(a_{i+1}b_i) = \left\lceil\frac{m+i}{4m}\right\rceil \quad \text{for } 1 \le i \le n,$$
$$\beta(a_{i+1}c_i) = \left\lceil\frac{i}{4m}\right\rceil \quad \text{for } 1 \le i \le n.$$

Evidently, the edge labels being used are not greater than $\left\lceil\frac{3m+n}{4m}\right\rceil$.

We shall show that $wt_\beta(SC_m^j) < wt_\beta(SC_m^{j+1})$ for $1 \le j \le n-m$. We obtain that

$$\begin{aligned} wt_\beta(SC_m^j) &= \sum_{i=j}^{j+m-1} (\beta(a_ib_i) + \beta(a_ic_i) + \beta(a_{i+1}b_i) + \beta(a_{i+1}c_i)) \\ &= \beta(a_jb_j) + \beta(a_jc_j) + \beta(a_{j+1}b_j) + \beta(a_{j+1}c_j) \\ &\quad + \sum_{i=j}^{j+m-1} (\beta(a_ib_i) + \beta(a_ic_i) + \beta(a_{i+1}b_i) + \beta(a_{i+1}c_i)) \end{aligned}$$

and

$$\begin{aligned} wt_\beta(SC_m^{j+1}) &= \sum_{i=j+1}^{j+m} (\beta(a_ib_i) + \beta(a_ic_i) + \beta(a_{i+1}b_i) + \beta(a_{i+1}c_i)) \\ &= \beta(a_{j+m}b_{j+m}) + \beta(a_{j+m}c_{j+m}) + \beta(a_{j+m+1}b_{j+m}) + \beta(a_{j+m+1}c_{j+m}) \\ &\quad + \sum_{i=j+1}^{j+m-1} (\beta(a_ib_i) + \beta(a_ic_i) + \beta(a_{i+1}b_i) + \beta(a_{i+1}c_i)) \\ &= 1 + \left\lceil\frac{3m+j}{4m}\right\rceil + \left\lceil\frac{2m+j}{4m}\right\rceil + \left\lceil\frac{m+j}{4m}\right\rceil + \left\lceil\frac{j}{4m}\right\rceil \\ &\quad + \sum_{i=j+1}^{j+m-1} (\beta(a_ib_i) + \beta(a_ic_i) + \beta(a_{i+1}b_i) + \beta(a_{i+1}c_i)) \end{aligned}$$

$$= 1 + \beta(a_j b_j) + \beta(a_j c_j) + \beta(a_{j+1} b_j) + \beta(a_{j+1} c_j)$$
$$+ \sum_{i=j+1}^{j+m-1} \left(\beta(a_i b_i) + \beta(a_i c_i) + \beta(a_{i+1} b_i) + \beta(a_{i+1} c_i)\right)$$
$$= 1 + wt_\beta(SC_m^j),$$

which implies that $wt_\beta(SC_m^j) < wt_\beta(SC_m^{j+1})$ for $1 \le j \le n-m$. Hence, β is the desired SC_m-irregular edge $\left\lceil\frac{3m+n}{4m}\right\rceil$-labeling of SC_n, and so $ehs(SC_n, SC_m) = \left\lceil\frac{3m+n}{4m}\right\rceil$. ■

For an illustration, an SC_2-irregular edge 2-labeling of the square chain graph SC_5 is given in Figure 3.

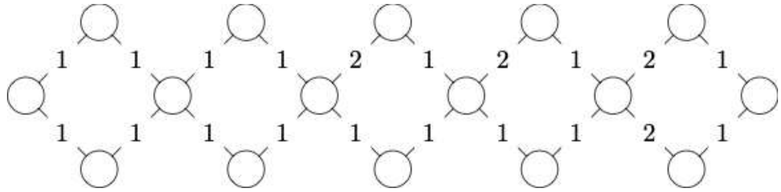

Figure 3: An SC_2-irregular edge 2-labeling of SC_5.

In next theorem, we give the total SC_m-irregularity strength of the square chain graphs.

Theorem 6. Let m, n be integers with $1 \le m \le n$. Then
$$ths(SC_n, SC_m) = \left\lceil\frac{6m+n+1}{7m+1}\right\rceil.$$

Proof. Let $1 \le m \le n$. As SC_n has $n-m+1$ subgraphs isomorphic to SC_m, and $|V(SC_m)| + |E(SC_m)| = 7m+1$, from Theorem 3 we have
$$ths(SC_n, SC_m) \ge \left\lceil\frac{n-m+1-1}{7m+1}\right\rceil + 1 = \left\lceil\frac{6m+n+1}{7m+1}\right\rceil.$$

We define a total labeling γ of SC_n in the following way.
$$\gamma(a_i) = \left\lceil\frac{6m+i}{7m+1}\right\rceil \quad \text{for } 1 \le i \le n+1,$$
$$\gamma(b_i) = \left\lceil\frac{5m+i}{7m+1}\right\rceil \quad \text{for } 1 \le i \le n,$$
$$\gamma(c_i) = \left\lceil\frac{4m+i}{7m+1}\right\rceil \quad \text{for } 1 \le i \le n,$$
$$\gamma(a_i b_i) = \left\lceil\frac{3m+i}{7m+1}\right\rceil \quad \text{for } 1 \le i \le n,$$
$$\gamma(a_i c_i) = \left\lceil\frac{2m+i}{7m+1}\right\rceil \quad \text{for } 1 \le i \le n,$$
$$\gamma(a_{i+1} b_i) = \left\lceil\frac{m+i}{7m+1}\right\rceil \quad \text{for } 1 \le i \le n,$$
$$\gamma(a_{i+1} c_i) = \left\lceil\frac{i}{7m+1}\right\rceil \quad \text{for } 1 \le i \le n.$$

It is easy to see that the maximum label used in the labeling γ is $\left\lceil\frac{6m+n+1}{7m+1}\right\rceil$.

Let us now consider the total SC_m-weights. We have that
$$wt_\gamma(SC_m^j) = \sum_{i=j}^{j+m} \gamma(a_i) + \sum_{i=j}^{j+m-1} \left(\gamma(b_i) + \gamma(c_i) + \gamma(a_i b_i) + \gamma(a_i c_i) + \gamma(a_{i+1} b_i) + \gamma(a_{i+1} c_i)\right)$$
$$= \gamma(a_j) + \gamma(b_j) + \gamma(c_j) + \gamma(a_j b_j) + \gamma(a_j c_j) + \gamma(a_{j+1} b_j) + \gamma(a_{j+1} c_j)$$
$$+ \sum_{i=j}^{j+m-1} \left(\gamma(b_i) + \gamma(c_i) + \gamma(a_i b_i) + \gamma(a_i c_i) + \gamma(a_{i+1} b_i) + \gamma(a_{i+1} c_i)\right)$$

and

$$
\begin{aligned}
wt_\gamma(SC_m^{j+1}) &= \sum_{i=j+1}^{j+m+1} \gamma(a_i) \\
&\quad + \sum_{i=j+1}^{j+m} \big(\gamma(b_i) + \gamma(c_i) + \gamma(a_i b_i) + \gamma(a_i c_i) + \gamma(a_{i+1} b_i) + \gamma(a_{i+1} c_i)\big) \\
&= \gamma(a_{j+m+1}) + \gamma(b_{j+m}) + \gamma(c_{j+m}) + \gamma(a_{j+m} b_{j+m}) + \gamma(a_{j+m} c_{j+m}) \\
&\quad + \gamma(a_{j+m+1} b_{j+m}) + \gamma(a_{j+m+1} c_{j+m}) + \sum_{i=j+1}^{j+m} \gamma(a_i) \\
&\quad + \sum_{i=j+1}^{j+m-1} \big(\gamma(b_i) + \gamma(c_i) + \gamma(a_i b_i) + \gamma(a_i c_i) + \gamma(a_{i+1} b_i) + \gamma(a_{i+1} c_i)\big) \\
&= 1 + \left\lceil \frac{6m+j}{7m+1} \right\rceil + \left\lceil \frac{5m+j}{7m+1} \right\rceil + \left\lceil \frac{4m+j}{7m+1} \right\rceil + \left\lceil \frac{3m+j}{7m+1} \right\rceil + \left\lceil \frac{2m+j}{7m+1} \right\rceil + \left\lceil \frac{m+j}{7m+1} \right\rceil \\
&\quad + \left\lceil \frac{j}{7m+1} \right\rceil + \sum_{i=j+1}^{j+m} \gamma(a_i) \\
&\quad + \sum_{i=j+1}^{j+m-1} \big(\gamma(b_i) + \gamma(c_i) + \gamma(a_i b_i) + \gamma(a_i c_i) + \gamma(a_{i+1} b_i) + \gamma(a_{i+1} c_i)\big) \\
&= 1 + \gamma(a_j) + \gamma(b_j) + \gamma(c_j) + \gamma(a_j b_j) + \gamma(a_j c_j) + \gamma(a_{j+1} b_j) + \gamma(a_{j+1} c_j) \\
&\quad + \sum_{i=j+1}^{j+m} \gamma(a_i) \\
&\quad + \sum_{i=j+1}^{j+m-1} \big(\gamma(b_i) + \gamma(c_i) + \gamma(a_i b_i) + \gamma(a_i c_i) + \gamma(a_{i+1} b_i) + \gamma(a_{i+1} c_i)\big) \\
&= 1 + wt_\gamma(SC_m^j),
\end{aligned}
$$

which implies that all the total SC_m-weights are distinct. Consequently, γ is an SC_m-irregular total $\left\lceil \frac{6m+n+1}{7m+1} \right\rceil$-labeling of SC_n, hence $ths(SC_n, SC_m) = \left\lceil \frac{6m+n+1}{7m+1} \right\rceil$. ■

An SC_2-irregular total 2-labeling of the square chain graph SC_5 is depicted in Figure 4.

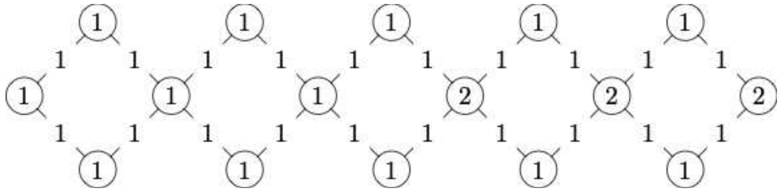

Figure 4: An SC_2-irregular total 2-labeling of SC_5.

The derived exact values for the SC_m-irregularity strengths highlight the critical role of structural symmetry and overlapping subgraph density in optimizing label distribution. While prior research has primarily addressed simpler planar structures like paths and cycles [1,2], our results confirm that square chain graphs still reach the established lower bounds, despite requiring a highly intricate labeling scheme due to higher vertex degrees and edge overlaps. Importantly, while the presented constructive proofs are specific to SC_n, the underlying technique of systematic vertex and edge partitioning is highly adaptable. This proof framework holds significant potential for generalization to other chain-like planar networks.

IV. CONCLUSION

In this article, we have established the exact values of the vertex, edge, and total SC_m-irregularity strengths for square chain graphs SC_n with $1 \leq m \leq n$. These findings demonstrate that the general lower bounds presented in Theorems 1, 2, and 3 are attained, providing a complete characterization of H-irregular labeling behavior in square chain graphs. Scientifically, this study advances the fundamental understanding of structural irregularity and optimal label distribution in structured graphs. However, our analytical techniques rely heavily on the specific symmetric properties of square chain graphs, limiting their direct generalization to highly asymmetric graphs. Therefore, future research should extend these methodologies to other generalized structures, such as hexagonal chains, and develop algorithmic approaches to compute H-irregularity strengths for broader graph families.

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Data Availability Statement

The study reported in this article did not involve the use of any external or empirical data.

Declaration of Interests Statement

The author declares no conflict of interest.

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