

A Hypothetical Learning Trajectory Design for Social Arithmetic to Fostering Computational Thinking and Self-Regulated Learning Abilities

Santika Lya Diah Pramesti¹, Scolastika Mariani², Putriaji Hendikawati³, Kartono⁴, Masrukan⁵

¹ Universitas Negeri Semarang, Indonesia; santikalyadiahpramesti@students.unnes.ac.id

² Universitas Negeri Semarang, Indonesia; mariani.mat@mail.unnes.ac.id

³ Universitas Negeri Semarang, Indonesia; putriaji.mat@mail.unnes.ac.id

⁴ Universitas Negeri Semarang, Indonesia; kartono.mat@mail.unnes.ac.id

⁵ Universitas Negeri Semarang, Indonesia; masrukan.mat@mail.unnes.ac.id

ARTICLE INFO

Keywords:

HLT;
MiMOPBL;
Computational thinking;
Self-regulated learning;
Predict-observe-explain (POE)

Article history:

Received 2025-06-03

Revised 2025-07-15

Accepted 2025-08-17

ABSTRACT

This study aims to design a Hypothetical Learning Trajectory (HLT) on the topic of social arithmetic to address learning obstacles related to students' computational thinking (CT) and self-regulated learning (SRL) abilities. Employing an Educational Design Research (EDR) approach, the study developed the Missouri Mathematics Open-Ended Problem-Based Learning (MiMOPBL) model supported by an e-module based on the Predict–Observe–Explain (POE) strategy. The HLT integrates CT and SRL by guiding students through prediction, observation, and explanation stages while solving open-ended contextual problems. Validation results confirmed that the instructional tools—including lesson plans, student worksheets, e-modules, CT and SRL instruments—are both valid and reliable. Content validity assessed using Aiken's V showed values ranging from 0.80 to 0.98, while reliability testing using Cronbach's Alpha indicated high internal consistency (SRL: $\alpha = 0.85$; CT: $\alpha = 0.89$; observation sheet: $\alpha = 0.88$). These results indicate that the tools are suitable for use in mathematics classrooms. Although limited to the design phase and one topic, the study offers a structured, competency-based learning design that supports student engagement, problem-solving skills, and independent learning. Future research should explore the effectiveness of this learning design across various topics and student groups.

This is an open access article under the [CC BY-NC-SA](https://creativecommons.org/licenses/by-nc-sa/4.0/) license.



Corresponding Author:

Scolastika Mariani

Universitas Negeri Semarang, Indonesia; mariani.mat@mail.unnes.ac.id

1. INTRODUCTION

Twenty-first century education presents major challenges in preparing young people to face rapid technological developments and increasingly complex demands of the global workforce. The development of technology and science cannot be separated from the role of mathematics in the process. According to Nurjanah, Nurcahyono, dan Imswatama (2022), mathematics is fundamental to the process of developing science, technology, and human thinking. In line with Sudrajat (2008), who explains the role and influence of mathematics learning on the development of science and technology.

There are five skills that can be developed through mathematics learning (NCTM, 2000), including: problem-solving skills, reasoning and proof skills, connection skills, communication skills, and representation skills. These skills are expected to help students solve various problems they will encounter in facing the developments of the times. In addition to the five skills mentioned by NCTM, there are other skills mentioned in the draft PISA 2021 framework, namely computational thinking skills.

Despite growing awareness of the importance of CT, Indonesia continues to struggle with low performance in mathematics and CT-related domains. The 2022 PISA results placed Indonesia 69th out of 81 participating countries, with an average mathematics score of 366, well below the OECD average of 472 (OECD, 2023). Furthermore, results from the 2024 *Bebras Computational Thinking Challenge* revealed that over 50% of Indonesian student participants scored zero, suggesting unfamiliarity with problem types that require computational strategies (Bebras Indonesia, 2024). These findings underscore a significant educational gap in both CT development and mathematical literacy.

Computational thinking in mathematics education can help students not only understand mathematical concepts more deeply but also develop critical and analytical thinking skills necessary for solving real-world problems (Sneider et al., 2014). CT has been widely recognized as a transversal skill necessary for students' cognitive and career readiness (Bouck & Yadav, 2022; Shute et al., 2017). This initiative was pioneered by Norway, which integrated computational thinking into mathematics education (Bocconi et al., 2016). In addition to Norway, CT integration in education has been implemented in several countries, such as Australia (Falkner et al., 2015), the United Kingdom (Brown et al., 2014), Finland (Mannila et al., 2014), Sweden (Bråting et al., 2022), and the United States (Fisher, 2016). In contrast, Indonesia has yet to adopt a comprehensive, pedagogy-driven framework that explicitly integrates CT into mathematics learning through student-centered, technology-supported instruction.

Developing CT skills in the classroom is not easy, especially if the approach is still based on conventional, teacher-centered instruction. Students are not accustomed to dealing with open-ended problems (Wiraharta et al., 2020). The potential of integrating CT into learning encourages active and independent learning, where the role of self-regulated learning is also important (Song et al., 2021). On the other hand, the development of self-regulated learning or the ability to manage learning independently is also an important factor in modern learning. SRL enables students to plan, monitor, and evaluate their learning progress consciously and actively. In the context of mathematics learning that requires exploration, reflection, and problem-solving, SRL is essential for students to overcome learning challenges with greater adaptability and resilience. Each individual has different characteristics that significantly influence their ability to understand the material being studied and impact the problem-solving analysis process (Hasdi et al., 2024; Prahmana et al., 2024).

Self-Regulated Learning, according to Schunk & Zimmerman (1998), is a process carried out by individuals in activating thoughts, behaviors, and feelings continuously in an effort to achieve predetermined goals. Students with good SRL abilities are better able to manage their learning time, recognize difficulties in understanding the material, and take the necessary steps to improve their

understanding. In the context of CT, SRL encourages students to internalize and reflect on complex CT concepts, which are crucial for problem-solving in today's technology-driven world (Paris & Paris, 2001). In a computational analysis approach, many indicators reveal significant differences between identified patterns of computational thinking skills and self-regulated learning (Kong & Wang, 2024; Song et al., 2021). With self-regulation, students can be more engaged in learning, manage their study time better, and evaluate their understanding of the material being studied. This, in turn, can improve their computational thinking skills, which are essential in today's digital age. McKnight et al. (2016) suggest that the use of technology in education, particularly electronic-based learning models, can be one solution to overcome these challenges. By utilizing technology, teachers can design more interactive and engaging learning experiences, thereby motivating students to develop their computational thinking skills.

To address this challenge, innovative pedagogical models that combine open-ended problem-based learning (OEPBL) and structured mathematical instruction are urgently needed. One promising approach is the integration of the Missouri Mathematics Project (MMP) a structured, phased mathematics teaching model with Open-Ended Problem-Based Learning (OEPBL) strategies. This hybrid model, referred to here as MiMOPBL, has the potential to enhance students' engagement with real-world, context-rich mathematical problems while supporting the development of critical and analytical thinking.

Open-ended problems in mathematics education have been shown to foster creativity, flexible thinking, and deeper understanding of mathematical concepts (Leikin, 2012; Sullivan et al., 2014). Open-ended problem-based learning can improve students' computational thinking skills (Santika Lya Diah Pramesti, Heni Lilia Dewi, 2024). Problem-based learning (PBL) allows students to learn independently by solving problems through observing and understanding real-world experiences through active participation in learning (Chen, 2017). This enables students to acquire the necessary skills and attributes, including knowledge, teamwork, and communication. To facilitate more interactive and problem-based learning, one interesting learning model is open-ended problem-based learning (Steck et al., 2012). The Missouri Mathematics Project (MMP) model was developed as an approach that emphasizes mathematics learning through the exploration of real-world problems, as well as the development of critical and analytical thinking skills (Bangun & W. L. Sihombing, 2023; Indrawati et al., 2024). However, the effectiveness of this approach can be further enhanced through digital learning technologies, particularly electronic modules (e-modules).

E-modules serve as digital learning tools that provide interactive content, immediate feedback, and learner autonomy. In particular, the Predict–Observe–Explain (POE) model has proven effective in fostering scientific reasoning and computational thinking by encouraging students to anticipate outcomes, engage in inquiry-based observation, and construct logical explanations (Jasdilla et al., 2019; Pektaş et al., 2024; Tuysuz & Özdemir, 2025). When embedded into e-modules, the POE model facilitates active, inquiry-driven learning that supports both CT and SRL development (McKnight et al., 2016; Sneider et al., 2014).

The integration of MiMOPBL with POE-based e-modules in mathematics education represents a novel and under-researched pedagogical framework. While prior studies have investigated CT development through PBL (Kong & Wang, 2024; Yadav et al., 2016) or e-learning tools (Kwon & Lee, 2019), few—if any—have systematically designed and evaluated a learning trajectory that combines MiMOPBL, POE, and digital e-modules to address both CT and SRL in a unified instructional model, particularly within the context of Social Arithmetic in Indonesian junior high schools.

To address this gap, this study aims to design a Hypothetical Learning Trajectory (HLT) that integrates MiMOPBL with POE-based e-modules for mathematics instruction. The HLT is developed to provide a coherent sequence of instructional activities that promote the development of CT and SRL

while aligning with curriculum goals and addressing common barriers in mathematics learning. This design is expected to offer a practical and innovative instructional model that supports 21st-century competencies and enhances students' ability to solve real-world problems in a digitally connected society.

2. METHODS

This study uses an Educational Design Research (EDR) approach with development studies and validation studies models as described by Cobb, Confey, Disessa, Lehrer (2011). The primary goal of EDR is to design and refine innovative learning interventions that are not only effective in practice but also theoretically grounded. In this study, the intervention aims to improve students' Computational Thinking (CT) and Self-Regulated Learning (SRL) through a learning model that integrates Missouri Mathematics Open-Ended Problem-Based Learning (MiMOPBL) with POE-based e-modules.

The research was carried out through the three iterative phases described by McKenney & Reeves (2018): (1) Analysis and Exploration, (2) Design and Construction, and (3) Evaluation and Reflection. The research was conducted through three main phases as outlined by McKenney, S., & Reeves, (2018) as follows:

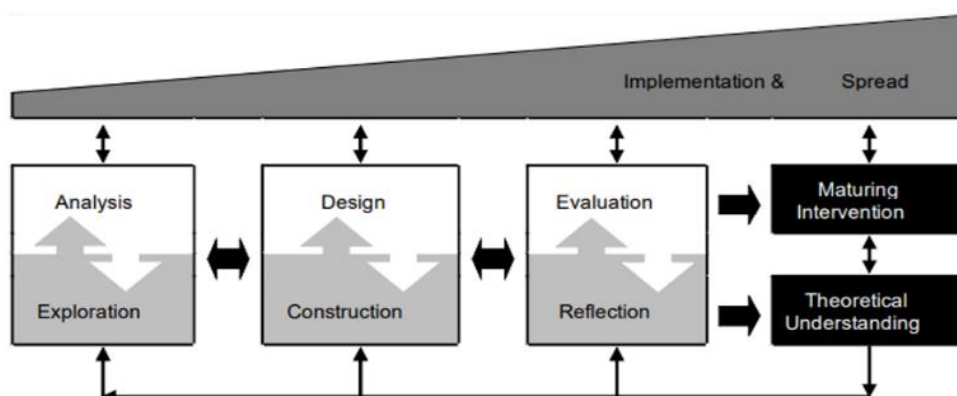


Figure 1. Educational Design Research (EDR) by Mc Kenney & Reeves

Analysis and Exploration

This phase involved a comprehensive contextual study at SMP Negeri 6 Kota Pekalongan. Qualitative data were collected through:

1. Classroom observations of Grade IX mathematics lessons,
2. Semi-structured interviews with mathematics teachers, and
3. Documentation analysis, including lesson plans, student worksheets, and assessment instruments.

The qualitative data were analyzed using Miles et al. (2014) milesinteractive model—comprising data condensation, data display, and conclusion drawing. These analyses helped identify instructional challenges, student learning behaviors, and gaps between expected and actual learning outcomes.

Findings from this phase were used to construct the initial Hypothetical Learning Trajectory (HLT), consisting of:

1. Observation data on student learning behavior and engagement, especially with open-ended problems, informed the design of learning activities and anticipated student responses.
2. Interviews revealed teacher challenges in implementing problem-based and technology-assisted instruction, guiding the inclusion of scaffolding strategies within the e-module.
3. Document analysis identified misalignments between existing assessments and the expected CT and SRL outcomes, prompting a redesign of learning objectives.

The initial HLT comprised:

1. Clear learning goals aligned with CT and SRL indicators,
2. Carefully sequenced learning activities using the MiMOPBL and POE based e-modules,
3. Anticipated learning processes and potential student difficulties with appropriate support strategies.

Design and Construction

The researchers developed a prototype design for MiMOPBL assisted by e-modules based on the Prediction–Observation–Explanation (POE) approach. This initial product was designed based on the results of the exploration and developed with consideration of content validity (by subject matter, media, and pedagogical experts). Validation was carried out through expert judgement. The validated initial product was then implemented on a limited basis through a pilot test, then improved and refined through a teaching experiment.

To collect both qualitative and quantitative data, several research instruments were developed and adapted from validated sources. These include: SRL Questionnaire, CT Test Instrument, Observation Sheet, and Interview Guide. The SRL questionnaire was adapted from Schunk & Zimmerman (1998), addressing metacognitive strategies such as planning, monitoring, and evaluation. The CT test was constructed based on the frameworks of Grover & Pea (2013); Voogt et al. (2016); Wing (2006), covering decomposition, abstraction, algorithmic thinking, and pattern recognition. The observation sheet was based on observable indicators of MiMOPBL and POE implementation. The interview guide was constructed to capture teacher/student perceptions regarding instructional strategies, self-regulation, and cognitive engagement with open-ended tasks.

Content Validity using Aiken's V

Content validity was assessed using **Aiken's V** coefficient with input from **three expert validators**: a mathematics educator, an educational psychologist, and an instructional technology specialist. Experts rated each item on a **5-point Likert scale** (1 = not relevant, 5 = highly relevant).

Table 1. Results of the Aiken's V analysis

Instrument	No. of Items	Aiken's V Range	Remarks
SRL Questionnaire	25	0.83 – 0.97	All items valid; 3 revised for clarity.
CT Test	8	0.80 – 0.95	1 item revised (instructions and scoring clarification).
Observation Sheet	12	0.86 – 0.94	Phrasing improved; all indicators retained.
Interview Guide	10	0.84 – 0.98	2 prompts refined for alignment with CT/SRL constructs.

All instruments exceeded the **minimum acceptable threshold of 0.75**, indicating strong content validity (Azwar, 2012).

Reliability Testing using Cronbach's Alpha

Following validation, reliability testing was conducted using Cronbach's Alpha to assess the internal consistency of the instruments. A pilot study involving 33 Grade IX students.

Table 2. Results of the reliability analysis

Instrument	Cronbach's Alpha (α)	Reliability Level
SRL Questionnaire	0.85	Very High Reliability
CT Test Instrument	0.89	Very High Reliability
Observation Sheet	0.88	Very High Reliability

These findings confirm that all instruments possessed excellent internal consistency and were suitable for use in the main study.

Evaluation and Reflection

An evaluation was conducted to assess the practicality and effectiveness of the learning design through classroom trials. Reflections on the implementation process were used to refine the HLT into a revised learning trajectory (Callejo et al., 2022).

The evaluation focuses on the achievement of students' CT and SRL abilities and the implementation of MiMOPBL syntax. The research subjects are ninth-grade students at SMP Negeri 6 Kota Pekalongan. The subjects were selected purposively based on their suitability for the characteristics and needs of the research. The research took place in the even semester of the 2024/2025 academic year.

Data collection techniques include: semi-structured interviews, open observation, documentation study (lesson plans, student worksheets), expert judgment (media and material validation), questionnaires to measure self-regulated learning, and tests to measure students' computational thinking. The instruments used include interview guidelines, observation sheets, SRL questionnaires based on Schunk & Zimmerman's (1998) indicators, and CT ability test questions based on Wing, (2006) indicators, (Grover & Pea, 2013; Voogt et al., 2016)Voogt et al. (2016), and Grover & Pea (2013). Data validity was ensured through triangulation of sources and methods, expert validation of instruments and learning tools, member checking, and review by mathematics lecturers and learning design experts. Data analysis was conducted using qualitative and quantitative analysis. Qualitative analysis: using Miles, Huberman, & Saldaña (1992) techniques, including data reduction, data presentation, and conclusion drawing for observational data, interviews, and documents. Quantitative analysis: normality tests, N-gain calculations, and t-tests to assess the effectiveness of the intervention on CT and SRL abilities.

This study is part of the initial stage of the *Educational Design Research (EDR)* approach, which focuses specifically on the process of needs analysis and initial learning design. Therefore, the scope of this study is limited to the formulation of a *Hypothetical Learning Trajectory (HLT)* for Social Arithmetic material, which includes the formulation of learning objectives, the development of learning activities, and the prediction of student learning processes. This study does not include the implementation or evaluation of the effectiveness of the HLT in the classroom. This limitation is set to maintain the focus and depth of the study on the design phase of the MiMOPBL-based learning model assisted by POE-based e-modules.

3. FINDINGS AND DISCUSSION

This study uses an Educational Design Research (EDR) approach based on the McKenney and Reeves (2012) model, which consists of three main stages: (1) analysis and exploration, (2) design and construction, and (3) evaluation and reflection. However, this study is limited to the design and construction stages, specifically the development of the Hypothetical Learning Trajectory (HLT) and the validation process of the learning tools and research instruments. The implementation stage in the

classroom and the evaluation of effectiveness have not been conducted due to the limitations of the scope and time of the study.

3.1 Research Results

3.1.1 Analysis and Exploration Stage

In the analysis and exploration stage, an in-depth identification of mathematics learning problems was conducted, especially in social arithmetic material. The main findings obtained from literature reviews, surveys, interviews, and observations are as follows:

1. Students' interest in mathematics learning is still low, especially in social arithmetic, which is considered boring and difficult due to its lack of connection to real life.
2. Limited use of innovative teaching materials, such as e-modules and technology-based LKPD, which can increase student active involvement.
3. The learning approach used is not yet optimal in developing computational thinking (CT) and self-regulated learning (SRL) in students.

To address these issues, the researcher designed an E-Module-assisted MiMOPBL Learning Design based on POE, which aims to increase student active involvement and develop CT and SRL skills in mathematics learning, particularly in social arithmetic material.

3.1.2 Design and Construction Stage

In the design and construction stage, the researchers developed a learning prototype based on the analysis results. Several main activities in this stage are as follows:

1. Solution Exploration: The MiMOPBL model was chosen because of its systematic structure and support for repetitive learning stages and concept reinforcement. In addition, the open-ended PBL approach allows students to think reflectively, while the POE strategy provides active learning experiences through prediction, observation, and explanation.
2. Solution Mapping: Learning is designed with problem-based activities, open-ended questions, and the use of interactive e-modules based on POE that support the development of CT and SRL.
3. Development of HLT (Hypothetical Learning Trajectory): HLT is developed to predict the learning process of students. This learning design includes phases such as orientation, open-ended problem exploration, group work, discussion, and presentation (Febriani & Sidik, 2020). HLT also identifies the predicted learning process, including predictions of student responses, potential misconceptions, and required assistance.

HLT was developed for the topic of Social Arithmetic, beginning with the following activities:

1. Creating and solving problems based on real-life contexts.
2. Overcoming learning obstacles.
3. Developing CT skills through activities such as decomposition and abstraction, and improving SRL through independent and reflective activities.

Learning activities are designed in the Missouri Mathematics Open Ended Problem Based Learning (MiMOPBL) sequence, which allows students to think analytically from the start and reflect on their results.

Table 3. Illustrates the learning stages in the Hypothetical Learning Trajectory design for Social Arithmetic.

Learning Stage	Learning Objective	Learning Activities	Predicted Student Response	Anticipation of Student Difficulties
Introduction to Social Arithmetic	Students can understand the basic concepts of Social	Using simple examples such as price comparisons, discounts, and interest	Students will easily grasp basic concepts and provide relevant examples.	Some students may struggle with new terms like <i>discount</i> and <i>interest</i> . The teacher needs to provide additional

Learning Stage	Learning Objective	Learning Activities	Predicted Student Response	Anticipation of Student Difficulties
Introduction to Discount Calculation Procedures	Arithmetic and its relevance to daily life.	to explain the concept. Students are asked to identify real-life problems.		examples and facilitate discussion to ensure understanding. To enhance self-regulated learning, students are asked to reflect on their understanding after the activity.
	Students can apply basic procedures in Social Arithmetic (percentage, discount, interest).	Students are given exercises on calculating discounts based on item quantity and category, such as questions from the STAR TOYS store. They calculate total price after discounts.	Students will begin to understand how to calculate discounts based on quantity and item type. They will try to solve the problems using systematic steps.	Some students may find it difficult to calculate two types of discounts simultaneously. Teachers need to clearly explain how to combine discounts by category and quantity. To enhance computational thinking, students are encouraged to break down problem-solving steps into manageable parts.
		Students are given a problem: "A child buys 6 toys with specific prices and quantities. Calculate the total price to be paid after applying quantity and category discounts." Students solve the problem in groups and discuss their solutions.	Students will attempt to identify which discounts apply based on quantity and category, and then calculate the total price after discounts.	Students may get confused managing discounts based on two factors (quantity and category). Teachers must emphasize how quantity and category discounts are applied together. To foster self-regulated learning, students evaluate the steps they took and verify if their solution is correct.
Understanding Complex Problems with Two Types of Discounts	Students can solve Social Arithmetic problems involving more than one type of discount.			Students may struggle to identify the pattern or relationship between quantity and discount. Teachers need to guide students on how to structure patterns from the given problem. To develop computational thinking, students will write down the steps they took and compare patterns found by other groups.
Solving Problems by Identifying Patterns	Students can identify patterns or relationships between quantity, item category, and discounts given.	Students are given a follow-up question: "Another child buys 4 Robot toys and 8 Puzzle toys. Calculate the total price after discounts and identify the pattern between quantity and discounts received."	Students will try to solve the problem and identify relationships between the number of items and the discounts received. They will start to relate discount patterns based on quantity and category.	
Applying Discounts in Real-Life Situations	Students can connect Social Arithmetic theory to real-world situations and maximize received discounts.	Students are asked: "What are the purchase criteria or item combinations that yield the maximum discount with the minimum number of items?" They propose optimal	Students will try to find item combinations that provide the highest discount, whether by quantity or category. They will analyze the results based on	Some students may struggle to understand how to optimize discounts based on certain criteria. Teachers should further explain how to find item combinations that provide the best discounts. To enhance self-regulated learning, students

Learning Stage	Learning Objective	Learning Activities	Predicted Student Response	Anticipation of Student Difficulties
		purchase combinations.	previous calculations.	are given time to evaluate and present their best combinations, and discuss their calculation processes.

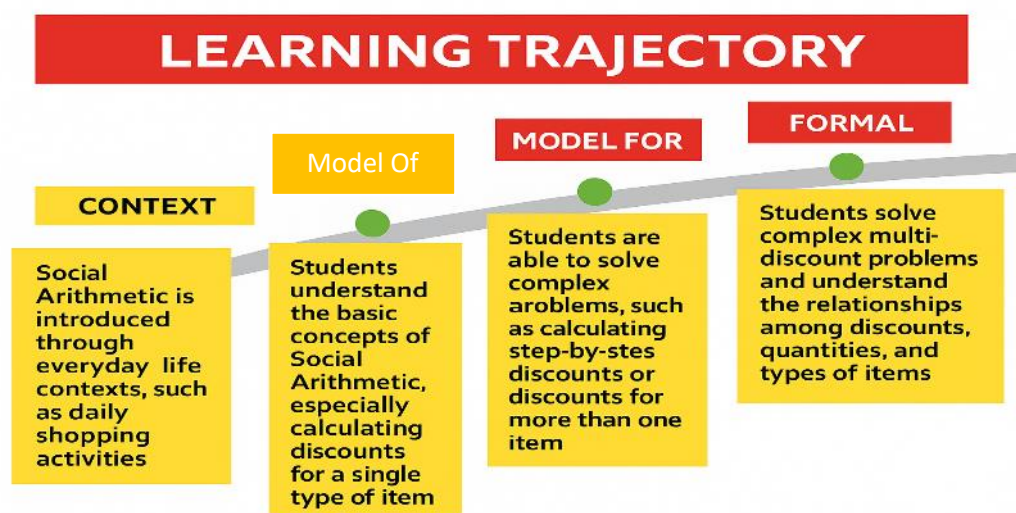


Figure 2. Hypothetical Learning Trajectory design for Social Arithmetic

3.1.3 Validation of Learning Instruments

Various instruments to measure learning success have been validated using the V-Aiken method (Merino-Soto, 2023). The validation results show that instruments such as CT ability tests, SRL questionnaires, and interview guides for teachers and students are valid and can be used with minor revisions.

The content validity of these instruments shows high validity criteria with V-Aiken scores indicating a very high category for most aspects of the assessment.

Validation of Computational Thinking (CT) Ability Test

The validation results show that the CT ability test is valid in terms of material suitability, construction, and language. Based on the validators' assessment, the test items have met content validity and can be used with minor revisions. Some comments and suggestions provided by the validators are as follows.

- a) The questions have included the main indicators of computational thinking (CT), namely decomposition, pattern recognition, abstraction, and algorithmic thinking in a proportional manner.
- b) The language of the questions is communicative, clear, and appropriate for the understanding level of junior high school students.
- c) Consider adding symbols or indicators of CT on each question in the teacher's work document to facilitate the analysis of achievement.
- d) Add stimuli or explicit instructions in some questions to encourage students to write general patterns or formulas as reinforcement of the abstraction indicator.

The results of the difficulty level and discriminative power tests have been met. The reliability results based on Cronbach's alpha score of 0.643 indicate that the questions have moderate reliability.

Table 4. Case Processing Summary Computational Thinking (CT) Ability Test

		N	%
Case	Valid	33	100.0
	Excluded ^a	0	.0
	Total	33	100.0

- a. Listwise deletion based on all variables in the procedure.

Table 5. Reliability Statistics Computational Thinking (CT) Ability Test

Cronbach's Alpha	N of Items
.643	3

Validation of the Self-Regulated Learning (SLR) Questionnaire

The validation results show that the self-regulated learning questionnaire is valid using the V-Aiken calculation. Based on the validator's assessment, the questionnaire items have met content validity and can be used with minor revisions. Some comments and suggestions provided by the validator are as follows.

- The language used is clear, simple, and appropriate for the level of understanding of junior high school students.
- Check the consistency of statement direction (positive and negative), and if there are negative statements, ensure they are explained in the scoring process to avoid confusion for respondents and researchers.

The reliability results based on Cronbach's alpha score of 0.900 indicate that the SLR questionnaire items have a high degree of reliability.

Table 6. Case Processing Summary Self-Regulated Learning (SLR) Questionnaire

		N	%
Case	Valid	5	100.0
	Excluded ^a	0	.0
	Total	5	100.0

- a. Listwise deletion based on all variables in the procedure.

Table 7. Reliability Statistics Self-Regulated Learning (SLR) Questionnaire

Cronbach's Alpha	N of Items
.900	3

Validation of Teacher Interview Guidelines

The validation results show that the teacher interview guidelines are valid using the V-Aiken calculation. Based on the validators' assessment, the teacher interview guidelines have met content validity and can be used with minor revisions. Some comments and suggestions provided by the validators are as follows.

- The interview structure is systematic, starting with general questions about learning, then moving on to specific aspects such as teachers' strategies in building student learning independence and CT skills. Avoid misspellings.

- b) Clarify the indicators or objectives of each question in a separate column so that the interviewer understands the direction of the expected answers in accordance with the research focus (e.g., SRL – planning; CT – decomposition).

Validation of Student Interview Guidelines

The validation results show that the student interview guidelines are valid using the V-Aiken calculation. Based on the validator's assessment, the student interview guidelines have met content validity and can be used with minor revisions. Some comments and suggestions provided by the validator are as follows.

- a) Add probe questions (in-depth questions) to each main question so that the interviewer can explore the students' answers more deeply (e.g., "Why did you choose that step?" or "What did you do when you felt stuck?").
- b) Include affective aspects in the interview, such as students' feelings when working on difficult questions or how they motivate themselves, to support a comprehensive analysis of SRL.

Content Validation of the Learning Module

The validation results show that the learning module is valid using the V-Aiken calculation. Based on the validator's assessment, the learning module has met content validity and can be used with minor revisions. Some comments and suggestions provided by the validator are as follows.

- a) The module has been designed with a systematic structure: it includes learning objectives, activity steps, core material, practice questions, and learning reflections.
- b) SRL and CT indicators need to be explicitly added at the beginning or in each activity so that teachers and students understand the skills being developed.
- c) Include a variety of question levels, ranging from simple to open-ended questions, so that students can practice generalization and abstraction skills.

Content Validation of the POE E-Module

After the POE E-Module was developed, content validity was conducted by qualitatively reviewing the E-Module to assess the quality of the questions in terms of content relevance, construction, and language. The validation results indicated that the POE E-Module is valid using the V-Aiken calculation. Based on the validators' assessment, the POE E-Module meets content validity criteria and can be used with minor revisions. Some comments and suggestions provided by the validators are as follows.

- a) The structure of the e-module clearly follows the POE stages: Prediction (initial stimulus and knowledge activation), Observation (independent exploration or experimentation), and Explanation (concept reflection and understanding reinforcement).
- b) The digital format of the e-module enables student-centered interaction that supports the principles of self-regulated learning, such as flexible navigation, independent tasks, and learning reflection.

Content Validation of the LKPD

After the POE LKPD was developed, content validity was conducted by qualitatively reviewing the LKPD to assess the quality of the questions in terms of content relevance, construction, and language. The validation results indicate that the POE e-module is valid using the V-Aiken calculation. Based on the validators' assessment, the POE e-module meets content validity criteria and can be used with minor revisions. Several comments and suggestions provided by the validators are as follows.

- a) Consider varying the types of questions, from exploratory to open-ended, to facilitate students with different learning styles and ability levels.
- b) Add SRL and CT skill indicators at the beginning or in each activity so that students and teachers are aware of the skills being developed.

Discussion

In the preliminary design stage of this Educational Design Research (EDR), the researcher developed a learning model integrating Missouri Mathematics Open-Ended Problem-Based Learning (MiMOPBL) with an e-module based on the Predict–Observe–Explain (POE) approach. The central aim of this design was to foster two essential 21st-century competencies—Computational Thinking (CT) and Self-Regulated Learning (SRL)—through the contextual topic of Social Arithmetic, which includes real-world problems such as discounts, taxes, and sales transactions.

As described in the theoretical framework, CT encompasses decomposition, pattern recognition, abstraction, and algorithmic thinking (Wing, 2006; Grover & Pea, 2013; Voogt et al., 2016). SRL, on the other hand, emphasizes students' ability to plan, monitor, and reflect on their learning processes (Schunk, 1998; Zimmerman, 2002). These competencies are operationalized through the POE model, where students are encouraged to make predictions, observe phenomena or processes, and explain outcomes based on their reasoning. This sequence not only promotes logical reasoning and problem-solving but also activates metacognitive processes essential to SRL.

To support the learning design, a Hypothetical Learning Trajectory (HLT) was constructed. The HLT outlines learning goals, predicted student strategies, and anticipated responses at each stage of the POE framework. For example, in the predict stage, students are expected to activate prior knowledge and estimate possible outcomes—such as estimating discounted prices in a real-world scenario. During the observe stage, they calculate or simulate outcomes, and in the explain stage, they reflect on the correctness of their predictions and rationalize their problem-solving steps.

The development of the HLT was informed by the assumption that students would begin to exhibit CT processes such as breaking down problems into manageable components (decomposition), recognizing repetitive structures (pattern recognition), and—although still developing—attempting generalizations (abstraction). Algorithmic thinking was expected to emerge gradually as students began to organize logical steps for solving problems. In terms of SRL, the POE-based design was predicted to foster planning, monitoring, and reflective evaluation through the structure of learning tasks and teacher prompts.

The validation of the learning design, including the HLT and accompanying instruments (lesson plans, e-modules, student worksheets, CT tests, SRL questionnaires, interview guidelines, and observation instruments), showed that the design was theoretically sound and practically applicable. Lesson plans integrated CT and SRL indicators clearly into each POE stage. The e-modules and student worksheets facilitated exploration of open-ended problems using multimedia support and guided reflection. Instruments for assessing CT and SRL were judged by experts to be aligned with learning objectives and appropriate for junior high school students.

However, critical reflection on the validation process revealed several limitations. First, while the tasks successfully prompted students to engage in decomposition and pattern recognition, abstraction and algorithmic thinking appeared more fragile and less spontaneously demonstrated. Students often relied on concrete examples rather than attempting generalizations, indicating a need for more explicit scaffolding in that area. Furthermore, validators noted that students occasionally misunderstood the intent of prediction tasks, treating them as guesswork rather than reasoned estimations. This highlights the necessity of incorporating explicit metacognitive prompts to guide students in making informed predictions. Additionally, observation data indicated that not all students consistently demonstrated self-regulated behaviors, especially in monitoring their own performance. Some students required significant teacher prompting to reflect effectively, suggesting that SRL development may demand more sustained instructional support beyond single POE cycles.

Another limitation lies in the contextual scope of the HLT itself. Since it was developed for a specific topic—social arithmetic—it may require adaptation when applied to other mathematical domains with different cognitive demands. While the underlying design principles are transferable, the specific tasks, scaffolds, and indicators may need modification. This also points to an opportunity for future research to extend the model across various mathematical topics to assess its generalizability.

From a theoretical perspective, the HLT developed in this study contributes to the growing body of knowledge on the integration of CT and SRL within mathematics learning design. While most existing HLT models focus on conceptual or procedural development in mathematics (e.g., fractions, geometry), this design explicitly integrates 21st-century cognitive and metacognitive competencies into the trajectory. It demonstrates how the POE structure can serve as a pedagogical vehicle to activate both computational and self-regulatory processes in a cohesive instructional model. Thus, the present study offers a conceptual contribution by situating CT and SRL as designable learning outcomes, not merely by-products of problem-solving tasks.

Moreover, the articulation between the predicted learning processes in the HLT and the validated assessment tools enables a clearer alignment between learning design, implementation, and evaluation—an area often underdeveloped in mathematics education research. This alignment enhances both the theoretical robustness and practical utility of the model and lays a strong foundation for further iterations in the next EDR phases.

In conclusion, while the preliminary design of the MiMOPBL model supported by POE-based e-modules shows promise in enhancing CT and SRL, the validation process highlights the importance of continuous refinement, especially in guiding abstraction and self-monitoring skills. The contributions of this study are both practical, in terms of validated instructional tools, and theoretical, through the development of an HLT framework that integrates cognitive and metacognitive competencies in mathematics learning. Future studies should focus on broader implementation and longitudinal evaluation to investigate the model's long-term impact and scalability.

4. CONCLUSION

Based on the results of the preliminary design stage, it can be concluded that the Missouri Mathematics Open-Ended Problem-Based Learning (MiMOPBL) model assisted by Predict–Observe–Explain (POE)-based e-modules has been systematically and theoretically developed to enhance students' Computational Thinking (CT) and Self-Regulated Learning (SRL) skills. The instructional design is underpinned by a clearly articulated Hypothetical Learning Trajectory (HLT) that outlines students' progressive learning processes through stages of prediction, observation, explanation, and reflection.

The findings demonstrate that the integration of POE within the MiMOPBL framework has strong potential to facilitate the development of CT components, including decomposition, pattern recognition, abstraction, and algorithmic thinking. Additionally, the structured learning process encourages students' engagement in planning, monitoring, and evaluating their learning, thus fostering SRL competencies. All developed learning tools—including syllabi, lesson plans, student worksheets, POE-based e-modules, CT test instruments, SRL questionnaires, interview guides, and observation protocols—have been validated by experts, ensuring theoretical alignment and consistency with HLT predictions and instructional goals.

However, this study has several limitations. First, it is confined to the context of social arithmetic in junior high school and has not been tested across other mathematical domains or different educational levels, limiting the generalizability of the findings. Second, while the design phase showed positive alignment between tasks and learning outcomes, some students exhibited misconceptions—particularly in interpreting prediction tasks—and required significant scaffolding, especially in abstraction and self-monitoring activities. These observations suggest that further refinement of instructional supports is necessary to maximize the impact of the learning model.

In light of these findings, future research should proceed to the prototyping and implementation stages of Educational Design Research (EDR). Classroom-based experimental or quasi-experimental studies are recommended to empirically assess the effectiveness and scalability of the MiMOPBL learning model across varied mathematical topics and learner profiles. It is also suggested that the POE-based e-modules be further developed to enhance interactivity, adaptiveness, and responsiveness to diverse student needs. Moreover, refinement of evaluation instruments is necessary to capture the complexity of students' CT and SRL processes more comprehensively.

This study not only offers validated instructional resources for classroom practice but also contributes to the theoretical advancement of mathematics learning design by embedding 21st-century competencies such as CT and SRL into the instructional architecture. It opens new avenues for integrating these competencies in diverse subjects and educational settings, thereby supporting the broader goal of preparing students to face complex challenges in the digital age.

Acknowledgments: In this section, you can acknowledge any support given, which is not covered by the author's contribution or funding sections. This may include administrative and technical support, or donations in kind (e.g., materials used for experiments).

Conflicts of Interest: Declare conflicts of interest or state "The authors declare no conflict of interest." Authors must identify and declare any personal circumstances or interests that may be perceived as inappropriately influencing the representation or interpretation of reported research results.

REFERENCES

- Bangun, S. E., & W. L. Sihombing. (2023). The Effect of the Missouri Mathematics Project (MMP) Learning Model on Students' Mathematical Problem Solving Ability in Algebra Material in Class VII Budi Murni 2 Catholic Private Middle School Medan. *Jurnal Ilmiah Pendidikan Holistik (JIPH)*, 2(1), 1–16. <https://doi.org/10.55927/jiph.v2i1.2827>
- Bebras Indonesia. (2024). *Pengumuman Hasil Bebras Indonesia Challenge 2024*. Bebras Indonesia. Keterampilan berpikir komputasi diperkirakan memiliki peran penting di hampir setiap bidang dan profesi di masa depan.
- Bocconi, S., Chiocciariello, G. A., Dettori, A. F., & Engelhardt, K. (2016). Developing Computational Thinking in Compulsory Education. In *Joint Research Centre (JRC)* (Issue June). <https://doi.org/10.2791/792158>
- Bouck, E. C., & Yadav, A. (2022). Providing Access and Opportunity for Computational Thinking and Computer Science to Support Mathematics for Students With Disabilities. *Journal of Special Education Technology*, 37(1), 151–160. <https://doi.org/10.1177/0162643420978564>
- Bråting, K., Kilhamn, C., & Rolandsson, L. (2022). *Mathematical Competencies and Programming: The Swedish Case BT - Mathematical Competencies in the Digital Era* (U. T. Jankvist & E. Geraniou (Eds.); pp. 293–310). Springer International Publishing. https://doi.org/10.1007/978-3-031-10141-0_16
- Brown, N. C. C., Sentance, S., Crick, T., & Humphreys, S. (2014). Restart: The Resurgence of Computer Science in UK Schools. *ACM Trans. Comput. Educ.*, 14(2). <https://doi.org/10.1145/2602484>
- Callejo, M. L., Pérez-Tyteca, P., Moreno, M., & Sánchez-Matamoros, G. (2022). The Use of a Length and Measurement HLT by Pre-Service Kindergarten Teachers' to Notice Children's Mathematical Thinking. *International Journal of Science and Mathematics Education*, 20(3), 597–617. <https://doi.org/10.1007/s10763-021-10163-4>
- Chen, G. (2017). *Learning*. 156(Seiem), 128–131.
- Cobb, Confey, Disessa, Lehrer, & S. (2011). *Design experoment in educational research*.
- Falkner, K., Vivian, R., & Falkner, N. (2015). Teaching computational thinking in K-6: The CSER digital technologies MOOC. *Conferences in Research and Practice in Information Technology Series*, 160(May), 63–72.
- Fisher, L. M. (2016). A decade of ACM efforts contribute to computer science for all. *Commun. ACM*, 59(4), 25–27. <https://doi.org/10.1145/2892740>

- Grover, S., & Pea, R. (2013). Computational Thinking in K-12: A Review of the State of the Field. *Educational Researcher*, 42(1), 38–43. <https://doi.org/10.3102/0013189X12463051>
- Hasdi, H., Manuharawati, M., & Sulaiman, R. (2024). Kemampuan Pemecahan Masalah Matematika Siswa dengan Gaya Kognitif Field Dependent dan Field Independent. *EDUKASIA: Jurnal Pendidikan Dan Pembelajaran*, 5(1), 1393–1398. <https://doi.org/10.62775/edukasia.v5i1.1040>
- Indrawati, Zaida, S., & Lestari, P. E. P. (2024). Pengaruh Model Missouri Mathematics Project (MMP) terhadap Hasil Belajar Matematika ditinjau dari Kemandirian Belajar. *Circle: Jurnal Pendidikan Matematika*, 4(1), 52–64. <https://doi.org/10.28918/circle.v4i1.6897>
- Jasdilla, L., Fitria, Y., & Sopandi, W. (2019). Predict Observe Explain (POE) strategy toward mental model of primary students. *Journal of Physics: Conference Series*, 1157(2). <https://doi.org/10.1088/1742-6596/1157/2/022043>
- Kong, S.-C., & Wang, Y.-Q. (2024). Dynamic interplays between self-regulated learning and computational thinking in primary school students through animations and worksheets. *Computers and Education*, 220. <https://doi.org/10.1016/j.compedu.2024.105126>
- Kwon, H., & Lee, E. (2019). Research trends and issues of education for sustainable development-related research in South Korea. *Journal of Baltic Science Education*, 18(3), 379–388. <https://doi.org/10.33225/jbse/19.18.379>
- Leikin, Mark. (2012). The effect of bilingualism on creativity: Developmental and educational perspectives. *International Journal of Bilingualism*, 17(4), 431–447. <https://doi.org/10.1177/1367006912438300>
- Mannila, L., Dagiene, V., Demo, B., Grgurina, N., Mirolo, C., Rolandsson, L., & Settle, A. (2014). Computational Thinking in K-9 Education. *Proceedings of the Working Group Reports of the 2014 on Innovation & Technology in Computer Science Education Conference*, 1–29. <https://doi.org/10.1145/2713609.2713610>
- McKenney, S., & Reeves, T. C. (2018). *Conducting educational design research*. (2nd ed.). Routledge. <https://doi.org/10.4324/9781315105642>
- McKnight, K., O'Malley, K., Ruzic, R., Horsley, M. K., Franey, J. J., & Bassett, K. (2016). Teaching in a Digital Age: How Educators Use Technology to Improve Student Learning. *Journal of Research on Technology in Education*, 48(3), 194–211. <https://doi.org/10.1080/15391523.2016.1175856>
- Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative Data Analysis*.
- Nurjanah, A., Nurcahyono, N. A., & Imswatama, A. (2022). Penerapan Model Problem Based Learning terhadap Kemampuan Pemecahan Masalah Matematis Ditinjau dari Gaya Belajar Siswa SMP. *Prisma*, 11(2), 406. <https://doi.org/10.35194/jp.v11i2.2420>
- OECD. (2023). *PISA 2022 Results (Volume I)*. <https://doi.org/https://doi.org/https://doi.org/10.1787/53f23881-en>
- Paris, S. G., & Paris, A. H. (2001). *Classroom Applications of Research on Self-Regulated Learning*. 36(2), 89–101.
- Pektaş, H. M., Çelik, H., & Karamustafaoğlu, O. (2024). *P OE Activity Enriched with Augmented Reality : Conceptual Change on Pressure in Liquids*. 20(December), 124–138. <https://doi.org/10.15294/jpfi.v20i2>
- Prahmana, R. C. I., Kusaka, S., Peni, N. R. N., Endo, H., Azhari, A., & Tanikawa, K. (2024). Cross-cultural insights on computational thinking in geometry: Indonesian and Japanese students' perspectives. *Journal on Mathematics Education*, 15(2), 613–638. <https://doi.org/10.22342/jme.v15i2.pp613-638>
- Santika Lya Diah Pramesti, Heni Lilia Dewi, N. A. (2024). *View of Analysis of Students' Computational Thinking Processes in Merdeka Curriculum Differentiation Learning using The Open-Ended Problem Based Learning Model.pdf*. <https://doi.org/https://doi.org/10.18326/hipotenus.v6i2.1899>
- Schunk, D. H., & Zimmerman, B. J. (Eds.). (1998). Self-regulated learning: From teaching to self-reflective practice. In *Self-regulated learning: From teaching to self-reflective practice*. (pp. xii, 244–xii, 244). Guilford Publications.
- Shute, V. J., Sun, C., & Asbell-clarke, J. (2017). *Version of Record*:

- <https://www.sciencedirect.com/science/article/pii/S1747938X17300350>.
- Sneider, C., Stephenson, C., Schafer, B., & Flick, L. (2014). Computational Thinking in High School Science Classrooms: Exploring the Science “Framework” and “NGSS.” *Science Teacher*, 81(5), 53–59. <https://www.learntechlib.org/p/155904>
- Song, D., Hong, H., & Oh, E. Y. (2021). Applying computational analysis of novice learners’ computer programming patterns to reveal self-regulated learning, computational thinking, and learning performance. *Computers in Human Behavior*, 120, 106746. <https://doi.org/https://doi.org/10.1016/j.chb.2021.106746>
- Steck, T. R., DiBiase, W., Wang, C., & Boukhtiarov, A. (2012). The Use of Open-Ended Problem-Based Learning Scenarios in an Interdisciplinary Biotechnology Class: Evaluation of a Problem-Based Learning Course Across Three Years. *Journal of Microbiology & Biology Education*, 13(1), 2–10. <https://doi.org/10.1128/jmbe.v13i1.389>
- Sudrajat. (2008). *Pengertian, Strategi, Metode, Teknik, dan Model Pembelajaran*. Sinar Baru Algensindo.
- Sullivan, A. M., Johnson, B., Owens, L., & Conway, R. (2014). Punish them or engage them? Teachers’ views of unproductive student behaviours in the classroom. *Australian Journal of Teacher Education*, 39(6), 43–56. <https://doi.org/10.14221/ajte.2014v39n6.6>
- Tuysuz, A., & Özdemir, Ö. faruk. (2025). An experimental study exploring the effects of predict–observe–explain method supported with simulations. *Research in Science & Technological Education*, 43(2), 512–524. <https://doi.org/10.1080/02635143.2023.2296458>
- Voogt, J., Fluck, A., Webb, M., Cox, M., Malyn-Smith, J., & Zagami, J. (2016). *A K-6 Computational Thinking Curriculum Framework: Implications for Teacher Knowledge*. 19, 47–57.
- Wing, J. M. (2006). Computational thinking. *Commun. ACM*, 49(3), 33–35. <https://doi.org/10.1145/1118178.1118215>
- Wiraharta, I. P. R. N., Yudiana, K., & Kusmariyatri, N. N. (2020). Meningkatkan Hasil Belajar Matematika Siswa Melalui Model Pembelajaran Open Ended Berbasis Tri Kaya Parisudha. *Jurnal Adat Dan Budaya Indonesia*, 2(1), 41–51. <https://doi.org/10.23887/jabi.v2i1.28907>
- Yadav, A., Hong, H., & Stephenson, C. (2016). Computational Thinking for All: Pedagogical Approaches to Embedding 21st Century Problem Solving in K-12 Classrooms. *TechTrends*, 60(6), 565–568. <https://doi.org/10.1007/s11528-016-0087-7>