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Exploring the Critical Thinking Structures of Low-Achieving Students in Solving Algebra HOTS Tasks

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Abstract

Purpose: Low achievement in mathematics remains a critical concern in supporting students' meaningful learning and cognitive development. This study explored the critical thinking skills of junior high school students with low mathematics achievement. Specifically, the study aimed to identify the structures of students' critical thinking based on the FRISCO framework, encompassing Focus, Reasons, Inference, Situation, Clarity, and Overview. **Methodology:** This study adopted a qualitative case study design to provide an in-depth understanding of students' critical thinking structures. Ten seventh-grade students with low mathematics achievement were purposively selected as participants. Data were collected through open-ended algebraic Higher-Order Thinking Skills (HOTS) tasks and semi-structured follow-up interviews. The collected data were subsequently analyzed using content analysis to identify patterns and characteristics of students' critical thinking processes. **Findings:** The findings indicate that most participants encountered substantial difficulties in identifying core problems, recognizing procedural errors, providing justification, and formulating valid conclusions. Two dominant patterns emerged: (1) absence of focus accompanied by invalid reasoning and false inference; and (2) presence of focus without adequate reasoning, with limited inferential coherence. **Significance:** These findings indicate that students' critical thinking remains insufficiently developed and point to the need for more systematic and structured pedagogical support in algebraic HOTS contexts.

Keywords: Algebra; Critical thinking; Difficulty level; Problem solving; Task analysis; Thinking skills.

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Introduction

In the twenty-first century, marked by the rapid advancement of science and technology, the demand for individuals who can think critically, identify problems accurately, and generate appropriate solutions has become increasingly important (Arisoy & Aybek, 2021). Within this context, critical thinking is increasingly emphasized in educational discourse and practice, including mathematics teaching, where teachers' perceptions and readiness shape the extent to which such skills are promoted in classrooms (Ismail et al., 2022). In mathematics learning, critical thinking has also been linked to students' engagement in analysis, communication, and problem solving (Artuz & Roble, 2021). Recent work has also examined how students' prior mathematical knowledge relates to their critical thinking skills (Kania et al., 2023).

In mathematics education, the development of critical thinking has become a central concern because it supports students in reflecting on how they learn mathematics and in engaging with mathematical reasoning more deliberately (Sachdeva & Eggen, 2021). Research has also investigated students' errors in solving mathematics tasks that require critical thinking, revealing difficulties in interpretation, conceptual understanding, procedural application, and technical execution (Umam & Susandi, 2022). These challenges contribute to low achievement in both qualifying and terminal mathematics examinations, as students generally perform better on numerical problems than on word problems (Osei et al., 2026).

In the Indonesian context, recent research has likewise analyzed students' critical thinking in relation to their initial mathematical ability, underscoring the importance of this construct in mathematics learning (Harahap et al., 2024). Mathematical critical thinking also aligns with broader twenty-first-century competencies and has been discussed in relation to academic ability and learning outcomes across a range of educational contexts (Mahanal et al., 2019; Wang & Abdullah, 2024). Empirical studies have reported positive associations between critical thinking and academic achievement or other learning outcomes, although these studies were conducted in contexts beyond junior high school mathematics (D'Alessio et al., 2019; Siburian et al., 2019). Nevertheless, the level of mathematical critical thinking among Indonesian junior high school students remains relatively low (Pramasdyahsari et al., 2023).

Differences in critical thinking are also evident among students with different levels of mathematical ability. Existing evidence suggests that critical thinking and mathematics achievement are related, although the strength of this relationship may vary across ability groups (Duru & Obasi, 2023). Students with low mathematical ability may therefore experience greater difficulty in applying critical reasoning in problem-solving contexts, particularly in analytically demanding topics such as algebra and HOTS-oriented tasks. In addition, Er (2024) reported positive relationships among secondary students' mathematical thinking, critical thinking, and academic achievement, further suggesting that these constructs are closely connected.

Furthermore, differences in the mathematics learning processes and underlying cognitive characteristics between students with mathematical learning disabilities and those with low mathematics achievement should be carefully considered in designing effective learning support (Kivirähk-Koor & Kiive, 2025). Variations in students' critical thinking development also contribute to disparities in mathematical performance, with low-achieving students often experiencing greater difficulties in developing mathematical critical thinking skills than their high-achieving peers. Consistent with this view, Monrat et al. (2022) reported that low-achieving students demonstrated limited mathematical critical thinking abilities. Consequently, educators are challenged to develop instructional approaches that not only enhance students' critical thinking skills but also reduce the achievement gap between high- and low-performing students (Mitani, 2021).

Previous studies have consistently reported positive associations between critical thinking and mathematics achievement and have identified various instructional approaches for promoting critical thinking skills. However, much of this research has focused on measuring the level of critical thinking,

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examining its relationship with academic performance, or evaluating instructional interventions. Comparatively less attention has been given to understanding how low-achieving students structure their critical thinking when engaging with specific mathematical tasks. In particular, existing studies rarely examine the underlying reasoning processes and patterns of argumentation that characterize low-achieving students' responses to algebraic HOTS problems. Consequently, the cognitive structures that constrain these students' critical thinking remain insufficiently understood.

Although extensive research has addressed critical thinking in mathematics learning, the internal structure of critical thinking processes among students with low mathematical ability remains insufficiently explored, particularly in the context of solving algebraic HOTS problems. A more detailed understanding of how students in this group reason, evaluate solution steps, and construct mathematical arguments is essential for designing more adaptive and effective pedagogical interventions.

Tasks that require students to examine and evaluate incorrect mathematical procedures may be particularly valuable for eliciting critical thinking, because error analysis can prompt students to justify, reflect on, and reconsider mathematical reasoning (Rushton, 2018). For this reason, such tasks provide a useful context for investigating how students identify errors, evaluate procedures, and articulate mathematical justifications. However, the assessment of critical thinking (CT) skills remains a persistent challenge, primarily due to the absence of a standardized and robust evaluative framework, thereby constraining systematic efforts to effectively integrate, monitor, and advance CT instruction within educational settings (Alsaleh, 2020).

In response to this gap, the present study aims to identify the critical thinking structures of junior high school students with low mathematical ability when solving algebraic HOTS problems through an analysis based on the FRISCO framework. The findings are expected to contribute both to theoretical understanding and to pedagogical practice, particularly by informing more adaptive efforts to support the development of mathematical critical thinking at the junior high school level.

Literature Review

The conceptualization of critical thinking (CT) has generated a wide range of definitions and theoretical models. Despite this diversity, there is broad consensus that CT encompasses both a dispositional dimension—reflecting an individual's inclination to engage in critical reflection—and a set of general cognitive skills and intellectual competencies (Van Le & Chong, 2023). In its most widely cited formulation, CT is understood as reasonable and reflective thinking directed toward determining what to believe or how to act (Ennis, 2002). From this perspective, CT can be viewed as a complex intellectual activity that involves the coordinated use of cognitive and argumentation skills to evaluate the validity of claims, distinguish truth from falsity, identify fallacious reasoning and unwarranted assumptions, examine inferential relationships among statements, and differentiate between reliable and unreliable information (Cananau et al., 2024).

From a contemporary perspective, CT can be viewed as a metacognitive process encompassing a constellation of skills and dispositions. Through purposeful and self-regulatory reflective judgment, this process enhances the likelihood of generating sound solutions to problems and well-founded conclusions to arguments (Dwyer, 2017; Quinn et al., 2020). Expanding this framework, Liu et al. (2014) identify five core dimensions of CT: the evaluation and use of evidence, the analysis of arguments, the interpretation of implications and consequences, the construction of coherent arguments, and the understanding of causation and explanation. Given its central role in evaluating claims, justifying conclusions, and scrutinizing reasoning processes, CT is inherently aligned with higher-order thinking skills (HOTS) (Kartika et al., 2026).

Within this broader domain, higher-order skills such as CT have been categorized into two principal contexts (Wheeler & Haertel, 1993). The first pertains to their application in problem-solving and decision-making in professional and everyday settings, including issues related to civic life and environmental

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concerns. The second involves their development through formal instruction, where learners engage in complex cognitive processes—such as comparison, evaluation, and justification—within specific disciplinary contexts. Across educational levels and fields of study, CT is indispensable, enabling learners to think independently, address complex problems, and make informed, reasoned decisions.

A growing body of research highlights effective pedagogical strategies for fostering CT (Abrami et al., 2014). These include engaging students in structured dialogue that encourages the exploration of diverse perspectives, situating learning within authentic, real-world contexts that require the active application of CT, and providing individualized guidance to support the progressive development of critical competencies. Importantly, the instructional goal should extend beyond mastery of discrete skills to ensuring that learners can recognize when and how to apply these skills across contexts (Halpern, 1998). Addressing this objective within educational programs remains a critical challenge. Empirical evidence suggests a positive association between CT and academic achievement, with higher levels of academic performance corresponding to more advanced development of critical thinking abilities (Ren et al., 2020).

In the present study, critical thinking is analyzed in relation to students' evaluation of solution steps, their justification of claims, and their drawing of conclusions in response to algebraic HOTS tasks. In this respect, critical thinking is closely related to problem solving and higher-order thinking skills (HOTS). Lewis and Smith (1993) noted that critical thinking has been understood in at least three ways: (a) as problem solving, (b) as evaluation or judgment, and (c) as a combination of evaluation and problem solving. This suggests that critical thinking and problem solving are conceptually related, particularly in tasks that require students to assess the validity of reasoning and arrive at justified conclusions.

Ennis (2002) defines critical thinking as reasonable and reflective thinking focused on deciding what to believe or do. In elaborating this definition, he explains that critical thinking involves a range of interrelated activities, including judging the credibility of sources, identifying conclusions, reasons, and assumptions, evaluating arguments and evidence, asking clarifying questions, defining terms appropriately, remaining open-minded, being well informed, and drawing conclusions with due caution. These activities collectively characterize critical thinking as a reflective, evaluative and reasoned process.

These elements are closely interconnected and together illuminate how individuals examine claims, justify positions, and evaluate arguments. For this reason, this study adopts the FRISCO framework as its analytical lens for examining students' critical thinking. The FRISCO framework provides a useful basis for analyzing students' argumentative processes in mathematical problem-solving contexts. In particular, the "R" (Reasons) component is especially relevant in the present study because students are required not only to determine whether a given solution is correct or incorrect, but also to justify that judgment with valid reasoning. This emphasis on justification makes FRISCO especially suitable for analyzing the structure of students' critical thinking in algebraic HOTS tasks. The FRISCO framework has recently been cited in studies on critical thinking (CT) in mathematics education contexts (Lukman et al., 2023), further supporting its relevance to the present study.

FRISCO was proposed by Ennis (1996) as a practical set of criteria for guiding critical thinking. It consists of six components: Focus, Reasons, Inference, Situation, Clarity, and Overview. A detailed description of these components is presented in Table 1.

Table 1
FRISCO Components Based on Ennis (1996)
Note. Descriptions reproduced from Ennis (1996, p. 20).

Component	Original Ennis Description	Operationalization in This Study
F	"For focus: identify the focus or central concern"	Identifying the central issue of the problem, recognizing the essential question, and maintaining attention on the key aspects of the task.

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R	"For reason: identify and judge the acceptability of the reasons"	Providing relevant and logically coherent justifications supported by appropriate mathematical evidence or principles.
I	"For inference: judge the quality of the inference, assuming the reasons to be acceptable"	Drawing conclusions that logically follow from the reasons and evaluating whether the conclusion is warranted.
S	"For situation: pay close attention to the situation"	Considering relevant mathematical concepts, assumptions, and relationships when making judgments or solving the problem.
C	"For clarity: check to be sure that the language is clear"	Expressing mathematical ideas, arguments, and procedures clearly, precisely, and unambiguously.
O	"For overview: step back and look at it all as a whole"	Reviewing the entire reasoning process to evaluate the coherence, consistency, and correctness of the solution.

Methods

This study adopted a qualitative case study design to investigate the critical thinking structures of junior high school students with low mathematical ability. A qualitative case study is appropriate when a researcher seeks an in-depth understanding of a bounded phenomenon in its real-world context through the use of multiple sources of evidence and detailed data collection over time (Creswell, 2014; Priya, 2021; Yin, 2018). This design was therefore well suited to examining how students with low mathematical ability demonstrated critical thinking while solving algebraic HOTS tasks.

Research Participants

The participants were purposively selected from Grade 7 junior high school students who scored low on a mathematics ability test and agreed to participate in the study. Ten students were included in order to obtain a focused qualitative picture of the critical thinking structures exhibited by students with low mathematical ability. The participant selection procedure is presented in Table 2.

Table 2

Participant Selection Process

No.	Steps	Description
1.	Determining the research location	The study was conducted in Karawang Regency, Indonesia.
2.	Defining the research population	The population comprised seventh-grade students from a junior high school in Karawang.
3.	Selecting the research sample	The sample consisted of six classes out of the total available classes.
4.	Administering the mathematics ability test	The test was undertaken by 225 students, including 90 male students (40%) and 135 female students (60%).
5.	Ranking students' mathematics ability scores	The test scores were arranged in descending order, from the highest to the lowest.
6.	Selecting the lowest-performing students.	Ten students with the lowest mathematics ability test scores were chosen.

As shown in Table 2, the study was conducted at a junior high school in Karawang Regency, Indonesia and all Grade 7 students at the school constituted the study population. Of the twelve Grade 7

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classes, six were randomly selected. This resulted in a sample of 225 students, comprising 90 males (40%) and 135 females (60%).

The students were then administered a Mathematics Ability Test (MAT). The MAT consisted of 23 multiple-choice items covering two content domains regarded as prerequisites for learning algebra at the junior high school level: (1) ratios and proportional relationships and (2) number systems. The instrument was developed by the researchers, reviewed by mathematics education experts, and pilot-tested prior to its implementation.

Item validity was examined using the point-biserial correlation coefficient. All 23 items met the established validity criterion, with item correlation coefficients exceeding the critical value (*r-table* = 0.2133, *df* = 83, *α* = 0.05). Internal consistency reliability was assessed using the Kuder–Richardson Formula 20 (KR-20), yielding a coefficient of 0.7304, which exceeds the minimum acceptable threshold of 0.60 and indicates satisfactory reliability (Frey, 2018).

The MAT was administered under standardized conditions, with each class supervised by a teacher. Upon completion, all tests were collected, scored, and ranked according to students' total scores. To identify participants for the subsequent qualitative phase, the ten students with the lowest MAT scores were selected. These students obtained scores ranging from 0 to 7, representing the lowest levels of mathematical achievement within the sample. Ten participants were considered sufficient because the analysis reached data saturation, with recurring FRISCO-based critical thinking patterns emerging and no substantially new patterns identified in the final cases.

Data Collection Procedures

Data were collected by the first author, with additional classroom teachers present solely to assist with classroom management during the data collection process. The study was conducted in Indonesian, and all participants completed the tasks individually in a quiet and controlled classroom environment. Data collection commenced with the administration of a Higher-Order Thinking Skills Problem-Solving Task (HPST) to ten voluntarily selected participants. The selected students were temporarily assigned to a designated classroom with prior approval from the school administration.

The HPST was developed by the research team and subjected to content validation by three doctoral-level experts in mathematics education. The overall Content Validity Index (CVI) across the dimensions of sufficiency, clarity, coherence, and relevance ranged from 0.90 to 1.00, while the modified Kappa coefficients ranged from 0.80 to 1.00. These values indicate excellent content validity and strong to almost perfect agreement among the expert validators, consistent with the criteria proposed by Polit and Beck (2006).

The instrument consisted of an open-ended task designed to elicit higher-order thinking by targeting the analysis and evaluation levels of the revised Bloom's taxonomy, which are widely recognized as core components of higher-order cognitive processes (Anderson & Krathwohl, 2001; Azid et al., 2022). Specifically, each task presented students with an incorrect mathematical solution procedure and required them to critically analyze the reasoning, identify errors, and evaluate the validity of the proposed solution. Through this design, the instrument aimed to capture students' critical thinking processes during mathematical problem solving. The HPST used in this study is presented in Figure 1.

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Figure 1
Higher-Order Thinking Skills (HOTS) Problem-Solving Task

Adi merupakan siswa kelas VII di salah satu SMP di Kota Karawang. Pada suatu ulangan matematika, Adi diberi soal $(4a + 3) \times (6a + 5)$.

Proses penyelesaian soal yang dikerjakan oleh Adi adalah sebagai berikut:
Langkah -1 $(4a + 3) \times (6a + 5)$.
Langkah -2 $(4a \times 6a) + (3 \times 5)$.
Langkah -3 $24a + 15$

Periksa langkah demi langkah proses penyelesaian soal yang dilakukan oleh Adi. Benar atau salahkah pekerjaan Adi? Jelaskan mengapa!

Translation:
Adi is a seventh-grade student at a junior high school in Karawang City. In a mathematics test, Adi was given the problem $(4a + 3) \times (6a + 5)$.
The solution process carried out by Adi is as follows:
Step 1: $(4a + 3) \times (6a + 5)$
Step 2: $(4a \times 6a) + (3 \times 5)$
Step 3: $24a + 15$
Examine, step by step, the solution process performed by Adi. Determine whether Adi's work is correct or incorrect, and justify your reasoning.

Before the HPST sheets were distributed, the researcher explained the purposes and procedures of the activity, provided instructions, and gave the students an opportunity to ask questions for clarification. The students completed the task individually using a paper-and-pencil format. The first session involved the administration of HPST 1 for forty minutes. After completion, the students' written responses were collected and documented. A 30-minute break was then provided. In the second session, HPST 2 was administered for forty minutes, after which the written responses were likewise collected and documented. The third session involved the administration of HPST 3, and the resulting written responses were again collected and documented. Each response sheet from HPST 1, HPST 2, and HPST 3 was coded to ensure accurate identification of the data sources.

HPST 2 and HPST 3 were structurally and conceptually equivalent to HPST 1 and differed only in the numerical values used in the problems. These additional tasks served as triangulation instruments to examine the consistency and credibility of the responses produced by the ten focal participants. Triangulation was further supported through interviews. For this purpose, the researcher selected two students. Two students were interviewed because they represented the two dominant critical thinking structures. Although this limited the breadth of perspectives, the interviews strengthened the credibility of the findings by triangulating the written data. Individual interviews were conducted with each selected participant, and all interviews were audio-recorded. The recordings were subsequently transcribed and systematically documented for analysis.

Data Analysis Procedures

Data were analyzed using the FRISCO framework developed by Ennis (1996), which comprises six interrelated components of critical thinking. The analysis involved systematically examining the written responses of the ten focal participants, with interview transcripts used to triangulate the interpretation of the data, according to the following criteria:

1. **Focus**: Whether the student demonstrated a clear understanding of the core problem presented in the task, including accurate identification of the essential question or issue to be resolved.
2. **Reasons**: Whether the student provided explicit justifications and logical reasons for the selected strategies or solution steps, supported by appropriate mathematical principles or evidence.

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3. **Inference**: Whether the student formulated a conclusion or final solution that followed logically from the preceding reasoning process.
4. **Situation**: Whether the student identified and appropriately applied relevant formulas, concepts, or mathematical relationships in a manner consistent with the chosen strategy and the contextual demands of the problem.
5. **Clarity**: Whether the student expressed mathematical terms, concepts, and procedures in a precise and explicit manner, including accurate explanations of terminology and its application (for example, explaining the commutative property and demonstrating its correct use).
6. **Overview**: Whether the student reviewed and verified the correctness of the reasoning process, decisions, conclusions, and final answers in order to ensure logical consistency and mathematical validity.

To ensure the credibility of the coding process, two researchers independently coded the data and subsequently compared their interpretations. Any coding discrepancies were discussed until consensus was achieved through iterative review of the data and coding framework. Trustworthiness was enhanced through analyst triangulation, peer debriefing, and maintaining detailed documentation of coding decisions throughout the analysis. In the final stage of analysis, the findings were presented descriptively, with particular emphasis on identifying patterns, processes, and the structure of the focal students' mathematical argumentation. Based on these analyses, conclusions were drawn regarding the nature and organization of critical thinking among students with low mathematical ability.

Results and Discussions

Students' responses to the question, "Examine step by step the problem-solving process carried out by Adi. Is Adi's work correct or incorrect?" were distributed as follows. A total of 80% of the participants (n = 8) stated that Adi's solution was correct, 10% (n = 1) stated that Adi's answer was incorrect, and 10% (n = 1) did not make any explicit claim.

These findings also indicate that students in the low mathematical ability group experienced difficulty in understanding both the question and the underlying mathematical structure of the problem. The predominance of incorrect evaluations suggests limitations in their ability to critically examine the validity of a solution procedure. At the same time, the fact that one student correctly identified the error indicates that the task remained cognitively accessible to this group and was therefore appropriate for eliciting critical analysis. This interpretation is consistent with prior work on error analysis in mathematics education, which suggests that engaging with incorrect solutions can support students' reflection on mathematical reasoning and justification (Rushton, 2018). This pattern also suggests difficulties in moving from arithmetic thinking to structural algebraic reasoning. Rather than examining the underlying distributive structure of the expression, many students appeared to rely on procedural cues and mechanical rules without a conceptual understanding of why the solution was incorrect.

With regard to justification, three of the ten participants (S223, S222, S220) articulated explicit reasons in support of their responses. Another five participants (S225, S221, S218, S217, S216) provided some form of supporting evidence, although such evidence was generally limited or insufficiently elaborated. Only one student (S219) failed to provide either a justification or supporting evidence, while another student (S224) merely rewrote the problem. Illustrative interview excerpts from participants with low mathematical ability are presented in Table 3.

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Table 3

Illustrative Interview Excerpts from Participants Regarding the Problem

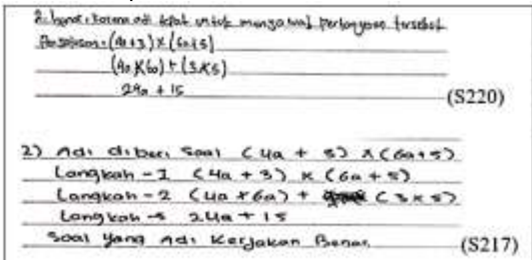
Label	Interview Transcript
S223	2. benar, dikarenakan Adi telah Menyelesaikannya dan menghitungnya. Translation: "Correct, as Adi had already solved the problem and performed the required calculations."
Researcher	In your test results you stated that Adi's problem-solving process was correct. Is that accurate?
S223	Yes, because Adi worked carefully and calculated thoroughly.
Researcher	So you consider it correct because Adi was careful in performing the calculations?
S223	Yes.
Researcher	In your opinion, was the mathematical procedure or rule applied also correct?
S223	Yes.
Researcher	Very well.
S224	-1 (4a+3) × (6a+5). -2 (4a×6a)+(3×5). -24a+15=39 Translation: $(4a+3)\times(6a+5)$ $(4a\times 6a)+(3\times 5)$ $24a+15=39$
Researcher	Could you explain what you wrote on your answer sheet?
S224	[The student remained silent.]
Researcher	You only rewrote the problem, correct?
S224	Yes.
Researcher	So you were not yet able to determine whether Adi's steps were correct or incorrect, is that right?
S224	Yes.
Researcher	Therefore, you did not understand whether the steps or the problem-solving process in the Question were correct or incorrect, and consequently you only rewrote the problem?
S224	Yes.

This study identified two distinct critical thinking structures exhibited by students when solving higher-order thinking skills (HOTS) problems involving the multiplication of algebraic expressions. The first structure is characterized by the absence of focus, invalid reasoning, false inference, invalid situational consideration, lack of clarity, and absence of overview. This structure can be represented as follows: **F_no → R_inv → I_f → S_inv → C_no → O_no**. The response was coded as invalid situational consideration **(S_inv)** because the student failed to recognize the mathematical relationships underlying the multiplication of two binomials and did not apply the distributive property appropriately. It was further coded as lack of clarity **(C_no)** because the explanations provided were vague, merely restated elements of the task, and did not explicitly articulate the mathematical concepts or reasons underlying the judgment.

The second structure is characterized by the presence of focus, absence of articulated reasoning, presence of inference, invalid situational consideration, lack of clarity, and absence of overview. This structure can be represented as follows: **F_with → R_no → I_with → S_inv → C_no → O_no**. Although the student correctly identified that Adi's solution was incorrect, the response was categorized as invalid situational consideration **(S_inv)** because the student did not demonstrate an understanding of the complete algebraic structure of the problem or provide a valid alternative solution procedure. The response was also classified as lack of clarity **(C_no)** because the explanation was incomplete and failed to clearly communicate why the omitted cross-products made the procedure incorrect. The written responses produced by S217 and S220, together with the corresponding critical thinking maps, are presented in Figures 2–5. Figures 2 and 3 illustrate the first structure, while Figures 4 and 5 illustrate the second structure.

Figure 2

Subject 1's Initial Response to the Problem



Translation:

(S220)

Correct, because Adi was able to correctly answer the question.

Solution:

$$(4a + 3) \times (6a + 5)$$
$$(4a \times 6a) + (3 \times 5)$$
$$24a + 15$$

(S217)

Adi was given the problem $(4a + 3) \times (6a + 5)$.

Step 1: $(4a + 3) \times (6a + 5)$

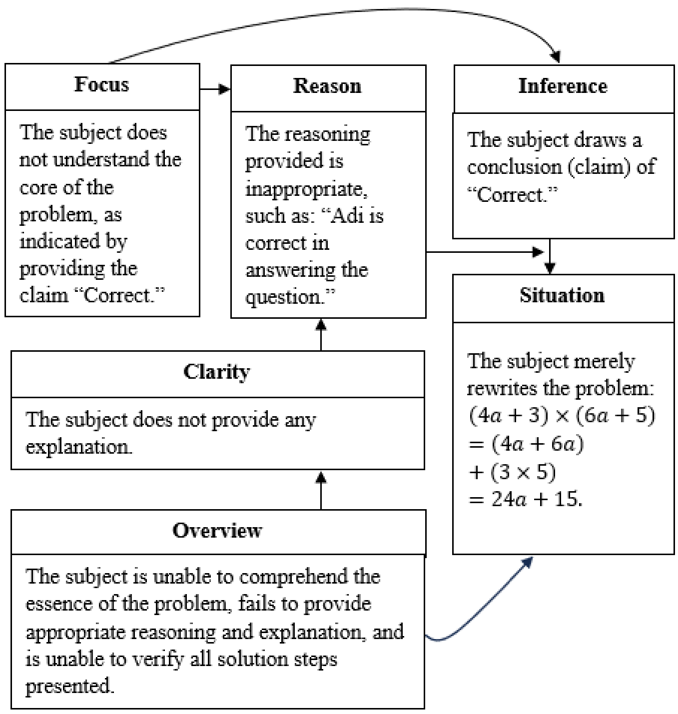
Step 2: $4a \times 6a + 3 \times 5$

Step 3: $24a + 15$

The solution completed by Adi is correct.

Figure 3

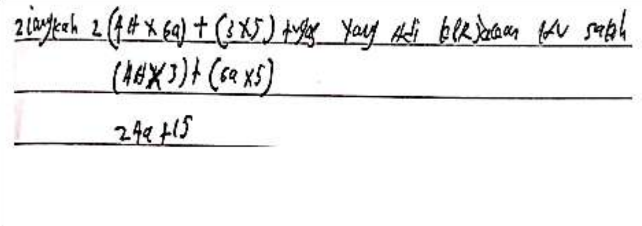
Initial Map of the Subject's Critical Thinking Process Regarding the Problem



As illustrated in Figure 3, the participant failed to grasp the core structure of the problem and consequently concluded that Adi's solution was correct, despite the presence of a clear procedural error. Moreover, the reasoning and explanations provided were vague and insufficiently articulated. The participant largely restated elements of the task without demonstrating conceptual understanding, particularly of the distributive property underlying the multiplication of two binomials, $(a + b)(c + d) = ac + ad + bc + bd$. These findings indicate that the participant was unable to identify the essential elements of the problem, construct coherent and relevant justifications, or critically evaluate the validity of the proposed solution steps.

Figure 4

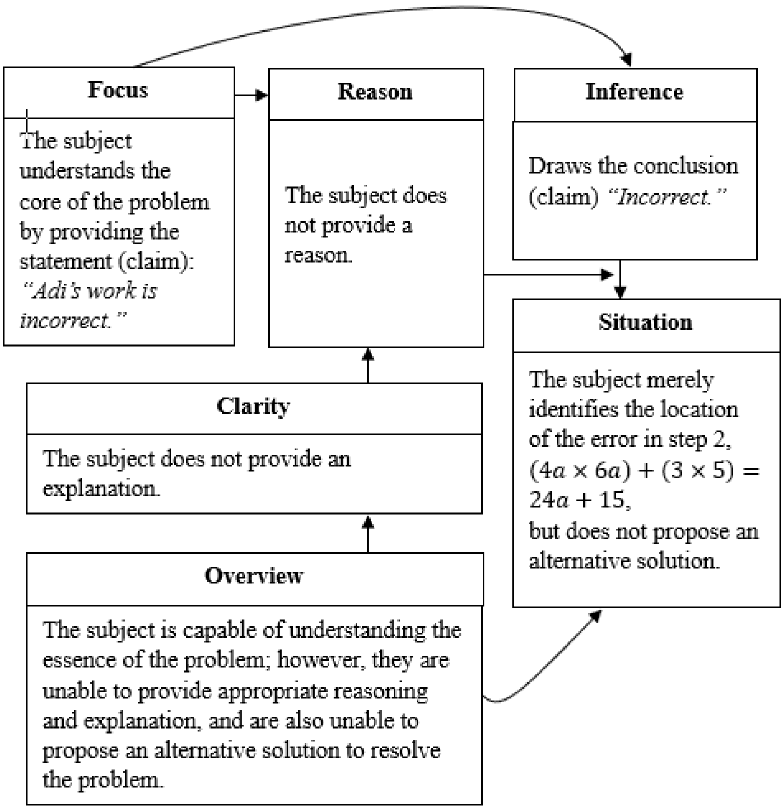
Subject 2's Response to the Problem



Translation:
Step 2:
$(4a + 3) \times (6a + 5)$
$(4a \times 3) + (6a \times 5)$
$24a + 15$
The assignment completed by Adi is incorrect.

Figure 5

Map of Subject 2's Critical Thinking Process Regarding the Problem



As shown in Figure 5, the second participant identified the central issue in the task by explicitly stating that Adi's work was incorrect. Nevertheless, the participant failed to substantiate this claim with a coherent justification. The error was correctly identified in the step yielding 24*a* + 15, where the cross-products were omitted; however, no alternative procedure or corrected solution was proposed. Thus, although the participant was able to recognize the core problem, the participant was unable to articulate an appropriate rationale or reconstruct a valid solution procedure.

Overall, the analysis of students' responses shows that most participants (80%, n = 8) failed to recognize the presence of an erroneous solution step in the task. Only one participant accurately identified the central issue, and even that participant did not provide a complete explanation or a correct reconstruction of the solution process. These findings suggest that the participants' mathematical critical thinking skills were still underdeveloped. The findings of this study are consistent with previous research indicating that students with low academic achievement tend to exhibit weaker mathematical critical thinking skills (Monrat et al., 2022) and that critical thinking is positively associated with academic achievement (Er, 2024). More specifically, the present study extends these findings by showing that low-achieving students not only struggle to produce correct solutions but also experience difficulties in identifying the core problem, providing valid justifications, and constructing coherent inferences within algebraic HOTS tasks. These findings suggest that limitations in mathematical achievement are accompanied by underdeveloped structures of critical reasoning and argumentation. Overall, the results underscore the need for instructional approaches that explicitly support students in developing conceptual understanding, mathematical justification, and reflective evaluation of solution processes, particularly in algebraic problem-solving contexts.

Conclusions

This study shows that seventh-grade students junior high school students with low mathematical ability exhibited underdeveloped critical thinking structures when solving higher-order thinking skills (HOTS) algebra problems. Using the FRISCO framework, two distinct critical thinking structures were identified. Both structures reflect limitations in students' ability to understand the core of the problem, construct sound reasoning or justification, draw accurate inferences, and review their solution processes and outcomes systematically. Even when some students were able to detect procedural errors, they were still unable to provide valid justifications or propose appropriate alternative solutions. These findings suggest that students' conceptual understanding and critical reasoning abilities remain insufficiently developed, highlighting the need for more deliberate and structured instructional support to strengthen mathematical critical thinking. The findings imply that instruction should move beyond procedural practice and provide opportunities for students to analyze errors, justify their reasoning, and evaluate alternative solutions. The use of error-analysis tasks, open-ended HOTS problems, and teacher scaffolding through probing questions may support the development of critical thinking and conceptual understanding among low-achieving students.

This study has several limitations. First, the relatively small number of participants and the exclusive focus on students with low mathematical ability limit the broader generalizability of the findings. Second, the study focused on a single mathematical domain, namely algebra, and on a specific HOTS task format, which may have constrained the range of critical thinking structures observed. Different mathematical domains and task types may elicit different forms of reasoning. Third, the limited number of interview participants restricted the depth of insight into the nuances of individual students' thinking processes.

In light of these limitations, future research should involve a larger and more diverse group of participants representing different levels of mathematical ability in order to provide a more comprehensive picture of students' critical thinking structures. Future studies are also encouraged to examine a wide range of HOTS problem types across algebra and other mathematical domains and to design or implement instructional models specifically intended to strengthen the FRISCO components in students' mathematical argumentation. In addition, longitudinal research could provide valuable insight into how students' critical thinking structures develop over time, thereby informing the design of pedagogical interventions that are more targeted, sustainable, and developmentally responsive.

Declaration of Conflicting Interests

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