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# Robust Queuing Systems Using Linear Programming to Accelerate The Queue of Patients (Applying to Marjan Hospital as a Sample)

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***Abstract***. *In this paper, we study the use of linear programming for the performance enhancement of fortified queues systems in which resources are unreliable in terms of variation and a case study on Marjan Hospital is considered. The quality management maturity level in the hospital was measured by an integrated inspection tool; senior management was found to be concerned for quality of service improvement and implementation of modern technology systems. Strengths and weaknesses in that activity are shown, and it is evidenced the need of boosting the quality of service activities and promoting the coordination between the different areas. From the findings, the study recommends the use of linear programming technique,Orgers should as well enhance proper training and development and Acquisition of modern day technology for effective performance improvement and quality of service. The study demonstrates that the adoption of linear programming techniques could be a viable solution to optimize resource allocation and enhance performance in the hospital's queue systems. Additionally, the research underscores the importance of investing in staff training and development programs, as well as the acquisition of state-of-the-art technology, to further improve service delivery and ensure sustainable quality improvement in the long term.*

***Keywords:*** *Alp, Basdogan [2] Queues Performance, Linear Programming, Robust Models Analysis For Robustness, Waiting Lines.*

## 1. INTRODUCTION

Queues are important in various applications e.g. customer services, communication and transport. These platforms are instrumental in service optimization for faster service response times combined with improved service delivery, even resulting in a satisfied customer experience leading to higher productivity. Nevertheless, the services offered by these systems are subject to multiple factors such as resource availability and demand variations.

This research attempts to use optimization to strengthen the performance of queuing systems, with a focus on accommodating variations in the resource availability. Linear programming is a very effective method with which to make decisions — we can use it to model complicated problems and figure out the best solutions given certain constraints. By means of linear programming we can study how resources can be sharing efficiently between increasing demand side and limited supply side.

In this study the challenges of queuing models when resources fluctuate, e.g. due to the lack of staff or changes in utilized technologies, will be faced. It will also analyze possible practical implications by applying linear programming models in order to improve the

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performance of these systems enhancing better outcomes, avoiding congestion and increasing customer satisfaction.

Through the investigation of these issues, this research aims to establish new knowledge which can support design and operation of queuing systems to deliver value, to improve business effectiveness and to generate good economic and or social outcomes.

**The Importance of the Study**

Line or queuing system is one of the most significant components of service operations. Its enhancements are aimed at minimising wait times, enhancing customer experience and maximising resource. Taking into account that the demand is varying all the time and resources have to be used and adjusted continuously it is essential to formally analyze and optimize such systems by means of mathematical methods (like linear programming). Knowledge generated in the research literature could help in finding ways for a better balance between demand and available resources to support organisations in the provision of high-quality services.

**Objectives of the study**

1. Analyze the design of the current systems as well as their effect on performance. Formulate mathematical models to facilitate the optimal allocation of resources in queuing systems.
2. Recommend effective solutions to optimize performance in the presence of resource variability.
3. Assess the effect of the proposed changes on waiting time and customers' satisfaction.

**Study Problem**

The crucial question is how to make queuing systems work better under the change of resource constraints. Variations in the availability of resources, including the shortage of labor or the fluctuation in demand, lead to longer queues and consequently lower consumer satisfaction levels. Thus, there is an imperative for developing effective strategies to enhance the performance and improve the quality of services.

**Study Hypothesis**

The research hypothesis is that using LP techniques for resource allocation in the context of queueing systems can help enhancing providers' performances in terms of minimizing waiting times and maximizing customer satisfaction in dynamic environments. What this means is that linear programming can optimize process efficiency, provide an agility in strategies that match demand, and much more.

**Research Methodology**

Data will be gathered and analyzed using a quantitative methodology. A mathematical modeling will be performed through linear programming and a statistical analysis will be made to interpret the results.

**Search limits**

**Study setting:** Study conducted in the waiting systems at Murjan Teaching Hospital

**Data constraints:** Historical data may not be readily available or accurate.

## 2. THEORETICAL ASPECT

**Linear Programming:**

Linear programming is a field of applied mathematics, which is directed at enabling better decision making in the presence of some constraints. A linear program is effective to resolve issues subject to the optimal allocation of certain allocations of resources, be it labor, money, materials, or time. (Schiltz, 2003, 38).

The idea of linear programming. Concept of Linear Programming L Linear programming can be expressed as - A mathematical model to distribute a given finite resources and limited activities to a number of competing demands (needs) for these resources among the optimal balanced point this is the point where distribution brings out best result i.e. the distribution of these resources (of any type) is most effective. Linear programming models are known as the most elementary mathematical models, that can be developed to resolve industrial and governmental problems, facilities and productive type of organizations and companies etc. (Kumar& Suresh,2008,132)

Thus generally the model is such a part which trasports the (the specifications) of whole, every that who is positive (have positive value) in model then in all and who is negative in model then in all and it may often have the form of a mathematical equation The working of specifications of a particular case through such a number of mathematical equations as represent the fact, or condition or the relationships to be studyed were understood in some way. In general, the purpose of constructing the model is to describe, analyze and understand the problem in a simplified manner. For the realisation original plan made a designer all aforesaid differences must know all number qualitative and quantitative advantages of the problem, remembering, by the way, on the very important not to forget the important phenomena and variable of the problem for the sake of simplification. The more the model represents the reality the more exact will be the analysis and forecast and vice versa. From the above, we can reduce the concept of the model as the composition or the concrete expression that is intended to represent reality to clear one of its appearances as it works (and generally the model is simpler than reality, but should be sufficiently complete as to be able to achieve the appearances of the problem to be solved).

For understanding the dynamics and types of mathematical models, their classifications and the division of the, the justifications for construction and constitution of models in general, and mathematical models in particular, will be discussed as follows: (Heizer & Render, 2008, 194)

The problem at issue being difficult to shift from place to place.

The problem of specifying, and then solving, the problem being discussed.

The large expense associated with the stating of the problem.

Hazards as a result of the issue in question.

The problem under discussion not being discussable directly,

Going by the summary the defined mathematical model could be as (a) a system of interrelated (interdependent) variables and factors which describe a specific issue (problem) and are interconnected with each other by means of certain mathematical relations (equations or inequalities of specific forms for the purpose of formulating the essence of the problem posed and specification of its internal and external variables), where. The following sections from mathematical models and operations research a subject by itself are indicated in the opinions and ideas in which are placed the QUANTITATIVE METHODS and the OPERATIONS RESEARCH: (Al-Bakri, 2005, 167).

**Literature Review**

Introduce a new way to optimize queuing models by casting them in a reinforcement learning setup. Their work motivates the need for learning optimal policies in an efficient way which relies on new parameterization taking into account the inherent properties of queuing network (QN) form. The reported experiments demonstrate the efficiency of their approaches in different traffic conditions: the traffic flows can be characterized by different densities, from light to heavy, which shows the practical utility of their research in real applications (e.g., communication networks and manufacturing systems) (H. Tran et al.,2022).

Continuing the tradition of previous works (Bogachev, 2022) meets a strong demand for successful applications of queuing theory in modern computer science, namely those aimed at reducing network congestion and delay. By concentrating on the optimization of processor sharing, his tales go far beyond usual queueing at the nodes of a network, as they allow for a single processor to simultaneously deal with several tasks. This work demonstrates the complex nature of the problem with variable arrival intensities and paves the way for further research on optimization problems under this framework.

Contribute to debates by critically reviewing integrated planning approaches within healthcare institutions for which the need to allocate resources is crucial. By an in-depth review of Operations Research and Management Science literature, it is evident that significant efficiency increases can be achieved through pooling of resources such as operating room and personnel. In doing so through pinpoint study gaps and future research, they offer insightful forward-looking perspective on how integrated methodologies can assist in improving the performance of queuing systems in health care settings (Rachuba et al.

The review of (K et al., 2023) on queueing-inventory systems presents a general survey of the latest developments in this composite discipline. They examine the queues over a wide range of topics from 2016–2023 and consider a variety of subjects like product-form solutions, and multi-commodity systems which demonstrate both the complexity and importance of queueing-inventory interactions. The observations in this survey reveal not only existing problems but also possible future research directions, towards an even more efficient framework for queueing in many applications.

First: systematic type classification in mathematical logic.

Second: classification of mathematical models.

Third: Sub-classification of mathematical models.

First: Uniform handling of mathematical models.

Model classes under this category are the following:

1. **Specific mathematical models**

They are models that are made up of 5– known and transparent elements and variables for the decision maker, or are geographically sprayed away from the ambiguous in- ternal and external virtual randomness that influences the structure of the investigated issue, and therefore in the formulation of its ma- thematical mathematical model, including linear programming models, transport models, transportation models and others models.

2. **Mathematical probabilistic models**

They refer to those mathematical models which contain in m input factors and probabilistic parameters, in relative the problem, which is inherently vague and is an outcome of a large number of external and internat forces, such as the models presented in the Quening Theory Models and the models developed in the Inventory Control Models etc. (Kotler and Armstrong, 2007, 502-506).

**3. Strategic models**

They are the models derived by the decision maker as a function of the location of the other competing decision maker which is operating in the same environment and the same field as the decision maker in question and the referred to location is called (strategy). It is worth noting that the judgement from this case is in favor of the first decision maker given the decision of the second decision-maker. If the competition is between only two decision-makers, then the strategic models become very simple, and if the competition is between many decision-makers, then it becomes remarkably more complex. of rivals such as mathematical game theory models (games) and etc.

**4. Mathematical model of statistical and accounting nature**

This kind of mathematical models represent models that have fixed and known uses described by simplicity and linearity and these statistical models are the models used in the arithmetic mean, correlation regression and other statistical methods. As for the accounting models, they are the models used in knowing the simple interest, complex interest, extinction premiums and others.

*Second: a primary classification of mathematical models

According to this classification, the mathematical models are divided to the following: *

1- Standard mathematical models –

These are mathematical structured models aiming to describe what the reality should be according to a form of a carefully and well-prepared mathematical equation.

2- Descriptive mathematical models –

Those models that describe the facts and relationships that connects among the components of the studied reality in a way that reflects what the reality of the situation is. You can notice that all the problems illustrated above can be solved through linear programming.

**Building-Formulation of Linear Programming Model**

Regarding the importance of linear programming, that's related to the importance of the issues that it can be solved generally speaking! As not every problem can be solved by linear programming where it requires the problem, concerning the linear tower method, to meet the conditions related to it and these are *:

Identify the objective function + the aim is what we need to accomplish ideally knowing how to express it in the form of a linear function and give him a value asshole and try to get this value to fire in the balance and find its utmost end to reach a maximum point if the goal is the profit or we shoot it and find the least end to reach a minimum end and a minimum point in case the target works Some produce for the least possible cost. The Objective Function The objective function are the variables and either the coefficient is how much profit one made if you are maximizing the objective function or it is the cost of one unit if you are minimizing the objective function.

Defining Constraints: Perhaps it is possible to describe the relationship between decision variables and available alternatives as linear constraints, that is, as linear inequality, or linear equations, or a combination of them and named structural constraints that shows how much of each required resource of limited resources each production unit needs in the form of linear constraints. 3 Negativity Non Conditions: The decision variables of the problem under consideration shall be all positive, zero and non-negative variables.

An industrial company, for instance, manufactures these kinds of products (1, 2, 3) and seeks to establish the number of units each product, has to be produced per day in order for you to obtain the maximum (the maximum possible profit. In order to produce one of each Product, these must go through three production processes (C, B, A). The following table gives the time taken (in minutes per unit) for each Product of each of the processes, and the profit per unit, and the total time available for the three operations. Model development A linear programming model that will earn the maximum profit with the provided information in the table above is given as follows: (Kotler and Armstrong, 2007, 513).

**First: Decision variables**

A certain amount of products of the three types must be produced, if the production of a given product is possible during the first, second and third operation, alternatively with the maximum profit within the time of the given operation.

If X is the number of items of a product to be produced, let's say 1.

Let us assume that quantity of units to be produced from product 2 = X2 Forever.

Assume that the product has the number of unit produced of 3 = X3.

It is not logical to have a negative number of each of the units X1 X2 X3 that will be produced to be produced in amount from 0 or not to be produced at all, that is, it has zero values representing the condition of non -negativity, ie X1, X2, X3 $\geq 0$

**Second: Restrictions:**

The first limitation (to the first operation The maximum time that can be allocated for the operation is 420 minutes per day (but not required for using all of days' available time.

This is because one unit of Product 1 requires time 2 to build the process 1, and one unit of Product 2 requires 2 minutes, one unit of Product 3 requires 3 minutes. Therefore, the first item may be expressed in the 118 The Source of Bliss (Al-Qaisi, 2008, 113-114)

$$2X1 + 2\,X2 + 3\,X3 \leq 420$$

Second restriction where the second process: The second process has a maximum time of 440 minutes per day, and a unit of the first product needs production in minutes in for the second process 5, and you need for one product in the second product does not need the second process (0), and you need to produce one product in the second process 4 minutes so the seconds restriction would be:

$$5X1 + 4X3 \leq 440$$

The third limitation (with reference to the third process-that the maximum time available in the second process for the first process is 456 minutes per day, one unit of the first product is manufactured in 3 minutes in the third process, 6 minutes to one unit of the second product, one unit of the third product does not produce the third process (0). So, the third constraint will be:

$$3X1+6X2 \leq 465$$

3: Objective function: The objective of the decision-maker of this problem is to Minimize the cost and the objective function is as follows:

Min Z = 0.04X1 +0.15X2 +0.40X3 Where Z = total cost of the system 0.04, 0.15 and 0.40 are the costs of X1, X2 and X3 respectively.

## 3. PRACTICAL SIDE

The use of linear programming to enhance the performance of fortified queuing systems under the situation of varying server and system resources at the research level

This topic seeks to employ the linear programming in the optimization of performance of fortified queue structure in Marjan Hospital. We shall consider examining data on waiting times, patient flow and resource and variable capacity constraints (e.g. through variation in numbers of available staff and equipment).

With the construction of a math potent as parser which is based on the linear programming, best strategies can be defined such to minimize the waiting time and to maximize the efficiency of the implemented service. This will involve dissecting various cases and their effect on the total performance of the system.

This includes gathering data from a range of sources including patient information and interviews with hospital staff, which will help to identify current difficulties and provide insight into potential solutions. Lastly, suggestions out of the model from findings will suggest the improvements for service quality and patient satisfaction in the hospital.

First: Implement Murjan Hospital linear programming list actions.

This theme is concerned with the role of linear programming techniques in Marjan Hospital to optimize quality and productivity. We will use the linear programming model to analyze various systems of operations in the hospital, including queue of patients, allocation of resources, and scheduling of the staff.

Information will be provided regarding waiting times, patient throughput and hospital resources utilised in the course of effecting the application. A model describing the interaction between these variables will be constructed and goals formulated such as a shortened wait time and improved patient satisfaction.

The application will allow the user to analyze alternative scenarios and see their influence on overall hospital performance. Conclusions Recommendation at the conclusion of this study, the following recommendations will be based on the findings obtained from the linear programming model and contribute to enhanced services quality and operational efficiency at Marjan Hospital, and Results concluded as follows:

The second level: organizational management.

Level 2 contains: 29 question mapped across six processes from the maturity levels categorized as: Quality of Service Policy, Requirements Management, Process Planning, Supplier Management, Quality of Service Assurance, and Quality of Service Dimensions.

From the contents of Table (2), the study can be having (21) questions that the respondent has issued a yes answer indicating that he is carrying out this activity in the process, as (8) questions has been answered by no answer indicating that he does not carry out this activity.

Finally, the outcomes reveal that quality policy is established at the senior management level and is spread across employees. It further specifies necessities for the customers and the desired outcomes develops plans referring to workactics for the operations. Nevertheless, although tools such as DSR can measure suppliers and determine what are the dimensions of quality of service, they are unable to document such dimensions.

all have got nothing as management has nothing to do with service quality assurance activity, as reflected in the questions asked or the area also include on any of such activities implemented as seen on quality assurance of services has not implemented any of such activities.

Table (1): Stage Framework for Level II (Organizational Management)

Third level: Quality Management System.

Level 1 This course is devoted to introduction of quality management system as per ISO 9001 and improvement in coordination among employees and work teams. Because Murjan

Hospital has not yet applied for ISO 9001, only three positive-response questions regarding teamwork among staff and teams were included in Table 2.

Table (2): Completion checklist for Level III (Quality Management System)

3-The fourth: the management of the quality improvement.

The forth level (13) is composed of questions concerning the three processes present in the maturity levels: quantitative process management, feedback loops, and benchmarking. As displayed in Table 3, there are three yes questions for feedback loop activities. The ten items about all process management and benchmarking practice were no.

Table (3): The maturity indicators for level IV (quality improvement management)

4- Level number five: managing for continuous improvement.

The fifth level (12) consists of questions in two processes: process improvement management and service process and, technology change management presented in maturity levels. From table (5), it is evident that the hospital management tends to pay more attention to enhancement of the service quality dimensions, and enhance the operations as well as the activities that concern technology management.

As the hospital management is unique in this system it continually looking for upgrading in the services offered and to coordination with the technological developments preparing the appropriate equipments and apparatus for the operations and the services matters. Eight therefore yes or noanswered in the affirmative, and four no.

Table (4): Level V Maturity Checklist Continuous Improvement Management(Level V) Note: After achieving Level V, further improvement is not possible Product Reengineering Systems-Version Control Process Engineering Quality Peer Objective Process product improvement library Defect Removal Training library Process management PSMs Quality management Roadmap for improvement Peer & management review Development Continues rallies Rework detection - elimination BDMS Process certification mutated Training certification mutated Peer - 127 -.

To evaluate levels of maturity in quality management in Marjan Hospital, maturity levels will be determined as follows: First maturity level in proposed method. Illustration of this approach can be represented by a criteria - table, containing the criteria of the assessment of performance and factors of linear programming, which contribute to the optimization of the queues and quality of the provided services. (5)

Table (5): List and Improvement of classroom systems in fix position marjan hospital and linear programming results

Table 1 shows no questions at level 1 (purpose-directed management), because the operations are performed purposefully. If the responses to the question being asked are both 0, the cheek-tumperasting organization will continue to stay at the first level (that is, the output (1)). For all other levels, the number of questions and the number of yes answers corresponding to each level is calculated.

The table also indicates the highest percentage after the first level with point (1) in the second level (organizational management) which was (0.724). This states that the operation activities are performed, excepting the quality-of-service assurances process. The fifth level (continuous improvement management) follows it that the implementation of most of the activities the service improvement process management of technological change has done. Marjan General Hospital is managed, and keen supervision is given to enhance and upgrade the service to a latest state of the art devices and equipment.

The third level that followed reached a percentage of (0.429) (Quality Management System) and was the result of a few coordination activities between employees and work teams. Finally, Feedback loop activities were implemented only at the fourth level (Quality Improvement Department) and thus obtained a score of (0.231).

Therefore, the QM maturity level of Marjan Hospital is represented by the sum of the weights of each of the five maturity levels as follows.

Linear programming level = 1+0.724+0.311+0.072+0.048=2.155.

Quality management maturity level in Marjan Hospital The quality management maturity is located at the first level of the starting point of the second level. MAIN AND MODULATING CRITERIA BASED ON QUALITY MANAGEMENT MATURITY 1. The hospital management team executes the second level operations (organisational operation) but not the service quality assurance process operations. Accordingly, the hospital management need to focus on the service quality assurance process and the ISO 9001:2008 quality management system implementation. By doing this, you are enabling yourself to achieve the relevant level three and subsequently better the Operations practices towards level five, a high level of maturity.

## 4. RESULTS & DISCUSSIONS

The authorities of Marjan Hospital should use linear programming techniques in order to model and allocate its resources more effectively and to enhance the level of service provided to its patients for queuing in varying source with a varying service rate.

QoS assurance process activities should be reinforced with clear task description and with training of the staff on the processes. The boss has to put his / her money where his / her mouth is, and invest in those things that will allow us our people to have a great quality of service and operational efficiency (yes, even something like a classroom control system).

"Continuous training and development for our workers in quality management and application of modern technology is really crucial.

Furthermore, collaboration and coordination among various teams should be enhanced to increase overall performance and thus facilitate high-quality healthcare services. Quality maturity level should be evaluated at regular intervals through performance indicators, to foster process improvement.

Higher degree of quality management maturity if the hospital can adopt and sustain the ISO 9001:2008 Quality management system performance is training, respectively.

## 5. CONCLUSIONS

Using linear programming techniques as described, the operation of Marjan Hospitalâ€™s queue system can be improved such that the service delivery is more efficient with shorter waiting times.

[It would seem the maturity level of quality management in the hospital needs to be] developed specifically in certain aspects, particularly within the system of quality of service assurance. Process performance should be enhanced using quantitative methods (eg, linear programming).

The findings highlight the need for applying contemporary technology for upgrading service through investment on sophisticated machines and systems to increase the job performance level.

Lack of Coordination Having a weak coordination among the teams would hamper the goal attainment. Better coordination lead to better service.

Feedback loops turn out to be useful in the vulnerability analysis of a plan, and can derive wider application towards the performance improvement of queuing systems.

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