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Development of a SEIR Model with Two Optimal Controls for Infectious Disease Control: Computational Simulation Using MATLAB

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ABSTRACT

This study develops a Susceptible–Exposed–Infectious–Recovered (SEIR) mathematical model with two time-dependent optimal control strategies educational campaigns and medical treatment to analyze Ebola transmission. The nonlinear system is examined using Pontryagin's Minimum Principle, and the optimality system is solved numerically through the Forward–Backward Sweep Method in MATLAB. Simulation results show that combined controls substantially reduce the peak number of infected individuals, decrease cumulative infection cases, and shorten the epidemic duration compared to the uncontrolled scenario. Early implementation of both interventions further improves epidemic suppression and reduces the overall cost functional value. These results demonstrate the effectiveness of integrated control strategies and highlight mathematical modeling as a decision-support tool in public health planning.

1. Introduction

Infectious diseases pose one of the greatest threats to global public health, particularly those caused by deadly viruses like Ebola. The Ebola virus first emerged in 1976 and has caused several major outbreaks with high mortality rates, in some cases reaching up to 90% (World Health Organization, 2018)(World Health Organization, 2018). Transmission of the virus occurs primarily through direct contact with the body fluids of infected individuals, making it particularly risky in areas with high population density and inadequate healthcare systems. In

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the context of globalization and high human mobility, the risk of this disease spreading is not limited to Africa but also has the potential to reach other countries, including Indonesia (Sariadji & Kambang, 2022)(K. Sariadji, 2022).

Various scientific approaches have been used to study the spread of infectious diseases, one of which is through mathematical modeling. The SEIR (Susceptible–Exposed–Infected–Recovered) model is a basic and widely used model to describe the dynamics of infectious disease spread (Li et al., 2015)(Li et al., 2015). This model divides the population into four compartments based on infection status and maps the transitions between these compartments over time. It is considered highly representative for modeling diseases like Ebola, which features a notable incubation period and high infection rate. In addition, several studies have shown that the SEIR model can capture local dynamics of transmission when applied to real epidemic scenarios, such as in the case of Ebola in Uganda (Chowell et al., 2004)(Chowell et al., 2004).

To strengthen the model's effectiveness and support real-world decision making, many researchers have adopted optimal control theory**.** The Pontryagin Minimum Principle (PMP) is one of the most widely used mathematical approaches to determine control strategies that can minimize infections and associated costs (Lenhart & Workman, 2007)(Lenhart & Workman, 2007). In epidemiological contexts, optimal control strategies often include public education campaigns aimed at reducing risky behaviors and treatment interventions to accelerate recovery in infected individuals (Funk et al., 2010; Okosun et al., 2013)(Funk et al., 2009)(Okosun et al., 2011). The combination of these two controls has been proven to provide a synergistic effect in suppressing the transmission of infectious diseases (Rachah & Torres, 2015)(Rachah & Torres, 2015).

Solving such complex models analytically is often infeasible due to the nonlinear nature of the equations involved. Hence**,** numerical methods are necessary to find approximate solutions. One of the widely used algorithms for solving optimal control problems is the Forward-Backward Sweep Method (Polak, 1997)(Polak, 1997). For implementation, MATLAB is commonly chosen because of its capability to handle complex differential systems and to produce high-quality visual simulations (Higham & Higham, 2016)(Higham & Higham, 2016).

This study builds upon and extends the SEIR optimal control framework previously introduced by Sumardi (2022)(Sumardi, 2022), which formulated an SEIR model incorporating two time-dependent control variables: public education and medical treatment. While the earlier study primarily focused on analytical derivation and theoretical optimality conditions, it did not provide an in-depth computational investigation of the dynamic behavior of the controlled system.

In response to this gap, the present study repositions the research by emphasizing a computational and quantitative analysis of the dual-control SEIR model. Specifically, this work integrates numerical simulations using MATLAB to systematically examine how the simultaneous implementation of public education and medical treatment influences the temporal evolution of each compartment (Susceptible, Exposed, Infected, and Recovered). Through this approach, the study provides a clearer evaluation of the effectiveness, interaction, and relative impact of the two control strategies in suppressing disease transmission. Therefore, the main contribution of this research lies in bridging the theoretical optimal control formulation with practical computational validation, offering deeper insight into policy-relevant disease mitigation strategies.

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Through this model, the dynamics of the Ebola outbreak can be visualized more realistically and measured in response to different intervention strategies. Therefore, this research contributes not only theoretically to mathematical epidemiology, but also practically to public health planning by providing insight into effective and cost-efficient control interventions that are applicable in real epidemic settings.

In accordance with academic writing conventions, this introduction clearly states the urgency of the study, the theoretical framework used, and the basis for the research. A preliminary literature gap is identified between analytical and simulation-based studies of SEIR optimal control. Therefore, the novelty of this study lies in its implementation of dual optimal controls within a realistic numerical framework, intended to support data-driven health policy.

2. Method

The subject of this study is a mathematical model of Ebola virus spread based on the SEIR (Susceptible, Exposed, Infected, Recovered) approach. This model is a system of nonlinear differential equations that represents the transition of individuals in a population through several health compartments, influenced by two types of optimal interventions or controls: education control for susceptible individuals and treatment control for infected individuals.

Data collection techniques were conducted through literature studies of relevant scientific publications, both national and international, that examine SEIR-based infectious disease modeling and optimal control theory. Parameter values such as infection rate, transition rate between compartments, recovery rate, and mortality rate were adopted from previous journals and references from official health organizations such as the WHO. Parameters were calibrated to match the realistic biological dynamics of Ebola in the simulation.

The initial step in data analysis is to construct a system of differential equations based on the modified SEIR model. The basic model used is as follows:

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \beta_1(1-u_1(t))SI - \beta_2(1-u_1(t))SE - \mu S + \xi R \\
\frac{dE}{dt} &= \beta_1(1-u_1(t))SI + \beta_2(1-u_1(t))SE - (\sigma+\mu)E \\
\frac{dI}{dt} &= \sigma E - (\gamma + \delta + \mu + u_2(t))I \\
\frac{dR}{dt} &= \gamma I - (\mu + \xi)R + u_2(t)I
\end{aligned} \tag{1}$$

Description:

S, E, I, R	: Number of susceptible, exposed, infected, and recovered populations
Λ	: Population entry rate
β_1, β_2	: Transmission rates from I and E
σ	: Transition rate from E to I
γ	: Recovery rate
δ	: Death rate due to disease

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μ	: Natural death rate
ξ	: Immunity loss rate
$u_1(t) \in [0,1]$	: Level of education, reduces the effectiveness of contact (infection).
$u_2(t) \in [0,1]$	: Treatment rate, accelerates recovery from I to R.

The equation 1, $u_1(t) \in [0,1]$ represents the time-dependent intensity of public education efforts aimed at reducing effective contact rates. The control variable is defined within the normalized interval [0,1] to reflect proportional implementation, where $u_1(t) = 0$ indicates the absence of educational intervention and $u_1(t) = 1$ represents the maximum feasible level of implementation under practical and resource constraints.

From a modeling perspective, restricting $u_1(t)$ to [0,1] ensures mathematical consistency and boundedness of the control, preventing unrealistic negative efforts or intervention levels exceeding full capacity. Epidemiologically, this normalization allows the control to be interpreted as the fraction or effectiveness of behavioral change induced by education (e.g., mask usage, hygiene practices, reduced physical contact), which proportionally decreases the transmission rate. Thus, the interval [0,1] provides both theoretical tractability in optimal control analysis and a meaningful practical interpretation in public health policy implementation.

Next, Pontryagin's Minimum Principle is applied to obtain optimal control $u_1(t)$ and $u_2(t)$ which minimize the number of exposed and infected individuals while also minimizing the associated intervention costs. The objective functional is defined as:

$$\boldsymbol{J(u_1, u_2) = \int_0^T [AI(t)^2 + B_1 u_1(t)^2 + B_2 u_2(t)^2] dt}$$

Description:

A	: weight associated with infected individuals,
B_1	: weight of public education cost,
B_2	: weight of treatment cost.

To enhance clarity, the computational procedure of the proposed optimal control strategy is encapsulated in the subsequent algorithm. Algorithm: Forward–Backward Sweep Method for Dual-Control SEIR Model.

Step 1: Initialization

1. Set parameter values and initial conditions for the SEIR system.
2. Choose final time $T$and discretize the interval $[0, T]$.
3. Initialize control functions $u_1^{(0)}(t)$and $u_2^{(0)}(t)$(e.g., constant initial guesses within $[0, 1]$).
4. Set convergence tolerance ε.

Step 2: Forward Sweep (State System)

5. Solve the state equations (SEIR model with controls) forward in time using the current controls $u_1^{(k)}(t)$, $u_2^{(k)}(t)$.
6. Obtain numerical solutions for $S(t), E(t), I(t), R(t)$.

Step 3: Backward Sweep (Co-state System)

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7. Using the transversality conditions $\lambda_i(T) = 0$, solve the adjoint (co-state) equations backward in time.
8. Obtain $\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t)$.

Step 4: Control Update

9. Update controls using the characterization derived from Pontryagin's Minimum Principle:

$$u_1^{(k+1)}(t) = \min\left\{1, \max\left\{0, \frac{\text{expression involving states and adjoints}}{2B_1}\right\}\right\}$$
$$u_2^{(k+1)}(t) = \min\left\{1, \max\left\{0, \frac{\text{expression involving states and adjoints}}{2B_2}\right\}\right\}$$

10. Ensure the updated controls remain within the admissible interval [0,1].

Step 5: Convergence Check

11. Compute the difference between successive controls:

$$\| u^{(k+1)} - u^{(k)} \| < \varepsilon$$

12. If convergence is satisfied, stop. Otherwise, set $k = k + 1$and repeat Steps 2–5.

Step 6: Evaluation of Effectiveness

13. Compare uncontrolled and controlled scenarios by analyzing:
    a. Peak and total number of infected individuals,
    b. Time to epidemic peak,
    c. Value of the objective functional $J(u_1, u_2)$.

All numerical simulations are implemented in MATLAB (version 2016a) using the Forward–Backward Sweep Method.

3. Results and Discussion

The simulation results clearly illustrate the effectiveness of implementing dual optimal controls—public education and treatment interventions—in minimizing the spread of the Ebola virus within the $SEIR$ model framework. In this study, we developed a SEIR model integrated with two optimal controls, namely u_1(education) dan u_2 (treatment) to control the spread of infectious diseases. Control u_1 is designed to reduce new infections by decreasing the transition rate from the Susceptible (S) to Exposed (E) compartment, representing preventive efforts such as education or contact restrictions.

Meanwhile, control u_2 aims to accelerate recovery by increasing the transition rate from the Infected (I) to Recovered (R) compartment, which can be interpreted as the effectiveness of treatment or clinical intervention. The results of computational simulations using MATLAB are presented in Figures 1, 2, and 3 to illustrate the impact of these optimal controls on population dynamics.

Figure 1 displays the population dynamics of exposed individuals (E) as a function of time (in months) between the no-control (dashed line) and optimal-control (solid line) scenarios. The vertical scale in this graph is in the order of 10^{-3}, indicating that the proportion of the exposed population is very small relative to the total population. In the no-control scenario, the exposed population (E) exhibits progressive exponential growth, reaching a value of approximately 5.5×10^{-3} at the end of the simulation period.

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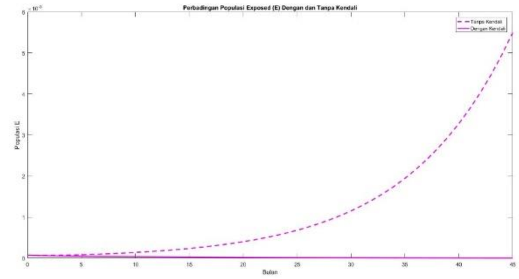

Figure 1. Comparison of Exposed Population (E) with and without Optimal Control

This growth indicates that without intervention, the number of individuals potentially becoming ill will continue to increase. In contrast, in the optimal-control scenario, the exposed population (E) is drastically suppressed to near zero throughout the entire simulation period. This nearly flat curve confirms the high effectiveness of the u_1 control in reducing the transition from S to E, indicating the success of the prevention strategy in suppressing the number of exposed individuals early in the transmission chain.

The nearly flat curve approaching zero indicates that outreach control has been significantly successful in reducing the probability of individuals moving from the vulnerable to the exposed group. This means that public education about the dangers of physical contact, as well as increasing awareness of health protocols, plays a key role in reducing the initial rate of disease spread.

Figure 1 ilustrates the significant reduction in the exposed population under optimal control emphasizes the role of educational campaigns in limiting new exposures. This aligns with findings from Funk et al. (2010)(Funk et al., 2009) and Seck et al. (2022)(Seck et al., 2021), who emphasized that public awareness and behavior change reduce contact rates and transmission pathways. The flatness of the exposure curve in our study suggests a near-elimination of early transmission chains, a result also observed in

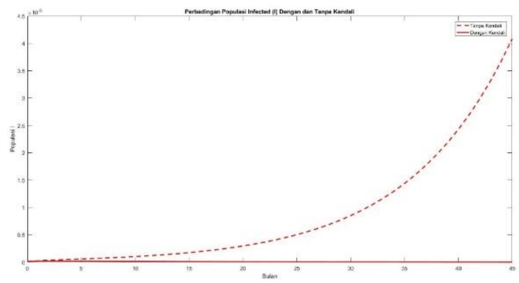

Figure 2. Comparison of Infected Population (I) with and without Optimal Control

Figure 2 shows the population dynamics of infected individuals (I) as a function of time, comparing the uncontrolled (dotted line) and optimally controlled (solid line) conditions. Similar to the exposed population, the vertical scale is in the order of 10^{-3}. In the uncontrolled scenario, the infected population (I) also shows exponential growth, reaching a value of approximately 4×10^{-3} at

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the end of the simulation. This indicates that without intervention, even on a small scale, the number of individuals actively transmitting the disease will continue to increase. However, with the implementation of optimal control, the infected population (I) is successfully suppressed significantly to near zero throughout the entire simulation period. This drastic suppression of I is a direct consequence of the success of the u_1 control in reducing the population E , as well as the contribution of the u_2 control in accelerating the recovery from I to R. These results demonstrate the comprehensive effectiveness of the control strategy in minimizing the burden of infection in the population.

The sharp decline in the infected population in the controlled scenario demonstrates that dual interventions, including education and treatment, can halt the rate of new infections and accelerate the transition to recovery. This demonstrates the synergy between controls, not only breaking the chain of transmission but also reducing the number of individuals experiencing severe symptoms.

Figure 2 shows the infected population drastically declines under optimal control. This supports the synergy between the two controls: public education reduces exposure, while timely treatment accelerates recovery. Similar outcomes were presented by Allali et al. (2024)(Allali et al., 2025), who concluded that the dual-strategy approach significantly lowers infection peaks and shortens epidemic durations. This outcome highlights the value of rapid and concurrent implementation of both control measures.

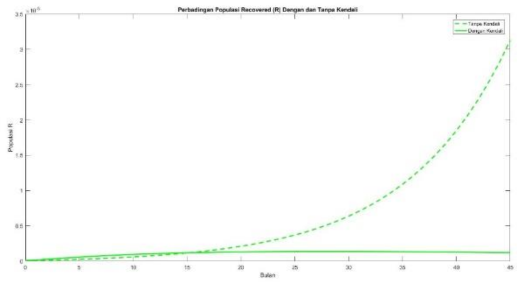

Figure 3: Comparison of Recovered Population (R) with and without Optimal Control

Figure 3 presents the population dynamics of recovered individuals (R) as a function of time, comparing the no-control scenario (dashed line) and the optimal control scenario (solid line). Similar to the E and I, compartments, the vertical scale is also on the order of 10^{-3}. In the no-control scenario, the R population shows continued exponential growth, reaching approximately 3.1×10^{-3} at the end of the simulation, indicating natural recovery. In the optimal control scenario, the recovered population remains low because very few individuals become infected due to early interventions that successfully contain the spread in the early stages. Although the u_2 control is designed to accelerate recovery ($I \rightarrow R$), the low R population in the optimal control scenario can be explained by the remarkable success of the u_1 control.

Since u_1 is highly effective in suppressing the number of exposed (E) and infected (I) individuals to near zero (as shown in Figures 1 and 2), it follows logically that very few individuals can enter the recovered (R). compartment. Therefore, the low R population does not mean that the u_2 control is ineffective,

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but rather is an indication of the overall success of the control system in preventing most individuals from becoming infected and then recovering, thereby reducing the total burden of disease in the population.

Figure 3 shows a lower number of recovered individuals in the controlled scenario. Although this might appear counterintuitive, it actually reflects the model's success in preventing individuals from becoming infected in the first place. This paradox has also been noted by Cacace and Oliviero (2023)(Cacace & Oliviero, 2024), who pointed out that effective early intervention can reduce the need for medical recovery entirely. The model implies that less recovery is not indicative of control failure, but rather a direct result of successful transmission suppression.

The discussion also reveals limitations in the model's structure. It assumes homogeneous mixing and static parameters, which are simplifications that may not reflect real-world epidemic dynamics such as social behavior shifts, viral mutations, or healthcare disparities (Zinihi et al., 2025)(Zinihi et al., 2025). Additionally, the model lacks spatial dynamics, which could affect accuracy in simulating real geographic spread (Hethcote, 2000; Wang & Zhao, 2012)(Hethcote, 2000)(Wang & Zhao, 2008).

Based on these observations, future research should incorporate real-time data integration, spatial heterogeneity, age structures, and adaptive controls. Simulation platforms should also accommodate real outbreak case data to improve the model's predictive accuracy and policy relevance. The implications of this study suggest that dual-control strategies, when initiated early, can be an effective framework for health policy planning, particularly in low-resource settings (Sumardi, 2022; WHO, 2018)(Sumardi, 2022)(World Health Organization, 2018).

4. Conclusions

This study confirms that implementing a dual optimal control strategy educational campaigns and medical treatment within the SEIR model framework can significantly reduce the number of exposed and infected individuals during the outbreak of infectious diseases such as Ebola. The simulation demonstrates that combining these controls from the early stage of an epidemic effectively suppresses transmission, preventing the disease from reaching wider segments of the population.

From a mathematical and epidemiological perspective, the use of Pontryagin's Minimum Principle and the Forward-Backward Sweep Method provides a reliable approach to identifying cost-effective intervention strategies. The findings support earlier research but further emphasize the importance of timing and synergy in public health responses.

Nevertheless, this research is not without limitations. The model assumes homogeneous mixing and constant parameters over time, which may oversimplify real-world complexities such as social behavior dynamics, mutation of the virus, or environmental changes. Additionally, the reliance on hypothetical parameter values without calibration using real outbreak data may limit the accuracy of the projections for policy application.

Future studies should consider integrating real epidemiological data, spatial dimensions, age structures, or stochastic effects to enhance model realism. The inclusion of adaptive control strategies that respond dynamically to data updates could also be explored. Furthermore, interdisciplinary collaboration with public

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health experts and policymakers is crucial to transform theoretical insights into practical action.

The implication of this research lies in its contribution to epidemic preparedness. Mathematical models, when paired with optimization techniques, can serve as decision-support tools for designing responsive, cost-efficient health interventions, particularly in regions with limited healthcare resources.

Author Contributions

The first author contributed to the construction of the model, theoritical analysis, and interpretation of the results. The second author contributed to the numerical analysis and simulation. The first author as corresponding author revised the article based on review from reviewer and journal editor. All authors provided critical feedback and helped shape the research, analysis and manuscript**.**

Declaration of Competing Interest

No potential conflict of interest was reported by the author(s).

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