Infinity Journal of Mathematics Education
Volume 14, No. 4, 2025
p-ISSN 2089-6867
e–ISSN 2460-9285
https://doi.org/10.22460/infinity.v14i4.p899-918

Integration of abstraction theory and TPACK framework in geometry learning to optimize prospective mathematics teachers' spatial abilities

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Received: Nov 15, 2024 | **Revised**: Jun 19, 2025 | **Accepted**: Jun 22, 2025 | **Published Online**: Oct 3, 2025

Abstract

This study aims to optimize the spatial abilities of prospective mathematics teachers (PMTs) using an integrated geometry learning module based on abstraction theory and TPACK, and to reduce the gap in geometry learning by integrating these theories. Epistemic abstraction refers to mental actions, including recognizing, building with, and constructing. Meanwhile, TPACK is introduced to use technology as a medium for prospective teachers to design geometry lessons and help learners understand geometry while considering pedagogical principles. We employed a research and development design by Borg and Gall, involving 30 participants selected using purposive sampling. Spatial ability data were collected using the Purdue Spatial Test and descriptively analyzed to assess the optimization of PMTs' spatial abilities before and after implementing the integrated geometry learning module. Findings demonstrated that the percentage of PMTs with beginner-spatial abilities decreased from 80% to 26.66%, while the rate of PMTs with advanced spatial skills increased from 20% to 73.33%. We concluded that integrating abstraction theory and TPACK in geometry learning produced an integrated geometry learning module capable of optimizing the spatial abilities of prospective teachers both didactically and pedagogically.

Keywords:
Abstraction, Geometry, Prospective teachers, Spatial ability, TPACK

How to Cite:
Sahrudin, A., Pamungkas, A. S., Pagiling, S. L., & Rosdianwinata, E. (2025). Integration of abstraction theory and TPACK framework in geometry learning to optimize prospective mathematics teachers' spatial abilities. *Infinity Journal*, *14*(4), 899-918. https://doi.org/10.22460/infinity.v14i4.p899-918

1. INTRODUCTION

The subtopic of geometry has a larger portion compared to other topics and has had a significant place in school mathematics education (Jablonski & Ludwig, 2023). The everyday activities for students in learning basic geometry involve studying points, lines, planes, and solid shapes (Sulistyowati et al., 2017), which later developed into concepts involving two-dimensional and three-dimensional objects (Lane & Sorby, 2021) and their applications in everyday life (Ariani et al., 2019). Potential actions, specifically abstraction, are necessary in studying geometry to understand and solve geometric problems.

There is a broad consensus that abstraction is paramount in mathematics teaching and learning (Dreyfus et al., 2015; Reinke, 2019; Scheiner, 2015). Numerous scholars have documented that abstraction could enable students to construct new mathematical ideas, reason, and engage meaningfully with mathematics (Dintarini et al., 2024; Hodiyanto et al., 2025; Hodiyanto et al., 2024; Lacey, 1994). Indicators of students' abstraction include: (1) clarifying geometric figures using attributes; (2) constructing definitions; (3) determining relationships between figures based on their characteristics; (4) visualizing geometric figures according to descriptions; (5) summarizing part or all of the information; and (6) creating connection diagrams (Budiarto, 2005; Budiarto et al., 2021).

The problem often encountered in geometry learning is that students do not fully understand the objectives and applications of geometry. This issue arises because geometric concepts are abstract, while real-life examples only present concrete forms of geometric objects (Lumbanbatu et al., 2023). For instance, introducing the definition of a cube, a cube is defined as an abstract concept that can only be imagined in the mind, whereas the cubes observed in real life are merely representations. Students often associate cubes with objects like boxes, dice, or other perfectly square items without understanding the definition of a cube. However, these square-shaped objects are merely representations of cubes resulting from abstraction (Sahrudin, 2024a). This misunderstanding of the cube's definition exemplifies how geometry learning is underdeveloped and still relies on traditional approaches (Bergstrom & Zhang, 2016). Geometry learning has not explicitly involved specific theories or technologies, despite technology's potential to enhance and advance learning (Dilling et al., 2024; Drijvers & Sinclair, 2023; Hollebrands & Okumuş, 2018), and assist students in solving geometric problems (Nindiasari et al., 2024; Sudirman et al., 2024; Weinhandl et al., 2023).

Geometry learning research around the world has mostly focused on improving students’ problem-solving skills in geometry (Bokosmaty et al., 2015), while some other studies concentrate on developing appropriate methods for implementing geometry learning (In’am & Hajar, 2017; Ishartono et al., 2019). Meanwhile, the development of abstraction theory in geometry learning is still centered on the discussion of the actions of recognizing, building-with, and constructing (Dreyfus, 2015; Hershkowitz et al., 2007). Integrating abstraction theory into geometry learning helps teachers and students conduct geometry instruction progressively, following the abstraction theory stages: recognizing, building-with, and constructing (Sahrudin et al., 2021). Recognizing refers to the activity of forming prior mathematical structures, where the student’s activities are stored and relevant to constructing new mathematical structures in the present situation (Hassan & Mitchelmore, 2006). Building-with involves the integration of elements to achieve a specific goal (Dreyfus et al., 2015). Constructing is the process of gathering structural information that allows the data to be reorganized vertically (Sumen, 2019).

The integration of technology and learning content, known as TPACK (Technological Pedagogical Content Knowledge), can assist students in solving geometric problems (Nindiasari et al., 2024; Sudirman et al., 2024). TPACK framework is a commonly used conceptual tool in studies that consider technology integration into classrooms (Olofson et al., 2016; Simsek & Clark-Wilson, 2024). The integration of TPACK into learning is expected to provide an effective solution to the issues in geometry instruction, making geometry learning more tangible (Bretscher, 2022). TPACK is crucial for offering specific projects and pedagogically-centered ICT intervention schemes (Pagiling et al., 2024). TPACK is a framework outlining the knowledge that teachers need to integrate technology effectively into learning (Petko et al., 2025) can be illustrated in Figure 1.

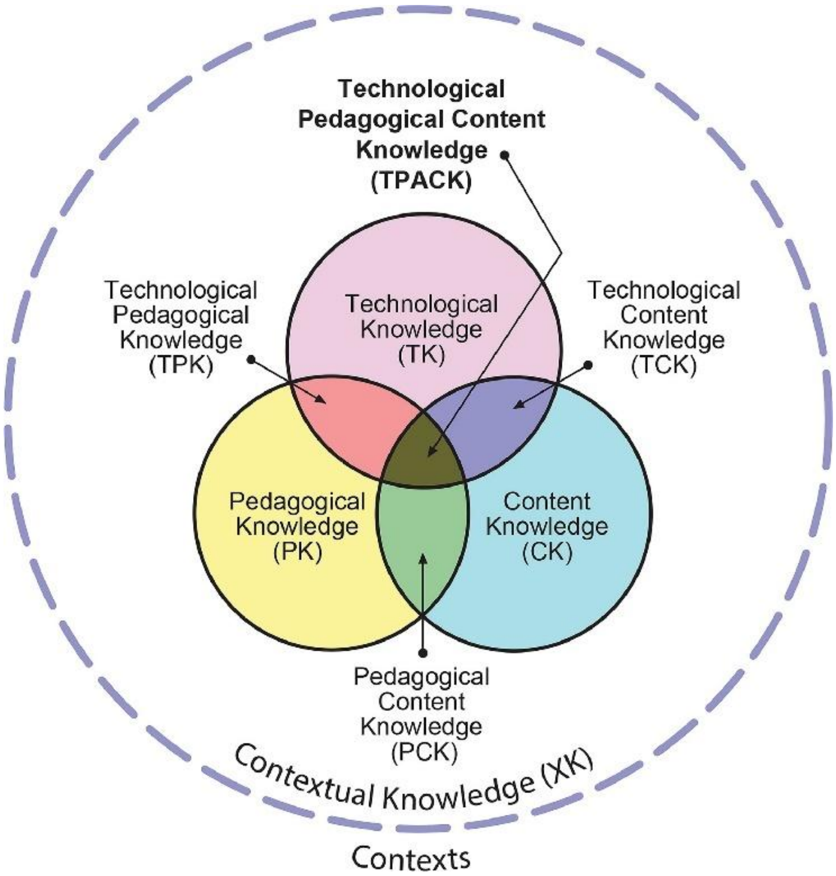

Figure 1. The updated TPACK model

The involvement of abstraction theory and TPACK in geometry learning is expected to provide a solution for various stakeholders, including prospective teachers, teachers, and lecturers, to conduct more appropriate teaching. This approach aligns with the readiness of students in terms of spatial, mental, and personal abilities to face technological advancements and future demands that involve geometric knowledge and spatial skills, such as graphic design, engineering, and architecture (Saeed et al., 2017). Therefore, geometry learning must challenge students to utilize the properties and content of modern geometry instruction and integrate geometric concepts with technology to make the learning process more tangible (Hollebrands & Okumuş, 2018).

The original TPACK did not specify a technology to look at (Kong & Lai, 2021) while in our framework, the technology is renewed geometry learning involves the use of technology in teaching (Ng et al., 2020; Pamungkas et al., 2023). The technology referred to in this study includes current technologies closely related to students' lives, such as the use of assembler applications, GeoGebra, Canva, 3D pens, and other graphic design applications. The use of pedagogy to teach concepts, practice how to build a cube net from the cube and understand the educational goal. The use of content knowledge for the development of geometry knowledge includes definitions, concepts, and their applications.

Abstraction is a critical process in mathematical thinking (Yilmaz & Argun, 2017). The primary foundation of abstraction used in this study involves mental actions, comprising three interrelated actions (Hershkowitz et al., 2007; Lacey, 1994). These three epistemic actions in geometry learning are utilized by students like a linear chain sequence (Budiarto et al., 2021). Previous research on abstraction revealed that in solving geometry problems, students employ abstraction encompassing recognizing, building-with, and constructing (Sahrudin et al., 2021). Students with beginner spatial abilities solve geometry problems through a fluctuating and complex abstraction process. In contrast, students with advanced spatial abilities address geometry problems using a simpler and more interconnected abstraction flow, covering recognizing, building-with, and constructing (Sahrudin et al., 2022). Recognizing is a mental activity that recalls past knowledge stored in memory to meet current needs. Building-with involves the mental activity of identifying and combining elements to produce new knowledge. Constructing is the integration of knowledge elements to form deeper, new mathematical structures (Sahrudin, 2024a).

Based on several studies conducted by previous researchers, it is concluded that in implementing geometry learning, a teacher must understand the abstraction process occurring in students, the technology related to the learning material, and the students' spatial abilities. Thus, the learning process should be prepared with content, technology, and spatial skills in mind (Lane & Sorby, 2021). Spatial ability is the capacity to generate, maintain, and manipulate images abstractly (Thissen et al., 2018) and includes the ability to accurately perceive the visual world. Spatial ability is essential in learning geometry as it involves spatial thinking that applies exploratory and understanding aspects (Fiantika et al., 2017) which can illustrate problem-solving situations and provide opportunities to visualize the position, size, color, and shape of spatial objects. Therefore, spatial ability is crucial in geometry learning (Yurt & Tünkler, 2016) and serves as a key to students' success in the technological era (Zurn-Birkhimer et al., 2018).

Based on the above explanation, the urgency of this research lies in integrating abstraction theory and TPACK into geometry learning to optimize the spatial abilities of prospective teachers so that geometry learning more tangible, and ensure that students gain authentic learning experiences (Abdul Hanid et al., 2022), and enable them to solve complex problems by utilizing various elements and technological content in geometry learning (Mandala et al., 2025; Nindiasari et al., 2024; Sudirman et al., 2024). This aligns with technological advancements and the educational needs of the independent curriculum (Sahrudin, 2024b).

If this research is not carried out promptly, prospective teachers intending to engage in teaching assistance activities, campus teaching programs, school field introduction programs, or field practice activities as part of the independent curriculum may lack the spatial readiness to teach geometry in schools and may be reluctant to apply teaching theories integrated with technology. Therefore, the primary goal of this research is to determine whether the integration of abstraction theory and TPACK into geometry learning can optimize the spatial abilities of prospective teachers.

2. METHOD

This study employs the Research and Development (R&D) method with a development design by Borg and Gall, conducted at a private university in Banten, Indonesia. The R&D method was chosen to assess the optimization of the spatial abilities of prospective teachers, requiring the development of a learning product in the form of an integrated geometry learning module. This module aligns with the independent curriculum and aims to optimize PMTs' spatial abilities. The research steps following the R&D method with reference to Borg and Gall's development framework (see Figure 2).

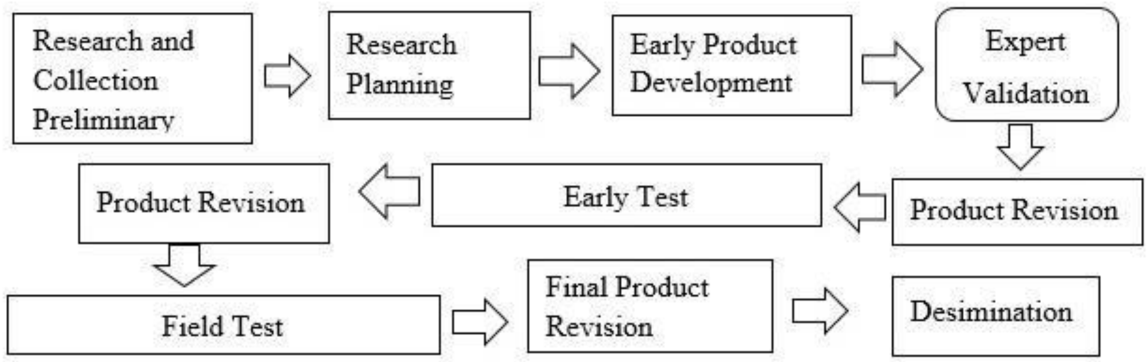

Figure 2. R&D procedures

The first step is research and collection preliminary; the second step is research planning based on the data that has been collected. The third step is early product development. The fourth step is expert validation. The fifth step is product revision. The sixth step is an early test in which researchers involved 3 PMTs as a limited trial participant. The seventh step is product revision. The eighth step is the field test that involved 5 PMTs as participants. The ninth step is final product revision. Finally, the last step is dissemination by conducting a focus group discussion with prospective teachers, teachers, and lecturers to discuss the shortcomings of the geometry learning module integrated with abstraction theory and TPACK.

Data spatial ability was collected using the Purdue Spatial Test instrument, which has been validated and proven reliable (Bodner & Guay, 1997). The Purdue Spatial instrument was administered to prospective teachers to assess their spatial abilities before and after implementing the learning activities using an integrated learning module. The integrated learning module refers to a geometry learning module covering flat-sided solid shapes and circles, which is integrated with abstraction theory and TPACK in accordance with the demands of the independent curriculum. The framework of this learning module includes general information, core components complete with steps and teaching methods relevant to abstraction theory and TPACK, learning evaluations, and student worksheets.

The participants in this study consist of 30 prospective mathematics teachers selected using the purposive technique, where they are chosen based on specific criteria determined by the researcher. The criteria for the participants' recruitment in this study include mathematics education students, having previously learned geometry, being prepared to undertake field practice and school field introduction, having a strong desire to apply renewed geometry learning by participating in the campus teaching program by the Ministry of Education and Culture in one semester and being willing to participate in the research and provide information during their involvement. To evaluate the practicality and effectiveness of the integrated geometry learning module, an experimental analysis was conducted by examining the improvement in the spatial abilities of PMTs before and after implementing learning with the module. This module integrates abstraction theory and TPACK. Overall, the integration of abstraction theory and TPACK into geometry learning has resulted in an innovative learning concept that applies technology that is aligned with the needs of future education and the aspirations of the independent curriculum.

3. RESULTS AND DISCUSSION

3.1. Results

3.1.1. Research and collection preliminary

The first step conducted in the research on integrating abstraction theory and TPACK into geometry learning to optimize the spatial abilities of prospective teachers was to measure the spatial abilities of the participants before implementing the learning activities using the integrated geometry learning module. The measurement was carried out using the purdue spatial test, which consists of 30 multiple-choice items with a correct answer score of 1 and a wrong answer score of 0. Participants who scored between 20 and 30 were categorized at the advanced level of spatial ability, while those who scored between 0 and 19 were categorized at the beginner level of spatial ability. The results of the spatial ability assessment of these 30 participants can be depicted in Table 1.

Table 1. Results spatial ability of prospective mathematics teachers

Prospective Teachers	Level of Spatial Ability		Average
	Beginner ($0 \leq$score≤ 19)	Advanced ($20 \leq$score≤ 30)	
Number of PMTs	24	6	16.47
Percentage (%)	80	20	

Table 1 shows that out of 30 participants, the majority had beginner-level spatal abilities before using integrated geometry module. Specifically, 80% have beginner-level spatial abilities, while 20% have advanced spatial abilities, with an overall average score of 16.47. Based on this, 80% of the participants need to optimize their spatial abilities to reach an optimal level with an overall average target of 20.00. Therefore, optimization is needed for the group of participants to achieve overall spatial ability improvement. Based on the collection of initial research data, the research continued to the second step, planning.

3.1.2. Research planning

The researcher plans to implement the necessary innovations by applying learning using the integrated geometry learning module that combines abstraction theory and TPACK. The abstraction theory referred to in this study is the abstraction defined by Lacey (1994) which states that abstraction in the context of epistemic actions involves the actions of recognizing, building-with, and constructing, which are interrelated. This was further developed by Budiarto (2005) and Dreyfus (2015), who concluded that the epistemic action of abstraction in students when learning geometry is like a chain sequence, meaning that recognizing, building-with, and constructing are linear activities. Both questions are closely related, as previous research by Sahrudin et al. (2021) found that epistemic actions of abstraction occurring in students are interrelated. However, the epistemic actions of abstraction in students with beginner-level spatial abilities tend to have a more variable and complex form, as illustrated in Figure 3.

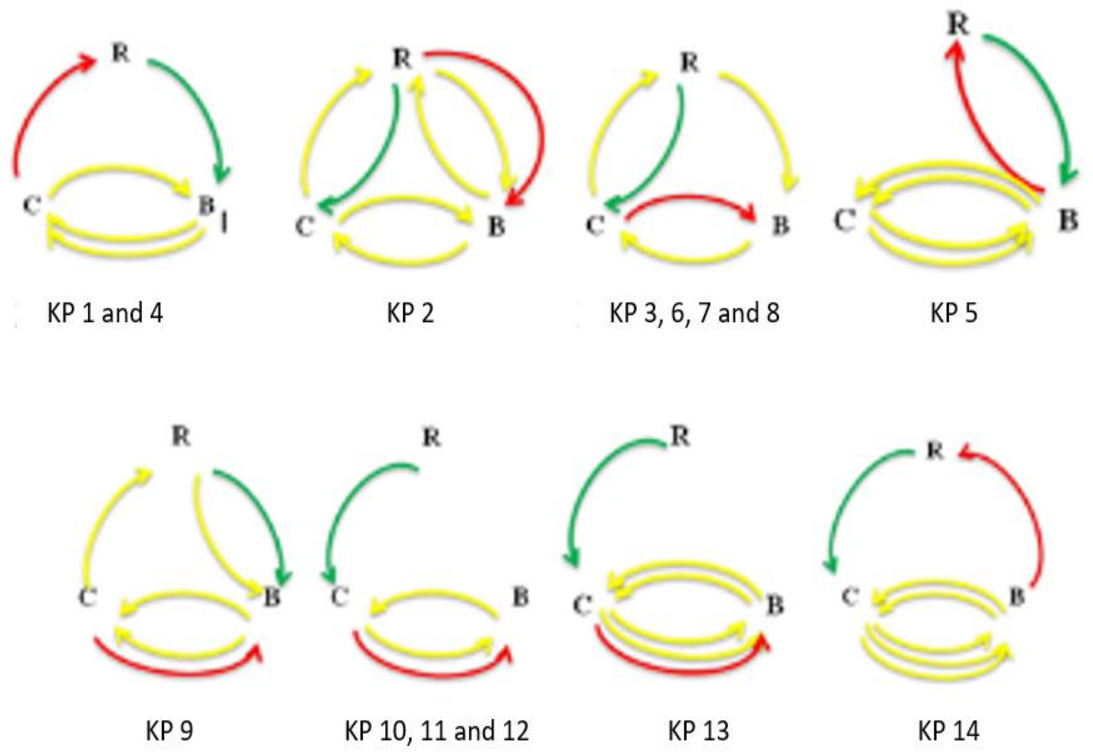

Figure 3. Form of abstraction in students with beginner-level spatial abilities

It contrasts with students who have advanced spatial abilities, where the form of epistemic action abstraction is simpler. The form of epistemic action abstraction in students with advanced spatial abilities is presented in Figure 4.

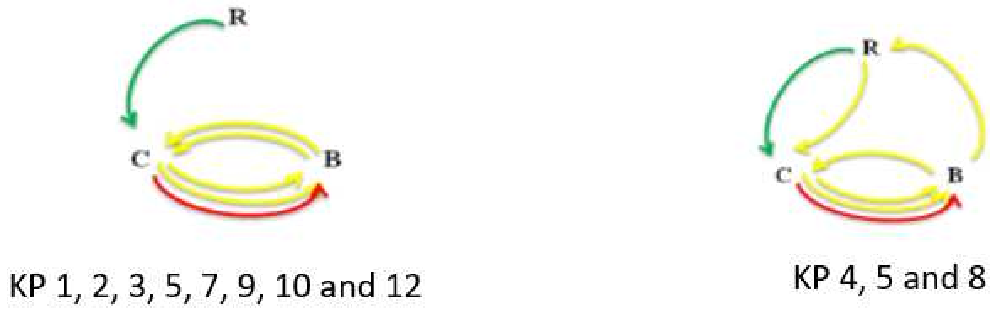

Figure 4. Form of abstraction in students with advanced spatial abilities

Based on Figure 3 and Figure 4, there are two possibilities for the epistemic action abstraction of individuals studying geometry: complex abstraction and simpler abstraction. The simpler epistemic action abstraction is carried out by individuals who have understood how geometry is used, how it changes, and its applications. This understanding is crucial for prospective teachers to optimize their spatial abilities and prepare them for teaching geometry effectively, whether in teaching assistance activities, campus teaching programs, school field introduction programs, or field practice experiences. In addition to understanding abstraction, the involvement of TPACK is also necessary for its development.

TPACK was introduced to utilize technology as a medium to facilitate students' understanding of abstract geometry content while still considering pedagogy. The integration of abstraction theory and TPACK into geometry learning for optimizing the spatial abilities of prospective teachers is implemented through a geometry learning module that adopts the independent learning curriculum developed by the research team in the third step, which is the development of the preliminary product.

3.1.3. Early product development

We designed a geometry learning module incorporating abstraction theory and TPACK and adopted the independent curriculum. It has a structure that includes a cover, general module information, module identity, initial competencies, the Pancasila student profile, facilities and infrastructure, target audience, number of students, learning model or method, core components, subheadings of core components, learning outcomes, learning objectives, activity objectives, guiding questions based on abstraction theory, learning materials following the stages of epistemic abstraction, and the use of technology applications, equipped with LiveWorksheet for LKPD (student worksheets). In addition, it also includes learning resources and technology-based media, as well as assessments.

3.1.4. Expert validation

Several steps are undertaken before using the geometry learning module integrated with abstraction theory and TPACK, including the preliminary product trial, which is the fourth step in this research. In the initial trial, validation is conducted by content validation involving a Universitas Negeri Surabaya lecturer who has expertise in geometry and a language expert from Universitas Sultan Ageng Tirtayasa, and through a limited trial. The result of content validation for the developed product can be illustrated in Table 2.

Table 2. Result content validation geometry learning module integrating abstraction theory and TPACK

No	Indicator	Score Max	Validator Score	
			1st Val.	2nd Val.
1	Conformity with curriculum	10	10	10
2	Adequacy of material	10	9	10
3	Evaluation	10	9	9
4	Readability and ease of understanding	10	10	10
5	Accessibility	10	10	10
6	Conformity with theory abstraction	10	10	10
7	Conformity with TPACK	10	10	10
	Total	70	68	69
	Mean	69		
	Final Score	98%		
	Criteria	Very valid		

The expert validation results for the integrated geometry learning module incorporating abstraction theory and TPACK that has been developed show a high validity of 98%. The expert validators recommended that the research continue to the fifth step: conducting a product revision with several expert recommendations for improved learning module development.

3.1.5. Product revision

The revised product developed and validated by experts indicates that the geometry learning module integrating abstraction theory and TPACK focuses on learning activities or core geometry materials that align with technological advancements and follow the stages of epistemic abstraction, which include recognizing, building with, and constructing. The module should also be equipped with LKPD (student worksheets) that can be accessed via a barcode, allowing tasks to be completed through typing or drawing using a Wacom pen. From a language perspective, the validators suggested that the language used should be adapted to the student's educational level and should refer to Piaget’s theory of development. After the product revision has been completed, the research continues to the sixth stage by conducting an initial product trial.

3.1.6. Early test

Early tests were conducted to evaluate the preliminary product trials of the integrated geometry learning module were conducted with 3 PMTs outside of the main subjects. The preliminary trial aimed to test practicality and demonstrate the use of the integrated geometry learning module. The preliminary group trial was conducted on Thursday, August 1, 2024.

The average scores for each aspect of the preliminary product trial results are portrayed in Table 3.

Table 3. Average scores for each component of preliminary trial results

Component	Average Score	Percentage (%)	Criteria
Interest	39	79	Practical
Material Presentation	16	82	Very Practical
Language	8	78	Practical
Average		**79**	**Practical**

Based on the results of a preliminary product trial, the component of PMTs interest received an average score of 39 with a percentage of 79%, the component of material presentation received an average score of 16 with a percentage of 82%, and the component of language received an average score of 8 with a percentage of 78%. Thus, the overall average score across all aspects was 79% with a "Practical" criterion. In the preliminary product trial, spatial ability was also assessed for the test. Comparing the spatial abilities of these PMTs before and after the learning activities using the geometry learning module integrated with abstraction theory and TPACK shows that their spatial abilities have been optimized. Based on the measurement of spatial ability conducted on the preliminary product trial subjects, the spatial ability of three PMTs in preliminary product trials exhibited advanced spatial abilities with an average score of 26.4. The measurement of spatial ability in the preliminary product trial phase indicates that the PMTs have optimal spatial abilities, falling within the average range of 20 to 30.

3.1.7. Product revision

After the preliminary product trial is completed, the seventh step is to revise the product based on the findings and notes from the early tests that evaluated the preliminary product. The preliminary trial results and the spatial ability measurements of the preliminary trial subjects indicate that the learning module can be utilized. Thus, after the early test, the revision activities only involved checking and revising minor imperfections. However, to ensure that the learning module achieves greater reliability, a broader trial was conducted with a larger group in the eighth stage, namely, conducting a field test.

3.1.8. Field Test

The activity continued to the eighth step, namely the field test, by conducting a main product trial after first completing revisions based on the recommendations and findings from the previous step. In the field test, a broader trial was conducted with a larger group of 5 subjects. This step aims to assess the practicality and effectiveness of the learning module so that it can function well to improve spatial ability on a larger scale. The main product trial was conducted on Tuesday, August 20, 2024. The average scores per aspect from the broader trial can be portrayed in Table 4.

Table 4. Average scores per aspect of the main product trial results

Aspect	Average Score	Percentage (%)	Criteria
Interest	39	78	Practical
Material Presentation	16	80	Practical
Language	8	82	Very Practical
Average		**80**	**Practical**

Table 4 depicts the main product trial results, the interest aspect of the subjects received an average score of 39 with a percentage of 78%, the material presentation aspect received an average score of 16 with a percentage of 80%, and the language aspect received an average score of 82%. Thus, the overall average score across all aspects is 80%, falling under the "Practical" criteria. In the main product trial, a measurement of spatial abilities was also conducted on the subjects. It is evident from the main product trial that all subjects (5 PMTs) possess advanced spatial abilities with an average score of 25.6. Based on the spatial ability measurements conducted during the main product trial step, it was shown that subjects have optimal spatial abilities, which fall within the average range of 20 to 30. This step indicates that regardless of the number of subjects involved in geometry learning, using a geometry learning module integrated with abstraction theory and TPACK can optimize the subjects' spatial abilities.

3.1.9. Final Product Revision

After the field test, which aims to evaluate the product, is completed, the ninth step is to revise based on the findings and notes from the main product trial research. The revisions include that the module must incorporate applications such as GeoGebra, Canva, Assembler-Edu, and other supporting tools like LiveWorksheet for student worksheets. The use of technology allows students to engage in deeper abstraction, thus optimizing their spatial abilities.

3.1.10. Dissemination

With the practicality and accuracy of the learning module established, the research continued to the tenth step, namely dissemination. The dissemination activity involved the education community, including mathematics teachers, mathematics education alumni, and students from the mathematics education study program who intend to implement or participate in campus teaching activities. The dissemination was held on Saturday, September 21, 2024, at the Mathematics Learning Development Laboratory. Figure 5 show the documentation of the FGD activity for the integrated learning module involving the education community, which includes mathematics teachers, mathematics education alumni, and PMTs who are planning to implement or participate in campus teaching activities.

Figure 5. FGD Activity on the integrated module with the mathematics education community

The FGD activity in Figure 5 was conducted with the mathematics education community and resulted in outstanding discussions and feedback for refining the integrated geometry learning module, incorporating abstraction theory and TPACK. Prior to the dissemination activity, the instructional module was implemented in the main subject, and tests were conducted on this subject using the purdue spatial est to evaluate the product's effectiveness in optimizing the spatial abilities of prospective teachers. The spatial ability scores of the research subjects are presented in Table 5.

Table 5. Spatial ability of prospective teachers after the learning season

Prospective mathematics teachers (PMTs)	Spatial Ability Level		Average
	Beginner	Advanced	
Number of PMTs	8	22	22.03
Percentage (%)	26.66	73.33	

In Table 5, it is shown that out of 30, 73.33% of PMTs have advanced spatial abilities, while 26.66% have beginner-level spatial abilities, with an overall average of 22.03, which exceeds the target average of 20.00. Based on the spatial ability measurements during the main product trials step, as illustrated in Table 1, there was an increase in the number of PMTs with advanced spatial abilities from 20% to 73.33%, while the number of PMTs with beginner-level spatial abilities decreased from 80% to only 26.66%.

3.2. Discussion

The integration of abstraction theory and TPACK into geometry learning for optimizing the spatial abilities of prospective teachers was carried out by first collecting initial data or detecting the spatial abilities of these PMTs. After obtaining data on the spatial abilities of prospective teachers through spatial ability measurements using the purdue spatial test, the majority had beginner-level spatal abilities before using integrated geometry module. Specifically, 80% have beginner-level spatial abilities, while 20% have advanced spatial abilities, with an overall average score of 16.47.

it was found that their spatial abilities were not yet optimal and remained at the beginner level. This condition reflects a suboptimal state of cognitive readiness among prospective teachers, indicating a critical need for targeted instructional interventions to support more effective and meaningful geometry learning. the research continued with efforts to comprehensively optimize these abilities by first developing a geometry learning module integrated with abstraction theory and TPACK. A geometry learning module grounded in abstraction theory and the TPACK framework was designed to address this pedagogical challenge as a strategic instructional solution. The module comprises several comprehensive components, including a cover page, general module information, module identification, prerequisite competencies, the Pancasila student profile, required facilities and infrastructure, target learner characteristics, expected number of students, instructional models or methods, core components and subcomponents, learning outcomes, instructional objectives, learning activity goals, abstraction theory-based triggering questions, and instructional content aligned with the phases of epistemic abstraction. Additionally, the module incorporates technology applications, live worksheets, digital media, and curated resources to enhance learning engagement. The geometry learning module that was developed has proven to be effective and falls into the practical category, with a practicality level of 80%. It has been validated by experts, with the product validity rated as very valid and a percentage score of 98%. The module also underwent an initial product trial, where its practicality was categorized as practical with a score of 79%, and a main product trial, which also placed the practicality level in the practical category with a percentage score of 80%. Thus, the integrated geometry learning module has gained reliability and practicality for optimizing the spatial abilities of prospective teachers. Comparing the spatial abilities of these students before and after the learning activities using the geometry learning module integrated with abstraction theory and TPACK, it is discovered that their spatial abilities have been optimized. This improvement is evident from the achievements observed before and after the learning activities. This is evident from the comparison of outcomes before and after the implementation of the learning process. The spatial abilities of prospective teachers improved significantly after participating in geometry learning (Bretscher, 2022) that incorporated technology, demonstrating that these abilities can be enhanced effectively (Abdul Hanid et al., 2022; Drijvers & Sinclair, 2023; Weinhandl et al., 2023).

The learning activities using the integrated geometry learning module based on abstraction theory and TPACK have shown that prospective teachers who possess advanced spatial abilities are more optimal than before. This indicates that the geometry learning module has positively impacted the optimization of the spatial abilities of prospective teachers. This positive outcome is reflected in the results of the Purdue Spatial Test, where previously only a small portion of prospective teachers had advanced spatial abilities, while most prospective teachers had beginner-level spatial abilities, as measured by the Purdue Spatial Test. It can be concluded that the spatial abilities of the prospective teachers were not yet optimal and required improvement through geometry learning using the integrated geometry learning module based on abstraction theory and TPACK. Therefore, geometry learning should not always rely on traditional approaches (Bergstrom & Zhang, 2016).

The optimization activities were conducted from June 22, 2024, to September 26, 2024. The optimization of the spatial abilities of prospective teachers at a private university in Banten was carried out on students who had not yet participated in teaching assistance, the campus teaching program, school field introduction, or field practice. It is expected that after the optimization activities, the spatial abilities of prospective teachers will be optimized, enabling them to be better prepared for teaching assistance, the campus teaching program, school field introduction, or field practice activities. As a result, geometry learning activities can become more meaningful and aligned with the demands of the independent curriculum.

After the learning activities using the geometry learning module were conducted, the researcher re-administered the Purdue Spatial Test to the prospective teachers to assess their spatial abilities. The research data indicated that after learning using the integrated geometry learning module based on abstraction theory and TPACK, only a small portion of the prospective teachers still had beginner-level spatial abilities. This means that the number of students with beginner-level spatial abilities decreased. The percentage of PMTs with beginner-level spatial abilities decreased from 80% to 26.66%, while the number of PMTs with advanced-level spatial abilities increased from 20% to 73.33%. The overall average score on the Purdue Spatial Test exceeded the set target. Thus, the involvement of abstraction theory (Budiarto, 2005; Sahrudin et al., 2021) and TPACK helps PMTs utilize various elements within TPACK to solve geometry problems (Bretscher, 2022; Radu et al., 2015) and optimize their spatial abilities (Sahrudin et al., 2022). Integrating abstraction theory and TPACK framework in geometry learning has significantly improved spatial abilities among prospective teachers.

The study's overall findings indicate that integrating abstraction theory and TPACK into geometry learning for optimizing the spatial abilities of prospective teachers was successful. The percentage of PMTs with advanced spatial abilities improved after the learning activities using the integrated geometry learning module based on abstraction theory and TPACK, compared to the number of PMTs with advanced spatial abilities before these learning activities. However, it is worth noting that some prospective teachers remained at the beginner level in terms of spatial abilities. Thus, the effectiveness of integrating abstraction theory and TPACK can be utilized by prospective mathematics teachers, teachers, and lecturers in developing geometry instruction by first assessing spatial abilities. This result corroborates previous research conducted by Lane and Sorby (2021), which states that in geometry instruction, a teacher must understand the abstraction process that occurs in students, the technology related to the learning content, and the spatial abilities of the students. Similarly, Abdul Hanid et al. (2022) emphasized that geometry learning should be authentic and concrete, utilizing technology. This also aligns with Nindiasari et al. (2024), who argue that the use of various elements and technology in geometry instruction can assist students in solving geometry problems. This research can be extended by exploring solutions to further optimize students' spatial abilities in learning geometry, as the outcomes of this study have not fully optimized all participants, leaving 26.66% who still require a solution.

These improvements were marked by an increased proportion of participants achieving advanced-level spatial proficiency and a corresponding reduction in those remaining at the beginner level. These findings underscore the module's effectiveness in preparing prospective teachers for professional activities such as microteaching, school induction programs, and teaching practicums. Pedagogically, the module equips future educators with the spatial competencies necessary to teach geometric concepts with clarity and contextual relevance. The integration of abstraction theory and the TPACK model enhances instructional quality, fosters deeper conceptual understanding, and supports the transition toward technology-integrated educational practices.

4. CONCLUSION

Integrating abstraction theory and TPACK into geometry learning to optimize prospective teachers' spatial abilities results in a learning product in the form of an integrated geometry learning module. This module includes a cover, general information about the module, module identity, initial competencies, Pancasila student profiles, facilities and infrastructure, target students, the number of students, learning models or methods, core components, subheadings of core components, learning achievements, learning objectives, learning activity objectives, triggering questions according to abstraction theory, learning material aligned with the stages of epistemic abstraction actions, and the use of technology applications, supplemented with live worksheet worksheets, sources, and technology-based media. The developed geometry learning module can optimize the spatial abilities of prospective teachers, The optimization of students' spatial abilities is demonstrated by an increase in the number of prospective teachers with advanced spatial abilities and a decrease in the number of prospective teachers with beginner spatial abilities. As a result, prospective teachers who are preparing for activities such as campus teaching, school introduction programs, and field practice programs will be better prepared after participating in integrated geometry learning. Therefore, this study should be further refined and examined in more detail regarding prospective teachers at the beginner spatial ability level to ensure that their spatial abilities are fully optimized and reach an advanced spatial ability level.

Acknowledgments

The author would like to thank the Rector of Mathla'ul Anwar University and colleagues who have contributed to this research serso that this research can be carried out properly. We also would like to thank the Ministry of Education, Culture, Research and Technology was funded this research through a regular fundamental research grant in 2024.

Declarations

Author Contribution	: AS: Conceptualization, Investigation, Visualization, Writing - Original Draft, and Writing – Review & Editing; ASP: Formal analysis, Methodology, and Writing – Review & Editing; SLP: Methodology, and Writing – Review & Editing; ER: Supervision, and Validation.

Funding Statement : This research was funded by the Ministry of Education, Culture, Research and Technology through a regular fundamental research grant in 2024.
Conflict of Interest : The authors declare no conflict of interest.
Additional Information : Additional information is available for this paper.

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