



Science Research Olympiad Preparation Assistance: Application of Differential and Integral in Radioactive Decay

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Abstract

Performance not only a source of pride for students, but also for schools. The Indonesian Student Research Olympiad (OPSI) is a national-level activity recognized by the National Achievement Center (PUSPRESNAS). Schools need to provide guidance to support their students in participating in OPSI competitions, one of which is in mathematics. The purpose of this community service is to provide new learning experiences to students at SMA Negeri 1 Mirit, Kebumen Regency, in managing real-world problems and using mathematics to find solutions. Examples given are the problem of medication timing and radioactive decay. The community service method consists of material presentation, class discussions, and providing examples of problems that can be solved using similar methods. The community service is carried out once for 2 hours, the results of the community service are measured by a questionnaire, and of the 20 students who participated in the activity, all agreed that this community service provided a new learning experience that was different from the learning experience during the lessons.

Keywords: mathematical models, olympiads, learning experiences, achievements, students, solutions.

1. Introduction

Students' research skills can be developed through collaboration between schools and universities. The government, through ministries, has organized numerous competitions at both the regional and national levels. One such event is the Indonesian Student Research Olympiad (OPSI), which focuses on mathematics.

The mathematics studied by high school students up to grade 11 is considered sufficient to provide the necessary tools for conducting simple research. Grade 11 students have learned about differentials and integrals. They are familiar with exponential, logarithmic, and natural logarithmic functions. The rules related to exponential and logarithmic functions have been provided through various exercises. Students have also learned to use differentials and integrals for these functions. Students' abilities are incomplete if they cannot apply their learning to solve real-world problems. These skills can be developed to foster a research atmosphere in the classroom, fostering students' awareness that learning mathematics can also be directed toward research.

This community service program was held to provide new learning experiences for students at SMA Negeri 1 Mirit, Kebumen Regency, in managing real-world problems and using mathematics to find solutions. The community service process utilized the students' existing knowledge by providing real-world problems that required the use of their existing knowledge. This mathematical knowledge included exponential functions, logarithms and natural logarithms, differential and integral functions.

In this community service, students are taught how to utilize their existing mathematical knowledge to solve real-life problems. Students will be introduced to research methodologies developed through mathematical modeling. In mathematical modeling, real-life problems are transformed into mathematical problems, and with the knowledge they already possess, students are guided, directed, motivated, and provided with assistance (scaffolding) to find mathematical solutions. The resulting mathematical solutions are then interpreted into real-world solutions that everyone can understand. Direct methods for finding solutions to real-world problems are very difficult to implement. Mathematics has abdicated from its position as king/queen to become a servant that benefits all humans. This is where mathematics—despite its abdication—proves its usefulness in human life. Furthermore, students' abilities and experience in finding solutions to real-life problems with the help of mathematics can be reused when students encounter similar or more complex problems.

2. Materials and Methods

The goal of community service is to provide students with new learning experiences, enabling them to find solutions to real-world problems using their existing mathematical knowledge. To achieve this goal, the community service method involves modeling a problem into a mathematical model, finding mathematical solutions to mathematical problems, and interpreting the resulting solutions into solutions that everyone can understand.

This community service program is designed solely to introduce new ways of understanding mathematics, providing students with a new learning experience. In the next phase, students will be fully guided in conducting research, writing articles, and delivering presentations.



Figure 1. Atmosphere in the Classroom During Community Service

The community service program lasted two hours on Wednesday, September 18, 2024, from 10:30 a.m. to 12:30 p.m. Twenty selected students, accompanied by their subject teachers, participated in the program. To determine the benefits of this community service, a short questionnaire with three questions was administered, one of which was, "Did you gain any new learning experiences after participating in this community service? Please explain!"

3. Results And Discussion

The Results and Discussion section presents sample questions related to medical issues, such as when a sick person should take medication again, whether once, twice, or three times a day is sufficient. The next section provides an example of a problem on radioactive decay. These two topics were covered during the two-hour community service. The final section presents questionnaire responses from 20 students regarding the presentation of both topics.



Figure 2. UNSOED Community Service Team and Teacher Partners of SMAN 1 Mirit

3.1 Mathematical Modeling

Mathematical modeling is the process of explaining real-world problems in a mathematical context to obtain solutions to those problems. Mathematical modeling begins with a real-world problem and ends with a solution to that problem. Obtaining a direct solution to a real-world problem is difficult. Therefore, a roundabout approach is taken by transforming the real-world problem into a mathematical problem and using existing mathematical knowledge to obtain

the solution. The resulting solution is called a mathematical solution. This solution must be interpreted into a real-world solution that is understandable to everyone. Figure 3 illustrates how the mathematical modeling process is carried out to obtain a solution to the problem at hand.

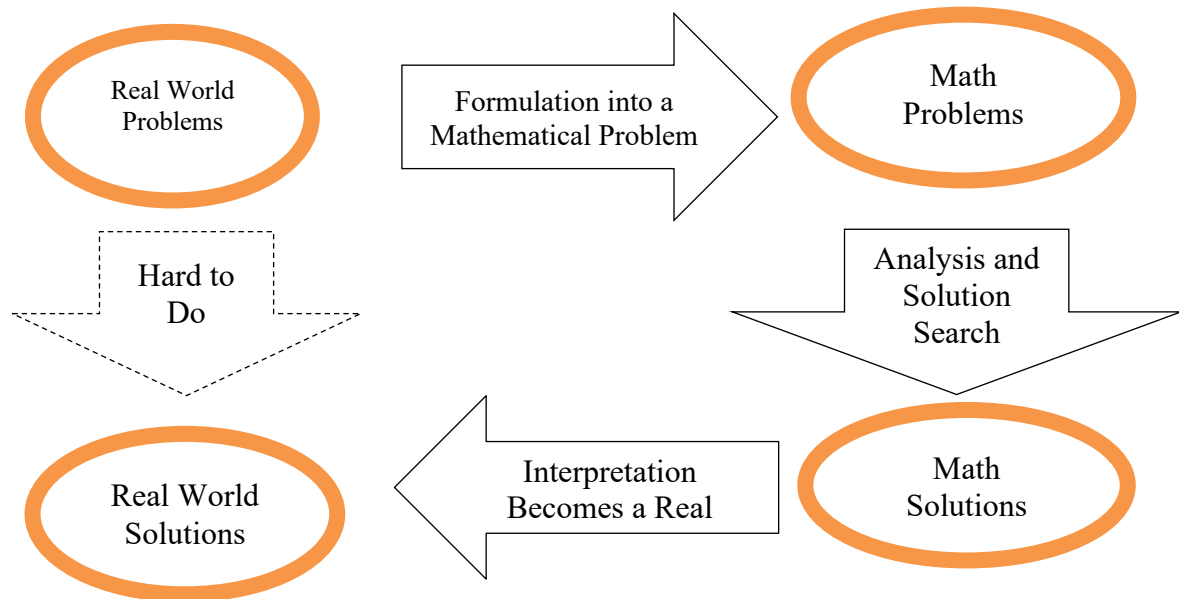


Figure 3. Mathematical Modeling Process

In Figure 3, The process of transforming a real-world problem into a mathematical problem requires knowledge of the variables involved. The process of discovering/finding solutions requires mathematical knowledge, mathematical skills, mathematical tools, and technology. The interpretation process requires the ability to transfer mathematical concepts related to the real-world problem being discussed.

3.2 The Problem of Repeated Medication

A sick person will be advised to take medication when they visit a doctor. The concentration of the medication will decrease over time. If the person takes the medication after breakfast at 7:00 a.m., when should they be advised to take the medication again?

Based on our own experience with illness, or that of those around us who are ill, doctors generally recommend taking medication three times a day—morning, noon, and night—preferably after meals. How do doctors know that medication should be taken five hours after the previous dose?

Mathematics has the answer to this problem. The question is very simple: What time should a patient take their next dose of medication after taking it at 7:00 AM? The answer is 12:00 PM, or five hours later. How does mathematics answer this question?

A doctor will advise a patient to restart their medication if the blood concentration of the drug approaches 0 mg/liter (not yet 0 mg/liter). When does the drug concentration approach 0 mg/liter? To find out, an experiment is conducted. The results of the experiment yielded the following data in Table 1:

Table 1. Drug Concentration

Time (t) minute	15	30	60	90	120	150	180	240	300
Drug Concentration (x) mg/liter	82	65	43	37	22	19	12	6	2

Based on the data in Table 1, it is clear that the drug concentration continues to decrease over time and after 300 minutes or 5 hours, it turns out that the remaining drug concentration is 2 mg/liter, a number close to 0. Thus, from the experimental data, the patient is advised to take the drug again after 5 hours.

The explanation based on the experimental data in Table 1 is sufficient to answer the question of how long a patient should be advised to take the medication again. If the question is what the drug concentration will be after 210 minutes (interpolation problem) and 305 minutes (extrapolation problem), then a mathematical model that fits the data in Table 1 is needed to answer this question. This question relates to our everyday world. Perhaps we can guess that after 200 minutes the drug concentration will be 10 mg/liter, and after 305 minutes it will be 1 mg/liter. How can we be sure that these are just guesses?

To answer this, we need to translate the real-world problem into a mathematical model. This problem can be characterized by two variables: time and drug concentration. The steps are as follows:

- 1) Create a scatter plot with the axes x (horizontal) for time variables and axes y (upright/vertical) for the drug concentration variable. From the scatter plot obtained, it can be seen that the drug concentration x continues to decrease over time t . The scatter plot also explains the relationship between time t with drug concentration x non-linear
- 2) Based on the curve generated by connecting the points of the scatter plot, we can infer that the concentration of the drug in the bloodstream decreases exponentially. In other words, we can infer that the equation relating the drug concentration to the bloodstream is x with time t is:

$$x = x(t) = Ae^{-kt} \quad ; \quad A > 0, k > 0. \quad (1)$$

Equation (1) is called a mathematical model.

- 3) Next, based on the data in Table 1, the value for the coefficient will be estimated A and k . We have many ways to estimate the coefficients. First, you can use the least squares method, the least squares method through the use of matrices, and third, with the help of technology such as spreadsheets like Excel.

- 4) The use of spreadsheets produces values $A = 101,56$ and $k = 0,012$. Thus, a mathematical model is obtained

$$x = x(t) = 101.56e^{-0.012t}. \quad (2)$$

From the mathematical model in equation (2), we obtain

$$x(200) = 101.56e^{-0.012 \cdot 210}.$$

$$x(305) = 101.56e^{-0.012 \cdot 305}.$$

- 5) To ensure that the mathematical model obtained is valid so that it can be used to predict drug concentration at any time, the average error is calculated using formula (3) and the average error is obtained to be very small. Thus, the mathematical model in equation (2) is valid.

$$E = \frac{1}{n} \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} = 0.99965 \quad (3)$$

- 6) Equation (2) is a mathematical model/solution. The mathematical solution obtained needs to be communicated to everyone in easy-to-understand language. This way, a solution to a real-world problem is obtained. Generally, real-world solutions are communicated to laypeople in the form of practical applications, such as stating that the drug concentration after 3.5 hours is 8 mg/liter.

The problem of repeating medication is an example of a real-world problem solved using empirical methods, as mathematical solutions are constructed from empirical experimental data. In addition to empirical methods, there are deterministic and probabilistic/stochastic methods.

The solution techniques used show that mathematical modeling techniques require knowledge of functions and curves, as well as skills to answer questions.

The mathematical modeling process doesn't end with a mathematical solution that can be transformed into a real-world solution. The resulting mathematical model can be tested and developed. Here are some questions that can be used to develop the resulting model:

- 1) Is there a theory/model that underlies each of the experimental data? In other words, is there any reason why the data collected in this example should be assumed to follow a decreasing exponential function?
- 2) What assumptions were used to build the model? Is it possible to review one or more of the assumptions so that a new model can be defined?
- 3) What are the parameters in the model? A dan k have a specific/special meaning? What are the implications if the parameter values are made larger (increasing) or smaller (decreasing)?
- 4) In the real world, someone might want to know the correct dose (concentration) to be given at the start of treatment so that the concentration of the drug in the body will produce an effective dose for healing. Provide an equation that is the answer to this problem!
- 5) In terms of curve fitting, apart from using the trendline facility in Excel, are there any other methods for obtaining parameter values? A dan k ? Can the method be used in various (all) situations or do special conditions need to be met before the method can be used.

3.3 The Problem of Radioactive Decay

Radioactive decay occurs naturally. For example, radioactive substances decay over time (think of sugar dissolving in water). The rate of decay is known to be negatively proportional to the amount of the substance present. Mathematically, this is expressed as:

$$\frac{dN}{dt} = -aN \quad ; a > 0$$

$$\frac{dN}{N} = -adt$$

$$\frac{1}{N} dN = -adt$$

Will be searched $N(t)$ namely the amount of radioactive substance at that time t or what remains after t time. Integrate the left and right sides.

$$\int \frac{1}{N} dN = \int -adt$$

$$\ln N = -at + C$$

$$\exp(\ln N) = \exp(-at + C)$$

$$N = e^{-at+C}$$

$$N = e^{-at} e^C$$

So, the amount of radioactive material remaining after t time is

$$N(t) = e^{-at} e^C$$

$$N(t) = e^{-at} e^C$$

It was known at the time $t = 0$ the amount of radioactive material is $N(0) = N_0$. Substitute $t = 0$ on $N(t) = e^{-at} e^C$ obtained $N_0 = e^C$. So, the amount of radioactive material remaining after t time is

$$N(t) = N_0 \cdot e^{-at} \quad ; a > 0, t \geq 0, N_0 > 0.$$

How to graph a function $N(t) = N_0 \cdot e^{-at}$? Take

$$t = 0 \quad N(0) = N_0 > 0$$

$$t = 1 \quad N(1) = N_0 \cdot e^{-a}$$

$$t = 2 \quad N(2) = N_0 \cdot e^{-2a}$$

$$t = 3 \quad N(3) = N_0 \cdot e^{-3a}$$

Because $a > 0$, so $N_0 > N_0 \cdot e^{-a} > N_0 \cdot e^{-2a} > N_0 \cdot e^{-3a} > \dots$

If time $t \rightarrow \infty$ so $N(\infty) \rightarrow 0$. An image of the function graph is given in Figure 4.

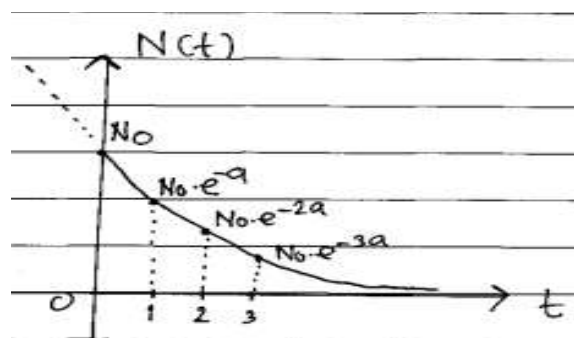


Figure 4. Curve $N(t) = N_0 \cdot e^{-at}$; $a > 0$.

3.4 Questionnaire Results Analysis

The community service process was recorded in the form of a three-question questionnaire. Table 2 provides concise answers to these questions.

Table 2: Questionnaire Answers

Question	Answer
Did you gain any new learning experiences after participating in this community service? Please explain!	Becoming familiar with the application of exponential and logarithmic functions (5 students). Became aware of how to use derivatives and integrals together (6 students) Becoming more complete and complex (9 students)
Do you want this service to continue? If so, in what way?	Continuous guidance (10 students). Direct tutoring and via zoom or g-meet (10 students)
Are you interested in conducting mathematical research and participating in the Indonesian Student Research Olympiad (OPSI)?	Interested in participating in the OPSI Olympiad (6 students) Want to know the application of mathematics with technology (6 students) Don't know yet (8 students)

Analysis of the questionnaire results demonstrates the need to continue this short community service activity with regular, periodic, and ongoing activities, both in-person and online. Student responses also suggest that some students are ready to continue and participate in OPSI. This demonstrates the emergence of self-confidence as a result of the case simulation-based learning model. These analysis findings align with the findings of Fardani et al. (2021).

The use of technology can be done with Excel or spreadsheets and students can be introduced to data collection techniques using the Publish or Perish (PoP) application as well as data analysis using bibliometric analysis assisted by VOS-Viewer.

5. Conclusion

This community service program successfully opened students' minds to opportunities to participate in competitions. Most students wanted the program to continue, and several expressed their readiness to participate in OPSI.

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