A memory improved proportionate affine projection algorithm for sparse system identification

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Article Info

ABSTRACT

For cluster sparse system identification, it is known that the cluster sparse improved proportionate affine projection algorithm (CS-IPAPA) outperforms the standard IPAPA. However, since CS-IPAPA does not retain past proportionate factors, its performance can be further improved. In this paper, a modification to CS-IPAPA is proposed by utilizing the past instant proportionate elements based on its projection order. Steady-state performance of the proposed memory cluster sparse improved proportionate affine projection algorithm (MCS-IPAPA) is studied by deriving the condition for mean stability. Different simulation setups show that the proposed algorithm outperforms different versions of IPAPA in terms of convergence rate, normalized misalignment (NM) and tracking, for different types of inputs like colored noise, white noise, and speech signal. By incorporating past proportionate factors, the proposed MCS-IPAPA significantly reduces computational complexity for higher projection orders.

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1. INTRODUCTION

Sparse system identification has gained much importance in the domain of adaptive signal processing. The system is said to have a sparse impulse response if the number of active or large coefficients is very significantly lesser than that of inactive or small coefficients [1], [2]. In a block-sparse or cluster-sparse system, the large coefficients are grouped in clusters or blocks. In proportionate normalized least mean square (PNLMS), the chosen step-sizes or adaptation gains are proportional to the individual filter coefficients, showing higher performance for sparse systems [3]. However, the PNLMS, shows poor performance for Speech and colored noise inputs [4], [5]. Over the years, several variants of PNLMS have been developed [6]–[9].

Affine projection algorithm (APA), a input-data reusing algorithm, exhibits good performance even for the colored noise and speech inputs [10], [11]. Several variants of the proportionate APA (PAPA) have been introduced in the last two decades. Improved PAPA shows good performance even for dispersive paths [12]–[15]. In memory IPAPA (MIPAPA), the proportionate elements track record was exploited to gain performance over the IPAPA [12]. In improved MIPAPA (IMIPAPA) [13], l0 norm was introduced to MIPAPA as a measure of sparsity, showing improved convergence performance over the MIPAPA.

Additionally, the memorized proportionate elements benefit computational complexity. The impulse response is segmented into many blocks, modifying PAPA or IPAPA update equation with l2,1 penalty [14] to give block sparse PAPA (BS-PAPA) or Block Sparse IPAPA (BS-IPAPA) that results in better convergence rate, tracking, reduced misalignment. In cluster sparse PAPA (CS-PAPA) or cluster sparse CS-IPAPA [16], [17], the norm l2,0 penalizes the PAPA or IPAPA as in CS-PNLMS or CS-IPNLMS to estimate sparse channels. It results in better convergence performance and tracking, and less misalignment. The improvisation of CS-PAPA by exploiting memory and its analyses was carried out in [17]. The l0-norm BS-PAPA (l0-BS-PAPA) [18] was introduced by incorporating the l0-norm sparsity measure into BS-PAPA. The added penalty l0-norm helps in the shrinkage of inactive coefficients thereby producing higher convergence rate than the BS-PAPA. Several improved versions of the PAPA and the IPAPA were presented over the years [19]–[23].

Memory CS-PAPA extends the idea of memory concept to the CS-PAPA. The performance of the MCS-PAPA is higher in terms of tracking, convergence rate and the misalignment, than the CS-PAPA. This research work is the sequel of [17] for further improvement of CS-IPAPA. In this paper, motivated by [12], [17], we propose the Memory CS-IPAPA (MCS-IPAPA) by extending the idea of memory in proportionate factors to the CS-IPAPA. In contrast to the CS-IPAPA, the past history of proportionate elements is incorporated in CS-IPAPA to enhance its performance.

The main contributions of this research paper are:
a. The manuscript is novel in the sense, for the first time, the memory characteristics of the proportionate coefficients are incorporated in an Improved proportionate affine projection algorithm, for cluster sparse channels. The mathematical analysis for the update equation of the proposed algorithm MCS-IPAPA is fully presented.
b. Steady- state performance study of the MCS-IPAPA is derived. The condition for the mean stability is derived to predict the steady-state performance. The condition shows that the mean stability depends on the input power level,
c. With different inputs, the superior performance of the proposed is shown over the competing algorithms like the BS-IPAPA and CS-IPAPA.
d. In terms of number of the multiplications, additions, divisions, memory spaces and comparisons, the time complexity of the proposed MCS-IPAPA is compared against existing algorithms. The proposed algorithm shows significant reduction in number of multiplications for higher projection order.

In this paper, lower case symbols in boldface and uppercase symbols in boldface are adopted for column vectors and matrices, *i.e.* x and X, respectively. Also, for scalars like, normal font lower case symbols are used. To denote the time dependency of scalars and vectors like $e(l)$ and $e(l)$, parentheses or round brackets are employed. The following notations are taken up in his research article: i) $(,)T$: Transpose of a vector; ii) $E(.)$: Expectation or statistical mean; iii) $||.||$: Euclidean norm of a vector; iv) $||x||_A^2$: Generalized inner product; and v) I_p: Identity matrix of dimension pxp.

The rest of the paper is organized as follows: section 2 briefly reviews the conventional IPAPA and CS-IPAPA. The proposed algorithm MCS-IPAPA and its update equation derivation part are presented in section 3. In section 4, the condition for mean-stability is derived. Section 5 presents the several simulation experiments carried out and the results are illustrated. The computational complexity and the transient performance of the proposed algorithm are studied in section 6. Finally, section 7 concludes the research paper.

2. BRIEF REVIEW OF IPAPA AND CS-IPAPA

The roadmap to the research work is given by the theoretical framework by presenting the exiting relevant theories in the literature. The echo cancellation is a challenging sparse system identification problem in which the canceller models the impulse response of the echo path. A typical modelling of echo canceller is shown in Figure 1. Here, the impulse response of the unknown echo path is given by h of length K. The signal at the far-end is given by its snapshot taken at time instant l as $x(l) = [x(l), x(l-1), x(l-2), ..., x(l-K+1)]^T$. Then the desired signal is expressed below, as the sum of the output of the unknown echo path and the near-end or additive noise a(l) [2],

$$d(l) = x^T(l)h + a(l) \tag{1}$$

The echo path is estimated by the adaptive filter as $i(l-1) = [i_0(l-1), i_1(l-1), ..., i_{K-1}(l-1)].^T$ The output of the adaptive filter is given by (2)

$$z(l) = x^T(l)i(l-1). \tag{2}$$

Then, the error in the estimation of the echo path is
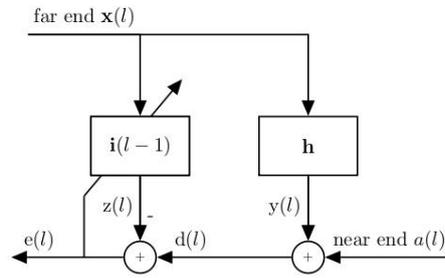
$$e(l) = d(l) - z(l) \tag{3}$$

Figure 1. An Echo canceller model

2.1. IPAPA

To further exploit the sparsity and improve the speed of convergence, the PAPA or the IPAPA recycles the input signal. The input data matrix is given as in [11].

$$X(l) = [x(l), x(l-1), \dots, x(l-\mathrm{p}+1)] \tag{4}$$

where 'p' denotes the projection order of IPAPA and p<K. The output vector, the desired signal vector, and the error vector of IPAPA are expressed as (5)-(7):

$$z(l) = \mathrm{X}^T(l)\mathrm{i}(l-1), \tag{5}$$

$$\mathrm{d}(l) = [d(l), d(l-1), \dots, d(l-\mathrm{p}+1)]^T \tag{6}$$

$$\mathrm{e}(l) = \mathrm{d}(l) - \mathrm{z}(l) \tag{7}$$

where d(l), y(l) and e(l) represent the desired signal vector, the output vector, and the error vector, respectively. Then the updated equation of the IPAPA is expressed based on [24] as (8):

$$\mathrm{i}(l) = \mathrm{i}(l-1) + \mu \mathrm{U}(l-1)\mathrm{X}(l)\left(\mathrm{X}^T(l)\mathrm{U}(l-1)\mathrm{X}(l) + \delta_{\mathrm{IPAPA}}\mathrm{I}_\mathrm{p}\right)^{-1}\mathrm{e}(l) \tag{8}$$

where $\mathrm{U}(l), \delta_{\mathrm{IPAPA}}, and$ μ denotes respectively, the proportionate matrix, regularization parameter, and the step size of IPAPA. Then U(l) is expressed based on [24] as (9):

$$\mathrm{U(l)} = \mathrm{diag}\{\mathrm{u}_0(\mathrm{l}), \mathrm{u}_1(\mathrm{l}), \dots, \mathrm{u}_{\mathrm{K-1}}(\mathrm{l})\} \tag{9}$$

where the elements are given by (10),

$$u_f(l) = \frac{(1-\alpha)}{2} + \frac{(1+\alpha)\|i_f(l)\|_2}{2\sum_{n=0}^{K-1}\|i_n(l)\|_2}, \quad 0 \le f \le K\text{-}1 \tag{10}$$

where α is a constant and is often selected between -1 and 1. The IPAPA acts like PAPA for α close to 1. If α=-1, IPAPA is same as APA. A good selection for α is either 0 or -0.5 [4].

2.2. CS-IPAPA

By incorporating a mixed-norm $l_{2,0}$ penalty into the IPAPA, the CS-IPAPA is presented, favoring the cluster-sparse characteristic of the echo path channel [16]. This algorithm is based on the fact that the l_0-norm is a better choice for exploiting the sparse characteristic than the l_1-norm. Here, the l_2-norm is used for segregating the channel into clusters of equal size. The update equation of the algorithm CS-IPAPA is same as the CS-PAPA, except for the definition of the gain distribution matrix.

$$\|_{2,0} = \left\| \begin{bmatrix} \|\mathrm{i}_{[0]}\|_2 \\ \|\mathrm{i}_{[1]}\|_2 \\ \vdots \\ \|\mathrm{i}_{[M-1]}\|_2 \end{bmatrix} \right\|_0 \tag{11}$$

In (11), 'M' denotes the number of clusters in the echo path channel, $i.e$, M=K/Q and 'Q' is the number of coefficients per cluster or cluster size.

Then the l_0-norm approximation [25] of the echo path or the weight estimate vector is

$$\|i(l)\|_0 \approx \sum_{t=0}^{M-1} (1-e^{-\beta\|i_{[t]}\|}),$$ (12)

and 'β' should be always greater than zero. Then, the $l_{2,0}$-norm approximation [16] of the echo path or weight estimate vector is

$$\|i(l)\|_{2,0} \approx \sum_{t=0}^{M-1} (1-e^{-\beta\|i_{[t]}\|_2}),$$ (13)

The update equation of the CS-IPAPA is as (14) [16].

$$i(l)=i(l-1)+\mu U_{CI}(l-1)X(l)(X^T(l)U_{CI}(l-1)X(l)+\delta_{CI}I_p)^{-1}e(l).$$ (14)

where δ_{CI} is the regularization parameter for CS-IPAPA. The diagonal matrix $U_{CI}(l-1)$ is

$$U_{CI}(l-1)=diag[u_0(l-1)1_Q,u_1(l-1)1_Q,..,u_{M-1}(l-1)1_Q],$$ (15)

wherein 1_Q is a row vector of Q ones. The tth cluster has the gain element $u_t(l-1)$ and is given by (10). Although CS-IPAPA demonstrates better tracking and convergence rates [16] compared to MIPAPA, IPAPA, and BS-IPAPA, it does not consider past proportionate elements when updating each adaptive filter coefficient, relying solely on the current time step.

3.   METHOD

The memory CS-IPAPA is introduced in this section, by extending the concept of memory of the proportionate elements to the CS-IPAPA. Because the algorithm APA considers the history of past 'p' moments of proportionate elements, it can be considered as an adaptive algorithm with memory, the history of the last 'p' proportionate elements is taken into account for updating each filter coefficient. Recursive implementation of the proportionate elements can be achieved using this approach. This approach leads to a significant reduction in the computational complexity in terms of multiplications for higher values of projection order [12]. The technique employed in this research work is incorporating memory to proportionate elements of CS-IPAPA that can improve its convergence performance.

Thus, the proposed MCS-IPAPA is derived by first starting with the optimization problem, then deriving the filter updating equation of the CS-IPAPA from the basis-pursuit and the method of Lagrange multipliers. Then, introducing the concept of memory into the cluster-sparse channel favors performance characteristics improvement and a reduction in number of multiplications.

The CS-IPAPA seeks an optimum solution for the following optimization problem.

$$\min_{i(l)} \frac{1}{2} \|i(l)-i(l-1)\|^2_{U_{CI}^{-1}(l-1)} \quad s.t \; d(l)-X^T(l)i(l)=0$$ (16)

where $U_{CI}^{-1}(l-1)$ is defined by (15). The above optimization (16) is based on the concept proposed in [26], [27]. A constrained optimization (16) is transformed into an unconstrained optimization problem by the method of Lagrange multipliers [26], [27] with many constraints.

The cost function of the CS-IPAPA, $i.e.$ J(l) is

$$J(l)=\frac{1}{2}[i(l)-i(l-1)]^T U_{CI}^{-1}(l-1)[i(l)-i(l-1)]+[d(l)-X^T(l)i(l)]^T\lambda(l),$$ (17)

where $\lambda(l)=[\lambda_0(l),\lambda_1(l),...\lambda_{p-1}(l)]^T$ is the Lagrange multiplier vector and $[i(l)-i(l-1)]^T U_{CI}^{-1}(l-1)[i(l)-i(l-1)]$ denotes the Riemannian distance between i(l) and i($l-1$).

Equating the first derivatives of the cost function to zero to give,

$$\frac{\partial J(l)}{\partial i(l)}=0,$$ (18)

$$\frac{\partial J(l)}{\partial \lambda(l)} = 0, \tag{19}$$

From (18), the updating equation of the CS-IPAPA becomes,

$$i(l) = i(l-1) + U_{CI}(l-1)X(l)\lambda(l). \tag{20}$$

The derivative in (19) gives,

$$d(l) - X^T(l)i(l) = 0. \tag{21}$$

As in [17], to further reduce the computational complexity, the cluster-sparse feature of the channel and sliding window technique can be included in (20). For inclusion of cluster-sparse feature, let $x_Q(l-tQ) = [x(l-tQ), x(l-tQ-1),.., x(l-tQ-Q+1)]^T$, with t= 0,1,.., M-1. The tQ term indicates the product of t and Q.

Then (20) becomes

$$i(l) = i(l-1) + P_{CI}(l)\lambda(l). \tag{22}$$

where $P_{CI}(l)$ is a K× p matrix, whose elements are determined by the cluster wise product of U and X. The subscript CI on $P_{CI}(l)$ implies CS-IPAPA.

Pre-multiplying (22) by $X^T(l)$ to get

$$X^T(l)i(l) = X^T(l)i(l-1) + X^T(l)P_{CI}(l)\lambda(l). \tag{23}$$

The error vector e(l) is

$$e(l) = d(l) - X^T(l)i(l-1) \tag{24}$$

Using (21) and (24) into (23) to obtain $\lambda(l)$ as (25).

$$\lambda(l) = (X^T(l)P_{CI}(l))^{-1}e(l). \tag{25}$$

Substituting (25) into (22), the CS-IPAPA updates filter coefficients as (26).

$$i(l) = i(l-1) + P_{CI}(l)(X^T(l)P_{CI}(l))^{-1}e(l). \tag{26}$$

Introducing the parameters like regularization parameter δ_{CI} and convergence rate μ in (31) gives control over the avoidance of numerical difficulty and the weight vector increment, respectively. Then, the update equation of CS-IPAPA becomes

$$i(l) = i(l-1) + \mu P_{CI}(l)(X^T(l)P_{CI}(l) + \delta_{CI}I_p)^{-1}e(l) \tag{27}$$

Direct computation of $U_{CI}(l-1)X(l)$ needs pK multiplications. By making use of the cluster-sparse feature of the channel, we can minimise the number of multiplications required, especially for higher values of projection order. $P_{CI}(l)$ just requires (Q+p-1) M multiplications.

As in [12], [17], the history of proportionate elements can be included to further decrease the computational complexity of CS-IPAPA. The matrix $P_{CI}(l)$ is selected in terms of the diagonal matrix $U_t(l-1)$ with t= 0,1,⋯, M-1. Choosing $U\_t(l-1) = u_t(l-1)I_p$, where $u_t(l-1)$ is the gain term for the tth cluster from (15). Expanding $P_{CI}(l)$ gives (3.32), where u($l-1$) has the gain terms for M clusters. The ⊙ operator denotes Hadamard product or element-wise multiplication, *i.e,* b⊙c=[b(1)c(1), b(2)c(2),⋯,b(K)c(K)]^T, where b and c are vectors of length K.

By using a modified gain matrix with t=0,1, ⋯, M-1,

$$U_t(l-1) = diag[u_t(l-1), u_t(l-2), \ldots, u_t(l-p)], \tag{28}$$

the history or memory of proportionate gain elements of M clusters can be incorporated in the CS-IPAPA. By this way, to iterate i(l) to i($l+1$), the last 'p' proportionate elements are considered. Thus, the matrix $P_{CI}(l)$ becomes $P'_{CI}(l)$ as shown in the (30).

$$P_{CI}(l)=[u(l-1)\odot x_Q(l\text{-}tQ) \ u(l-1)\odot x_Q(l\text{-}tQ\text{-}1) \cdots u(l-1)\odot x_Q(l\text{-}tQ\text{-}p+1) \ ] \tag{29}$$

$$P'_{CI}(l)=[u(l-1)\odot x_Q(l\text{-}tQ) \ u(l-2)\odot x_Q(l\text{-}tQ\text{-}1) \cdots u(l-p)\odot x_Q(l\text{-}tQ\text{-}p+1)] \tag{30}$$

3.1. Proposed algorithm

The proposed algorithm MCS-IPAPA has two advantages because of this modification. Firstly, as the proposed MCS-IPAPA takes into account the last 'p' proportionate elements, the convergence rate and tracking improve as 'p' increases. Another advantage is that the computational complexity of the proposed is lower than that of the CS-IPAPA. This advantage in terms of computational complexity is shown in Table 1. The equation of $P'_{CI}(l)$ is expressed in recursive realization as (31),

$$P'_{CI}(l)=\left[u(l-1)\odot x_Q(l\text{-}tQ)P'_{-1}(l-1)\right] \tag{31}$$

where in the matrix $P'_{-1}(l-1)$ is, $P'_{-1}(l-1)=[u(l-2)\odot x_Q(l\text{-}tQ\text{-}1)\cdots u(l-p)\odot x_Q(l\text{-}tQ\text{-}p+1)]$.

By substituting $P_{CI}(l)$ with $P'_{CI}(l)$ in (27), the update equation of the proposed algorithm is given by

$$i(l)=i(l-1)+\mu P'_{CI}(l) \ (X^T(l)P'_{CI}(l) +\delta_{CI}I_p)^{-1}e(l) \tag{32}$$

The matrix $P'_{-1}(l-1)$ has the first p-1 columns of $P'_{CI}(l-1)$. The first p-1 columns of $P'_{CI}(l-1)$ can be utilized directly for computing the matrix $P'_{CI}(l)$, thereby saving computations. For $P'_{CI}(l-1)$, the sliding window technique cannot be applied. From (34), it requires pK multiplications, to find $P_{CI}(l)$. However, from (30), to compute $P'_{CI}(l)$, it requires 'K' multiplications only. This advantage in computation becomes significant for higher projection order 'p'. Thus, from the cost function of the CS-IPAPA, the update equation of the proposed algorithm is derived. The condition for mean-stability is derived in the next section.

4. STABILITY OF MCS-IPAPA

The stability of the MCS-IPAPA is studied in this section. step-size μ plays a major role to ensure the convergence of any kind of adaptive algorithm. The higher the value of the step size, the higher the possibility of the adaptive algorithm to diverge from the optimum solution. Therefore, it is highly important to study the range of μ that confirms the convergence in the mean. *i.e.*, $E(\tilde{\imath}(l))\to 0$ as $l \to \infty$. This section finds the range of step-size that ensures stability in the mean. The weight-error vector is stated as $\tilde{\imath}(l)=h\text{-}i(l)$. In terms of the $\tilde{\imath}(l)$, the updating equation of the MCS-IPAPA, *i.e.*, (32) becomes.

$$\tilde{\imath}(l)=\tilde{\imath}(l-1)-\mu P'_{CI}(l)(X^T(l) P'_{CI}(l)+\delta_{CI}I_p)^{-1}e(l). \tag{33}$$

Substituting $e(l)=a(l)+ X^T(l)\tilde{\imath}(l-1)$ in (33).

$$\text{yields } \tilde{\imath}(l)=\tilde{\imath}(l-1)-\mu P'_{CI}(l)(X^T(l)P'_{CI}(l)+\delta_{CI}I_p)^{-1}\times(a(l)+ X^T(l)\tilde{\imath}(l-1)) \tag{34}$$

The following valid assumptions are made to make convergence analysis tractable [28]–[30].
Assumption 1. The noise $a(l)$ is assumed to be a zero-mean WGN.
Assumption 2. The weight-error vector $\tilde{\imath}(l)$, the input vector $x(l)$, and the noise $a(l)$ are statistically independent of each other.
Assumption 3. The algorithm converges to optimum in the mean-square sense.

By taking statistical mean or expectation on both sides of (34) to give

$$E(\tilde{\imath}(l))=E(\tilde{\imath}(l-1))-\mu E((P'_{CI}(l)(X^T(l)P'_{CI}(l)+\delta_{CI}I_p)^{-1}a(l))-\mu E((P'_{CI}(l)(X^T(l)P'_{CI}(l)+\delta_{CI}I_p)^{-1} X^T(l)\tilde{\imath}(l-1)) \tag{35}$$

Using Assumption 2 in (35), the evolution of the mean of the weight-error vector is given by (36).

$$E(\tilde{\imath}(l))=(I-\mu B)E((\tilde{\imath}(l-1)), \tag{36}$$

where $B=E(P'_{CI}(l)(X^T(l)P'_{CI}(l)+\delta_{CI}I_p)^{-1} X^T(l))$. $E((\tilde{\imath}(l-1))$ can converge if $\varphi(I-\mu B) <1$ wherein $\varphi(.)$ represents the spectral radius of the B.

By Eigenvalue decomposition,

$$B = W \Lambda W^T, \tag{37}$$

which leads to

$$I_K - \mu B = W(I_K - \mu \Lambda)W^T \tag{38}$$

The above equation becomes

$$\Phi(I_K - \mu B) = \Phi(I_K - \mu \Lambda) < 1 \tag{39}$$

where W is the eigenvector and Λ is a diagonal matrix having the eigenvalues of R. From (39), it can be seen that the convergence in the mean sense is ensured or guaranteed for the following range of μ.

$$0 < \mu < \frac{2}{\varphi(B)} \tag{40}$$

The above condition on μ in (40) gives the necessary and sufficient condition for the proposed MCS-IPAPA to be stable.

5. SIMULATION RESULTS AND DISCUSSION

Several simulation experiments are carried out to evaluate the performance of the proposed algorithm MCS-IPAPA. While performing simulations, the length of the adaptive filter N is set to 1024, and the unknown system is assumed to have the same length. The performance of the proposed algorithm is compared with that of existing algorithms like the IPAPA, MIPAPA, BS-IPAPA, and the CS-IPAPA with respect to convergence rate, NM, and tracking. Two different types of clusters, as shown in Figure 2, are used. Figure 2(a) is a single cluster which has 32 non-zero active taps in the range [281, 312] and a double cluster is shown in Figure 2(b), which has 64 non-zero taps, 32 each in the ranges [281, 312] and [793, 824]. For the network echo path and satellite echo path, the impulse response is regarded as a single cluster and a double cluster sparse system, respectively. In all the experiments, to study the tracking behavior of the proposed MCS-IPAPA, first, a single cluster is utilized and then abruptly, the simulation environment is shifted to a double cluster.

Colored noise, white gaussian noise (WGN), and speech are the three different input signals that are used in the simulations. By filtering the WGN through a system of first-order with a pole at 0.8, colored noise is obtained. A speech signal in real-time sampled at 8 kHz is utilized. An independent WGN with a signal-to-noise ratio, SNR=30 dB is added to the background of the unknown system. A common performance metric for adaptive algorithms is NM which is defined as $10\lg\{(\|h-i\|)^2/(\|h\|)^2\}$. In all the simulations, the entire process is divided into two halves, the first half allotted for the performance study of a single cluster and the second half for the double cluster. In the following sections, the impact of varying cluster size 'Q' and the parameter 'β' on the performance of the proposed is studied. Then the performance comparison study is done by comparing the proposed MCS-IPAPA with the existing algorithms in the literature.
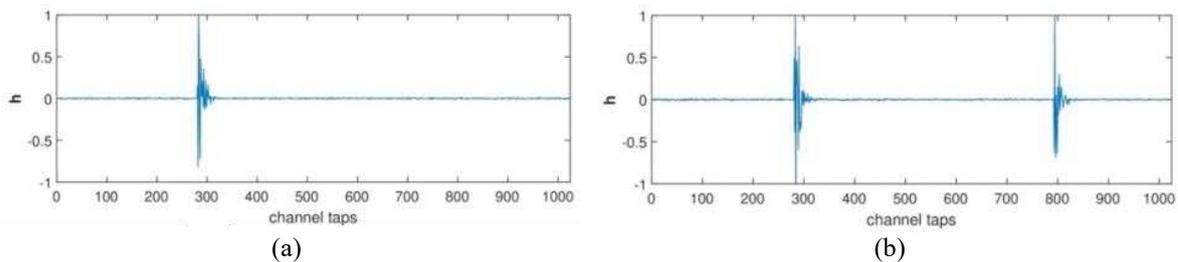

(a)   (b)

Figure 2. Cluster-sparse channel with its types (a) a single cluster and (b) a double cluster

5.1. Performance curves of MCS-IPAPA with different β

With inputs like colored noise, WGN and speech signal, the effect of varying 'β' on the performance of the proposed MCS-IPAPA is studied, with cluster size Q, set to 2. Low, medium and high values of β are chosen as in [16]. Figure 3 shows the effect of β on MCS-IPAPA. For different values of β like 2, 5, 10, and 20, the simulation results are shown in Figures 3(a) and 3(b) for colored noise and WGN, respectively.

The proposed MCS-IPAPA shows a reduction in NM with an increase in β for single and double cluster systems, for colored noise and WGN inputs. If β=10, the MCS-IPAPA shows the best NM for both the inputs. The MCS-IPAPA shows higher misalignment for other values of β. But, for speech signal, the MCS-IPAPA shows a decrease in NM with a decrease in β for both single and double cluster systems, attaining the minimum NM for β=2. So, for the performance comparison of the proposed with the algorithms like the IPAPA, MIPAPA, BS-IPAPA, and the CS-IPAPA, the parameter β is selected as 10 for colored noise and WGN, β as 2 for the speech signal.
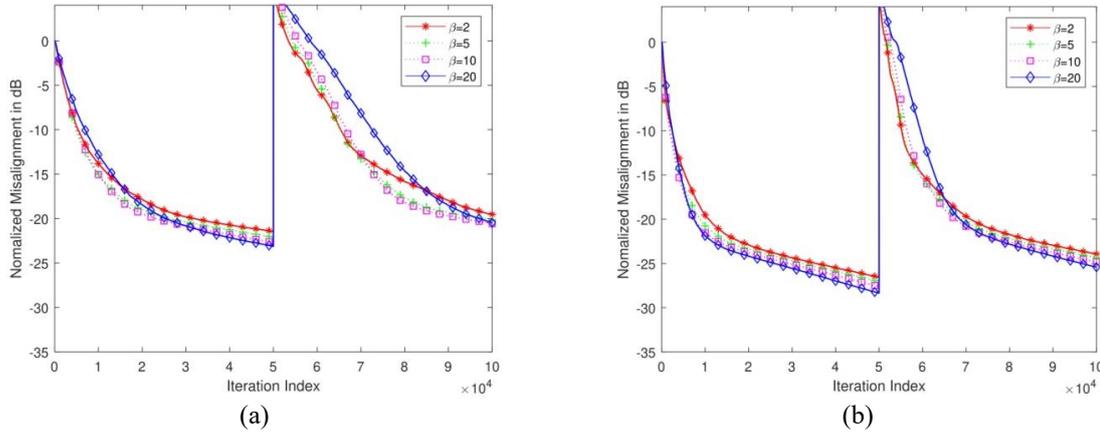

Figure 3. Effect of β with different values on MCS-IPAPA (μ=0.01; SNR=30 dB; p=2) (a) colored input and (b) WGN input

5.2. Performance curves of MCS-IPAPA with different cluster size Q

Varying the cluster size Q can affect NM. Low, medium and high values of Q are chosen as in [16]. Clusters of different sizes of Q=2, 4, 8, 16 are selected to study and evaluate the effect of Q on the performance characteristics of the MCS-IPAPA. For these studies, the parameter β is set to a value of 10 for both colored noise and WGN, 2 for speech input. Figure 4 shows the effect of varying Q on MCS-IPAPA. The results are shown in Figures 4(a) and 4(b) for colored noise and WGN input, respectively.
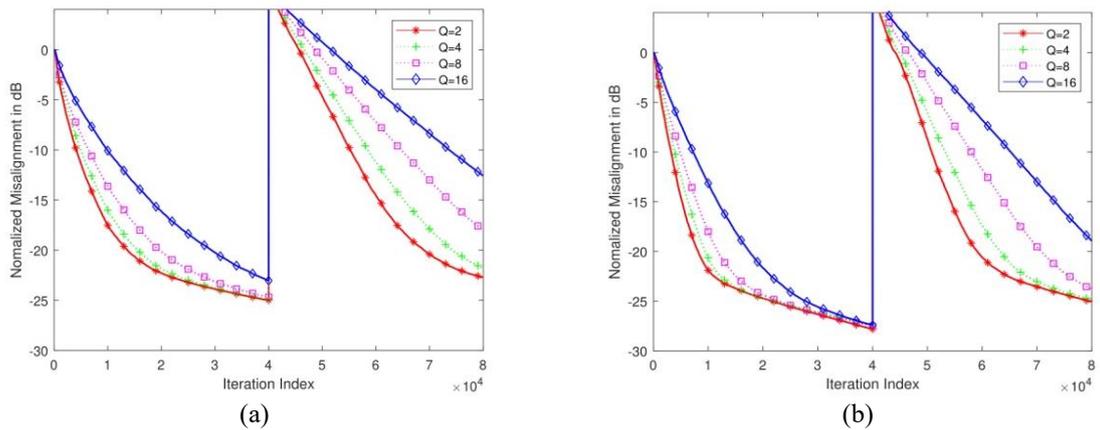

Figure 4. Effect of Q with different values on MCS-IPAPA (μ=0.01; SNR=30 dB; p=2) (a) colored input (pole at 0.8) and (b) WGN input

Figure 5 shows the parameter variation on MCS-PAPA with speech input. Figures 5(a) and 5(b) depict the effect of β and Q on MCS-IPAPA, respectively. The NM of the proposed MCS-IPAPA decreases with cluster size, for all three inputs, for both single and double clusters. The MCS-I PAPA shows the least NM for Q=2. Thus, the two parameters 'β' and 'Q' result in the performance improvement of the MCS-

IPAPA in terms of NM. By selecting suitable values to β and the cluster size Q, the MCS-IPAPA is shown to outperform the competing algorithms like the IPAPA, MIPAPA, BS-IPAPA, and the CS-IPAPA for identifying the single or double cluster channels.
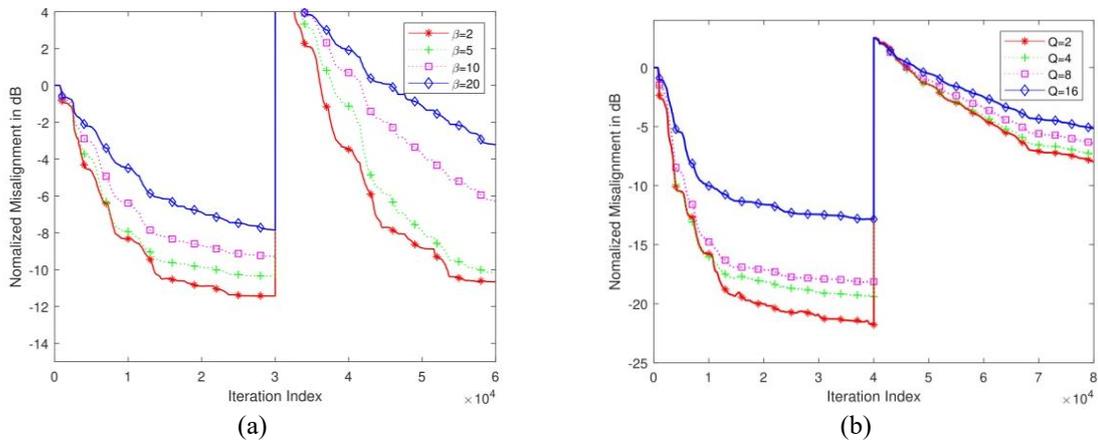

Figure 5. Effect of different β and Q on MCS-IPAPA with speech input (μ=0.02; SNR=30 dB; p=2): (a) varying β and (b) varying Q

5.3. Performance curves of MCS-IPAPA against existing algorithms

Figure 6 shows the performance curves of MCS-IPAPA and other algorithms. The performance evaluations are illustrated in Figures 6(a) and 6(b), with colored noise input and WGN input, respectively. With speech input, Figure 7 depicts the performance curves of MCS-IPAPA and other algorithms. For SNRs 30 dB and 15 dB, the performance evaluation plots are shown in Figures 7(a) and 7(b) respectively. The results of simulation trials that are independently repeated 15 times are ensemble averaged to obtain the plots. From the obtained plots, it can be seen that the MCS-IPAPA is showing higher performance than the algorithms like the MIPAPA, IPAPA, BS-IPAPA, and the CS-IPAPA. For the colored noise and WGN input, the parameter β is maintained at 10, for the MCS-IPAPA and β is set to 2 for the speech input. For all the three cluster or block algorithms namely the MCS-IPAPA, CS-IPAPA, and the BS-IPAPA, the same cluster size of Q=2 is assigned. The step-size for each algorithm is set to μ=0.01 for colored noise and WGN input, but μ is set to 0.02 for speech signal input. The regularization parameter for IPAPA is chosen as δ=0.01. The regularization parameter is set to 0.01 for the other four algorithms as well. The parameter α is chosen as 0. With β=10 and Q=2, the proposed MCS-IPAPA, for colored noise and WGN inputs, shows better tracking, lesser NM than the existing algorithms. Figures 6(a) and 6(b) show that the MCS-IPAPA shows higher convergence rate, better tracking, lesser NM, than the existing algorithms.
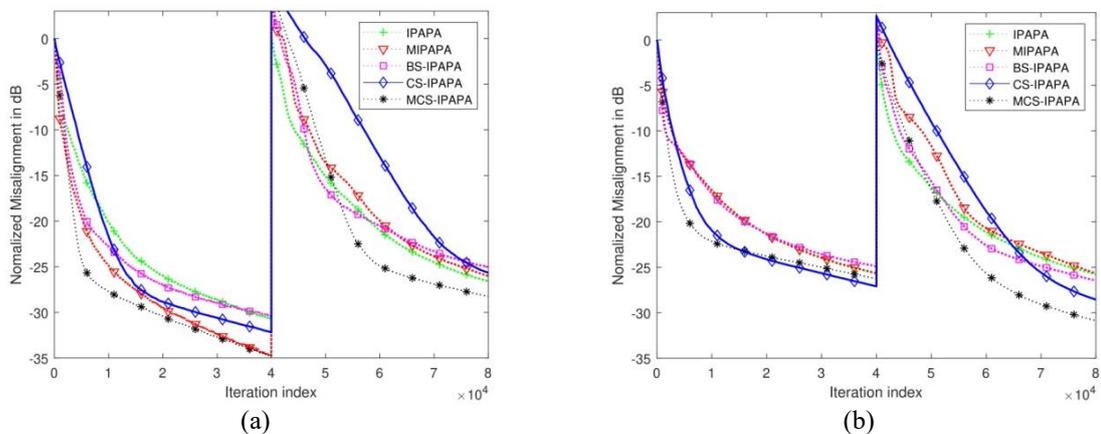

Figure 6. Performance curves of MCS-IPAPA and other algorithms (μ=0.01; SNR=30 dB; p=2): (a) colored input (pole at 0.8) (b) WGN input

From Figures 7(a) and 7(b), it is shown that the MCS-IPAPA shows higher convergence rate, better tracking, and lesser NM, than the CS-IPAPA for speech signal input as well. In Figure 7(b), improvement in NM is 20% and 29%, shown by MCS-IPAPA over the CS-IPAPA, for single cluster in [1000, 6000] and double cluster in the iteration range [41000, 47000], respectively. From the above discussed analyses, the chosen parameters of β and Q show higher performance for the MCS-IPAPA in terms of convergence rate, NM, and tracking. Hence, the simulation plots prove that the proposed MCS-IPAPA outperforms the existing algorithms like IPAPA, MIPAPA, BS-IPAPA, and CS-IPAPA for single and double cluster channels.
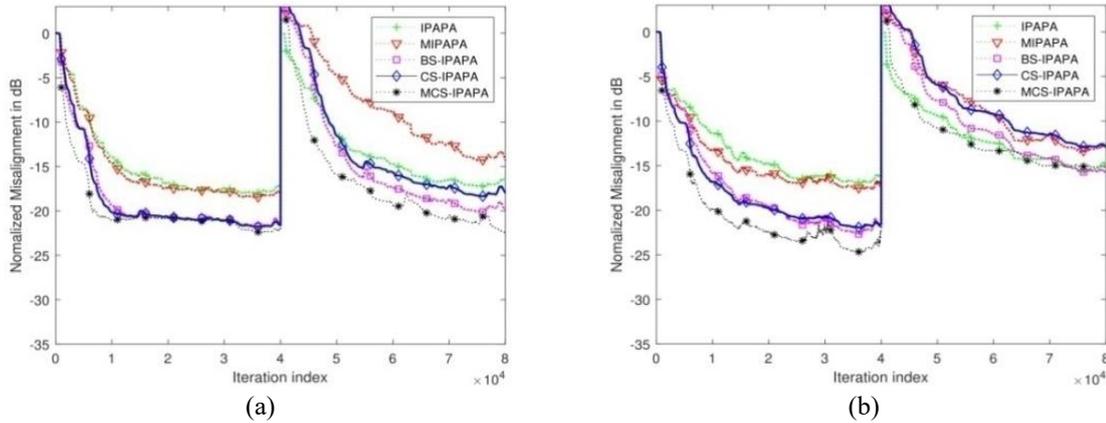

Figure 7. Performance curves of MCS-IPAPA and other algorithms for speech input (μ=0.02; p=2):
(a) SNR=30 dB and (b) SNR=15 dB

5.4. Performance curves for higher projection order and dispersive systems

In this section, a study is extended for higher projection order and dispersive system. Figure 8 shows the performance curves of algorithms for higher projection order and dispersive system. With speech as input, Figures 8(a) and 8(b) show the performance comparison of the MCS-IPAPA, over the CS-IPAPA, and the MIPAPA for higher projection order and dispersive system, respectively. By ensemble averaging of 10 independent trials, the simulation plots are obtained. The IPAPA behaves almost like an NLMS algorithm for $p = 2$. So, it is highly necessary to check the performance of the proposed MCS-IPAPA for higher projection order.

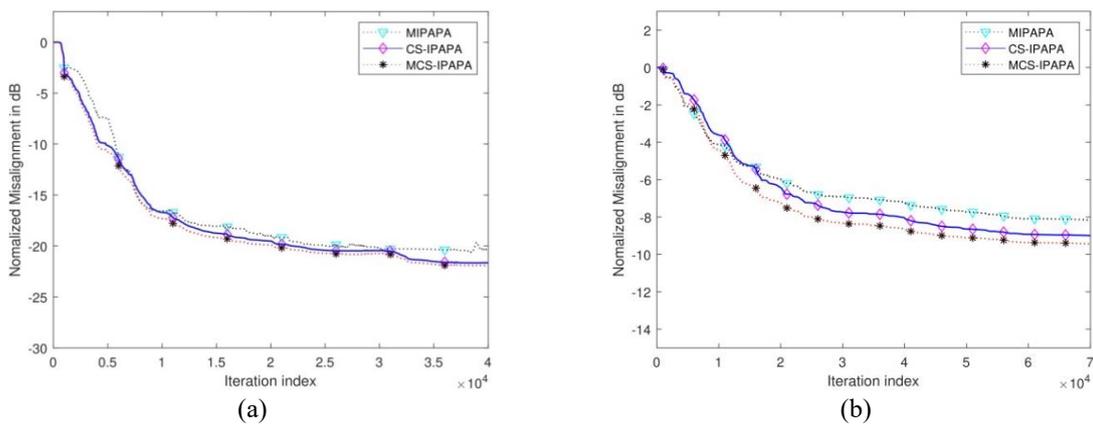Figure 8. Performance curves of MCS-IPAPA, CS-IPAPA and MIPAPA for speech input: (a) higher projection order (p=6, ξ=0.875, μ=0.02) and (b) dispersive system (ξ=0.484, μ=0.05, p=2)

6.   COMPUTATIONAL COMPLEXITY

In Table 1, the computational complexity of the proposed algorithm is compared against different algorithms namely the IPAPA, MIPAPA, BS-IPAPA, CS-IPAPA in terms of the total number of additions

(A), multiplications (MU), divisions (D), and memory spaces (ME). At every iteration, to update from i(l) to i(l+1), the proposed algorithm with length K and projection order 'p' requires (p^2+p+2) K-2 additions, (p^2+5) K+p^2 multiplications, 2 divisions, and K+M+p memory spaces. None of the algorithm listed in the Table 1 require any comparison as against the PAPA or its variant. Both the MCS-IPAPA and MIPAPA need extra 'p' memory spaces for storing the past 'p' proportionate factors. Compared to the IPAPA, the proposed MCS-IPAPA needs an additional memory of 'M' memory spaces for storing the $l_{2,0}$ norm of 'M' clusters. But it can save (p-1)K multiplications compared to the CS-IPAPA. This advantage will be more significant as the projection order increases. Also, as in the algorithm CS-IPAPA, the parameters Q and β have no effect on the computational complexity of the MCS-IPAPA.

Table 1. Computational complexity of MCS-IPAPA and various algorithms: additions (A), multiplications (MU), divisions (D), and memory (ME)

Algorithms	A	MU	D	ME
IPAPA	(p^2+p+1)K	(p^2+p+2)K+p^2	2	K
MIPAPA	(p^2+p+1)K	(p^2+3)K+p^2	2	K+p
BS-IPAPA	(p^2+p+1)K-2	(p^2+p+3)K+p^2	3	K+M
CS-IPAPA	(p^2+p+2)K-2	(p^2+p+4)K+p^2	2	K+M
MCS-IPAPA	(p^2+p+2)K-2	(p^2+5)K+p^2	2	K+M+p

6.1. Transient performance of the MCS-IPAPA against different SNRs

Figure 9 shows the learning curves or MSE curves of the proposed MCS-IPAPA for three different values of SNRs. To evaluate the performance of the MCS-IPAPA, the MSE curves, *i.e.*, 10lgE|e(l)|², are plotted against different values of SNR. The input signal used is a colored noise obtained using AR(1) process with a first-order pole at 0.8. In all the different simulations, the measurement noise added $a(l)$ is a zero-mean WGN with a suitable variance σ_v^2 to obtain the required SNR in dB.

For three different SNRs of 20, 30, and 40 dB, the simulation tests are carried out. The step-size μ is chosen as 0.3. The projection order and the regularization factor are selected as 2 and 0.001, respectively. The expectation or the statistical mean in the MSE expression is estimated by taking an average over 500 independent trials. As can be clearly seen from Figure 9, with an increase in the measurement noise level (as SNR decreases), the MSE increases for the proposed MCS-IPAPA. Also, the MCS-IPAPA with lesser SNR takes lesser number of iterations to reach the steady-state but with end up with a higher steady-state MSE.
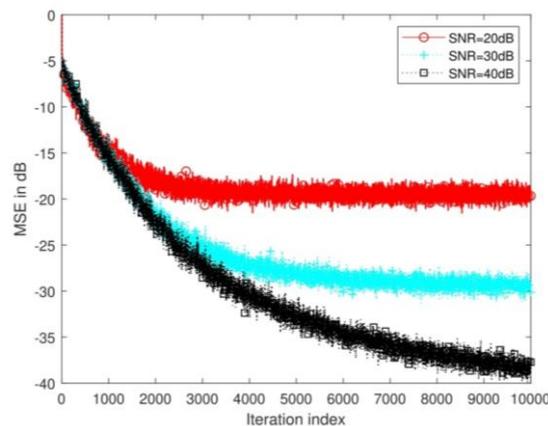

Figure 9. Learning curves of MCS-IPAPA for colored input (μ=0.3, p=2)

7. CONCLUSION AND FUTURE WORK

In this paper, the MCS-IPAPA has been proposed for cluster-sparse system identification. The implication of this research is by adding memory to the CS-IPAPA, its computational complexity is significantly reduced. The performance improvement for the CS-IPAPA is achieved in this work by considering the last 'p 'proportionate elements in estimating the current or present weights of the filter. Adding memory resulted in significant reduction in the number of multiplications and improvement in steady-state and convergence performance. Compared to the CS-IPAPA, the MCS-IPAPA saves pK-K

multiplications, indicating a significant reduction in the number of computations, mainly multiplications, for the higher values of p. Condition to guarantee mean stability was derived. Experimental simulations have been carried out to analyze how the proposed algorithm outperforms the existing algorithms like IPAPA, MIPAPA, BS-IPAPA, and CS-IPAPA in terms of convergence rate, NM, and tracking. With WGN input, colored noise, and speech signal, the proposed algorithm MCS-IPAPA significantly improves NM over CS-IPAPA, in estimating both single and double cluster channel.

Analysis of impact of additional memory usage will be carried out as future work. In addition, future work will focus on optimizing memory selection strategies in MCS-IPAPA. Applying MCS-IPAPA in real-world scenarios such as wireless communication and biomedical signal processing will be explored. Hardware implementation feasibility of MCS-IPAPA on FPGA/ASIC will be studied.

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C	:	Conceptualization	I	:	Investigation	Vi	:	Visualization
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CONFLICT OF INTEREST STATEMENT

The authors declare that there is no conflict of interest.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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