

Multi-Objective Portfolio Optimization Using Hybrid Ant Colony Optimization and Compromise Programming

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ABSTRACT

The increasing complexity of stock trading requires effective portfolio management to optimize returns while minimizing risks. Portfolio selection is critical in determining the most suitable combination of stocks, aiming to maximize expected returns and minimize risk within a given investment limit. This study constructs a mathematical model for portfolio optimization using six different stocks, incorporating constraints such as expected return, risk, and available investment. Given the multi-objective nature of the problem, a hybrid approach is proposed, combining Compromise Programming (CP), Nadir Compromise Programming (NCP), and Ant Colony Optimization (ACO) to address both minimization and maximization objectives. The ACO algorithm is applied to minimize deviation variables, which serve as the fitness function in the optimization process. The results demonstrate the effectiveness of the hybrid method in selecting portfolios that achieve minimal deviation, providing an optimal balance between risk and return. This research offers valuable insights for investors by illustrating the trade-offs between risk and reward in stock selection, contributing to more informed decision-making in portfolio management.



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1. Introduction

Investors generally seek to create portfolios that yield long-term benefits. Stocks, one of the most common assets, fluctuate in prices influenced by market demand and supply over time. Most investors aim to construct portfolios that maximize expected returns while minimizing risk within the constraints of available capital. Given the volatility in stock prices, portfolio selection becomes crucial to balance these objectives. Statistical measures such as return, expected return, and stock risk can be computed based on historical data, providing valuable insights for decision-making [1]. Various studies have explored portfolio models and their modifications. For example, models focusing on portfolio stability and minimizing risk have been developed [2]. Fuzzy preference techniques have been applied to portfolio selection [3], while other studies have constructed efficient portfolios with fewer stocks [4]. Furthermore, development



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costs associated with portfolio selection have been suggested for decision-makers [5]. Predicting expected returns and risk involves forecasting future stock values [6], and synergies between projects have been shown to impact portfolio decisions [7]. Extendable investments have been incorporated into portfolio models [8], and pre-selected assets have been used for portfolio optimization [9]. Additionally, multi-objective portfolio selection models have been proposed, considering variable risk [10] and return distributions [11].

Several previous studies have utilized different methods to estimate stock prices. For example, the Kalman Filter [12] and H-infinity [13] methods rely on predictor and corrector iterations to make estimations. Other approaches include Neural Networks [14] and Adaptive Neuro-Fuzzy systems [15], which train data and then apply the optimized parameters during testing with a set data proportion. The Autoregressive Integrated Moving Average (ARIMA) model [16], leveraging autocorrelation, has also been widely used. All these methods aim to minimize the error between actual stock prices and forecasted data. In this research, we develop a portfolio optimization model using Compromise Programming, which is well-suited for solving multi-objective problems by finding a compromise solution that balances two or more conflicting objectives [17]. The basic principles of Compromise Programming have been applied to a range of problems, including general portfolio selection [18], resource allocation [19], multi-objective shipment problems [20], multi-objective task assignment [21], and energy generation planning [22].

Ant Colony Optimization (ACO) is an optimization method inspired by the behavior of ants in searching for food and navigating their environment using pheromones. This method, developed by Dorigo in the 1990s, simulates how ants traverse through various nodes from their nest to a food source. ACO has been extensively researched and has proven to optimize search paths and resource allocation efficiently [23]. Additionally, ACO has been successfully combined with other algorithms such as Genetic Algorithm (GA) [24], Variable Neighborhood Descent [25], and Simulated Annealing [26] to improve outcomes. Applications of ACO span diverse areas, including traffic management systems [27], resource optimization [28], completion time minimization [29], vehicle routing problems [30], distribution planning [31], and open shop scheduling problems [32].

Previous studies have focused mainly on applying Ant Colony Optimization (ACO) to single-objective optimization problems, either addressing only minimization or maximization, which limits its applicability to more complex, real-world scenarios. Similarly, Compromise Programming has traditionally been solved through analytical methods, resulting in inefficient computations for large-scale problems. These limitations highlight the need for more robust and efficient methods capable of handling multiple objectives simultaneously, particularly in portfolio optimization, where both risk minimization and return maximization are critical. This research introduces a novel hybrid approach combining ACO with Compromise Programming and Nadir Compromise Programming to address these gaps. Given investment constraints, this study aims to develop an optimization model that minimizes portfolio risk while maximizing expected returns. By integrating these methods, the study seeks to overcome the limitations of previous research and provide a more efficient solution to the multi-objective optimization problem in portfolio management. The contributions of this research are twofold: (1) Practically, it offers investors valuable insights into managing the trade-offs between risk and return when selecting stocks, thus enhancing decision-making in portfolio construction; and (2) Theoretically, it demonstrates the potential of

ACO as a metaheuristic that can be effectively applied to multi-objective optimization problems, broadening its scope and applicability beyond single-objective cases.

2. Methods

This study begins with the statistical computation of expected return and risk for a set of stocks [1]. Before determining the expected return, we calculate each stock's return per time unit, as shown in Equation (1). If the return is positive, the stock generates a profit; otherwise, it results in a loss.

$$R_{it} = \frac{S_t - S_{t-1}}{S_{t-1}} \quad (1)$$

Where:

- R_{it} : i-th return of stock when time t
- S_t : the price of the stock when the time t
- S_{t-1} : the price of stock when time $t-1$

The expected return for stock i over the period is calculated as the mean return, or average return, over time. It is expressed in Equation (2). A positive expected return indicates profitability, while a negative value indicates a loss.

$$E(R_i) = \frac{\sum_{t=1}^T R_{it}}{T} \quad (2)$$

Where T is the number of periods.

To compute the risk B_i , we calculate the covariance between the stock's and the market returns, represented by the Indonesia Composite Index (ICI). The formula for risk is given in Equation (3), and the covariance is used to measure how changes in one variable are related to changes in another variable.

$$B_i = \frac{\text{cov}(R_i, R_h)}{\sigma_h^2} \quad (3)$$

The formula of covariance to stock can be seen in Equation (4), and market variance are in Equation (5), respectively. Return R_{ht} is the market return represented by ICI.

$$\text{cov}(R_i, R_h) = \frac{\sum_{t=1}^T (R_{it} - E(R_i))(R_{ht} - E(R_h))}{T} \quad (4)$$

$$\sigma_h^2 = \frac{\sum_{t=1}^T (R_{ht} - E(R_h))^2}{T} \quad (5)$$

The methods employed in this research involve a hybrid approach combining Ant Colony Optimization (ACO) with Compromise Programming (CP). A hybrid of ACO and Nadir Compromise Programming (NCP) is also used.

The process begins by generating a set of feasible solutions, i.e., the proportion of the selected portfolio and determining the number of ants. The pheromone parameters are initialized uniformly. During each iteration, the pheromone values are updated based on the deviation as the objective value, with each ant selecting candidate solutions based on the updated pheromone levels.

2.1 Compromise Programming

Zeleny introduced the Compromise Programming (CP) method in 1974 [17]. Compromise Programming can solve multi-objective problems by finding the best compromise solution in optimizing two or more objectives. In Compromise Programming, we optimize f_i , minimize f_j , and maximize f_k then they can be written in Equation (6)-(8):

$$\text{Opt } f_i (i=1,2,3,...,N_A) \quad (6)$$

$$\min f_j (j=1,2,3,...,N_B) \quad (7)$$

$$\max f_k (k=1,2,3,...,N_C) \quad (8)$$

Overall, the CP model for optimizing f_i , minimizing f_j , and maximizing f_k as follows:

$$\min \left(\sum_{i=1}^{N_A} w_i (\delta_i^+ + \delta_i^-)^p + \sum_{j=1}^{N_B} w_j (\delta_j^+)^p + \sum_{k=1}^{N_C} w_k (\delta_k^-)^p \right)^{\frac{1}{p}} \quad (9)$$

Subject to in Equation (10)-(14) :

$$f_i + \delta_i^- + \delta_i^+ = f_{(i)}, \quad i=1,2,...,N_A \quad (10)$$

$$f_j - \delta_j^+ = f_j^{\min}, \quad j=1,2,...,N_B \quad (11)$$

$$f_k + \delta_k^- = f_k^{\max}, \quad k=1,2,...,N_C \quad (12)$$

$$\delta_i^-, \delta_i^+, \delta_j^+, \delta_k^-, w_i, w_j, w_k \geq 0 \quad (13)$$

$$\sum_{i=1}^{N_A} w_i + \sum_{j=1}^{N_B} w_j + \sum_{k=1}^{N_C} w_k = 1 \quad (14)$$

where the $f_{(i)}$ is the target function of i-th objective, f_j^{\min} is the ideal minimum of j-th objective function, f_k^{\max} is the ideal maximum of k-th objective function, δ^- is the negative deviation and δ^+ is the positive deviation.

The mathematical formulation can be seen in Equation (15)-(20) when applied to the portfolio optimization model.

$$\min Z = \frac{1}{3}(\delta_1^- + \delta_1^+) + \frac{1}{3}(\delta_2^-) + \frac{1}{3}(\delta_3^+) \quad (15)$$

Subject to :

$$\sum_{i=1}^n x_i + \delta_1^- - \delta_1^+ = N_0 \quad (16)$$

$$\sum_{i=1}^n E(R_i)x_i + \delta_2^- = N_0 \bar{R} \quad (17)$$

$$\sum_{i=1}^n B_i x_i - \delta_3^+ = N_0 Z \quad (18)$$

$$x_i \geq 0 \quad i = 1, 2, \dots, n \quad (19)$$

$$\delta_j^-, \delta_j^+ \geq 0 \quad j = 1, 2, 3 \quad (20)$$

With decision variables and parameters :

x_i , $i = 1, 2, \dots, n$ is the proportion of selected portfolio as decision variable

N_0 : total investment

$E(R_i)$: expected return of i-th stock

B_i : the risk of i-th stock

Z : risk of portfolio

Equation (15) represents the minimization of deviation variables derived from constraints (16)-(18). Constraint (16) ensures that the total proportion of the selected portfolio equals the available investment. Constraint (17) requires that the expected return of all selected stocks be greater than the average return, incorporating a deviation variable $+\delta_2^-$ for maximization. Lastly, constraint (18) limits the total portfolio risk to be less than or equal to a predefined risk threshold, using the deviation variable $-\delta_3^+$ for minimization.

2.2 Nadir Compromise Programming

Nadir Compromise Programming (NCP) is an extension of the Compromise Programming (CP) method, introduced in 2011. NCP is designed to address multi-objective optimization problems by simultaneously optimizing, minimizing, and maximizing objectives. This approach modifies the original CP framework to improve performance in specific contexts [17].

The general NCP model is formulated as Equation (21)-(26):

$$\min \left(\sum_{i=1}^{N_A} w_i (\delta_i^+ + \delta_i^-)^p + \sum_{j=1}^{N_B} w_j (-\delta_j^-)^p + \sum_{k=1}^{N_C} w_k (-\delta_k^+)^p \right)^{\frac{1}{p}} \quad (21)$$

Subject to :

$$f_i + \delta_i^- + \delta_i^+ = f_{(i)}, \quad i = 1, 2, \dots, N_A \quad (22)$$

$$f_j + \delta_j^- = f_j^{\min}, \quad j = 1, 2, \dots, N_B \quad (23)$$

$$f_k - \delta_k^+ = f_k^{\max}, \quad k = 1, 2, \dots, N_C \quad (24)$$

$$\delta_i^-, \delta_i^+, \delta_j^+, \delta_k^-, w_i, w_j, w_k \geq 0 \quad (25)$$

$$\sum_{i=1}^{N_A} w_i + \sum_{j=1}^{N_B} w_j + \sum_{k=1}^{N_C} w_k = 1 \quad (26)$$

When applied to portfolio optimization, the NCP model can be seen in Equation (27)-(32):

$$\min Z = \frac{1}{3}(\delta_1^- + \delta_1^+) - \frac{1}{3}(\delta_2^+) - \frac{1}{3}(\delta_3^-) \quad (27)$$

Subject to :

$$\sum_{i=1}^n x_i + \delta_1^- - \delta_1^+ = M_0 \quad (28)$$

$$\sum_{i=1}^n E(R_i)x_i - \delta_2^+ = M_0 \bar{R} \quad (29)$$

$$\sum_{i=1}^n \beta_i x_i + \delta_3^- = M_0 S \quad (30)$$

$$x_i \geq 0 \quad i = 1, 2, \dots, n \quad (31)$$

$$\delta_j^-, \delta_j^+ \geq 0 \quad j = 1, 2, 3 \quad (32)$$

Equation (27) represents the minimization of slack and surplus variables derived from the constraints in Equations (28)-(30). Constraint (28) ensures that the total proportion of selected stocks equals the available investment. Constraint (29) requires that the expected return of all selected stocks exceed the average return, with the slack variable $-\delta_2^+$ addressing the maximization requirement. Constraint (30) ensures that the overall portfolio risk remains below a certain threshold, with the surplus variable $+\delta_3^-$ managing the minimization aspect.

2.3. Ant Colony Optimization

Ant Colony Optimization (ACO) is an algorithm inspired by the behavior of ants in their search for food and nesting sites. In this algorithm, ants depart from the nest and traverse through multiple nodes, starting from the first layer (nest) to the last layer (food), ultimately stopping at their destination [30]. This method was introduced by Dorigo in 1990. For the portfolio selection model, the ACO algorithm can be structured as follows:

1. Set the number of ants N and the pheromone decay factor ρ .
2. Generate P feasible solutions i.e. the proportion of selected portfolio $X^k, k = 1, 2, \dots, P$ with the design $X = x_i, i = 1, 2, \dots, n$ with n is the number of stocks.

In generating population, there are some constraints that should be satisfied like

$\sum_{i=1}^n x_i = M_0$, so that initialization of feasible solutions can be constructed as follows

:

for $k = 1 : P$

$p = 0$

while ($p = 0$)

$q = \text{rand}(0, 1)$

if ($q \leq 0.5$)

take two stocks randomly

else

take three stocks randomly

end

- $$x_i \leftarrow \frac{x_i}{\sum_{i=1}^n x_i}, i = 1, 2, \dots, n$$
- Compute δ_1^-, δ_1^+
- $$\text{Compute } \delta_2^+ = M_0 \bar{R} - \sum_{i=1}^n E(R_i) x_i$$
- $$\text{Compute } \delta_3^- = \sum_{i=1}^n \beta_i x_i - M_0 S$$
- if ($\delta_1^-, \delta_1^+, \delta_2^+, \delta_3^- \geq 0$)
 $p = 1$
 end
 end
 end
3. Give the uniform probability.

$$p(X^k) = \frac{1}{P}, k = 1, 2, \dots, P \quad (33)$$
 4. Calculate cumulative probability range $C_k, k = 1, 2, \dots, P$
 5. Generate random variable $r_s \sim U(0,1), s = 1, 2, \dots, N$.
 6. Determine selected variable $X^k, k \in \{1, 2, \dots, P\}$ for every ant s .
 7. Calculate objective function $f(X^k)$ for every ant s .
 8. Choose minimum fitness function $f_{best} = \min(f(X^k), k \in \{1, 2, \dots, P\})$, and count N_{best} , the number of f_{best}
 9. Set constant Q and calculate $\sum \Delta\tau(X^k), k = 1, 2, \dots, P$

$$\sum \Delta\tau(X^k) = \begin{cases} N_{best} \cdot \frac{Q}{f_{best}}, & \text{if } X^k \text{ is the best variable} \\ 0, & \text{otherwise} \end{cases} \quad (34)$$
 10. Update the pheromone based on Equation (35)

$$\tau_k = (1 - \rho)\tau_k + \sum \Delta\tau(X^k), k = 1, 2, \dots, P \quad (35)$$
 11. Update the pheromone probability based on Equation (36)

$$p(X^k) = \frac{\tau_k}{\sum \tau_k}, k = 1, 2, \dots, P \quad (36)$$
 12. Repeat step 3-10 until all ants choose the best path consisting pheromone and process converges.

2.4. Data

The data used in the experiments for the hybrid Compromise Programming and Nadir Compromise Programming models are obtained from six stock datasets covering the period from January 2016 to December 2018. The stocks analyzed include Kimia Farma (KAEF), Telekomunikasi Indonesia (TLKM), Gudang Garam (GGRM), Matahari Department Store (LPPF), Garuda Indonesia (GIAA), and Bank Central Asia (BBCA). For each stock, the expected return is computed using Equation (2) based on return data over the selected period, while the risk is calculated using Equation (3). The results of these computations are presented in Table 1.

Table 1. Expected return and risk of each stock

Stock	Expected Return	Risk
KAEF	0.047	2.040
TLKM	0.005	0.396
GGRM	0.012	0.968
LPPF	-0.023	1.226
GIAA	-0.004	0.552
BBCA	0.020	1.180

With total investment $N_0 = 1$, risk $Z = 0.9$, and average of expected return $\bar{R} = 0.0094$

In Ant Colony Optimization, parameters used both in Compromise Programming and Nadir Compromise Programming are :

The number of ants : 10, 20, 30

Maximum iterations : 25, 50 100

3. Results and Discussion

3.1 Simulation Result of Compromise Programming

After calculating each stock's expected return and risk, the Ant Colony Optimization (ACO) algorithm was constructed using the earlier parameters. In each iteration, ants randomly select candidate solutions. The best solution from all ants is then identified, and the pheromone level for this solution is increased, improving its likelihood of being selected in subsequent iterations. The results of the simulation for the hybrid Ant Colony Optimization and Compromise Programming are shown in Figure 1.

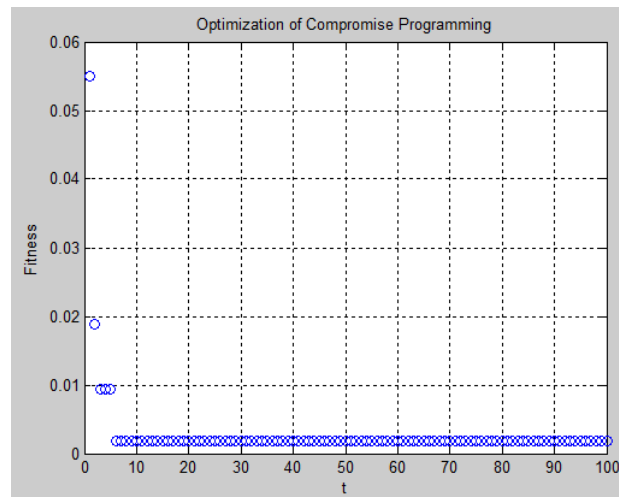


Figure 1. Simulation result of hybrid Ant Colony Optimization and Compromise Programming

The simulation shows that ants select candidate solutions randomly from the feasible set in the initial iteration, as the pheromone probability is evenly distributed. Once the best solution is identified, the pheromone probability for this solution is updated, increasing its chances of being selected in the next iteration. As the iterations progress, the algorithm converges, and after reaching the maximum iteration, the optimal investment proportions for each stock are determined. Table 2 shows that KAEF

and GIAA stocks are selected for investment, with proportions of 23.65% and 76.35%, respectively.

Table 2. Investment proportion for each stock

KAEF	TLKM	GGRM	LPPF	GGIA	BBCA
0.2365	0	0	0	0.7635	0

Based on the sum of deviation variables, the resulting fitness value is 0.00178. We further extended the experiment by varying the number of ants and iterations. Table 3 summarizes the results under different configurations. It can be seen that GGRM and GIAA stocks are frequently selected across different scenarios due to their relatively low risk, as indicated in Table 1, where their risk values are both less than 1.

The simulation results demonstrate that the hybrid ACO and Compromise Programming method can optimize portfolio selection by identifying stocks with favorable risk-return profiles. In particular, stocks like GGRM and GIAA exhibit lower risk. They are frequently selected across different iterations and ant configurations, indicating their robustness in various scenarios. This research provides valuable insights for investors seeking to balance risk and return in their portfolios. The hybrid approach offers a systematic way to minimize risk while maximizing returns, leading to more informed investment decisions. Additionally, the flexibility of the ACO algorithm in selecting optimal solutions based on pheromone probabilities highlights its potential in complex multi-objective optimization problems. The findings suggest that ACO, combined with Compromise Programming, can significantly improve portfolio optimization processes, offering theoretical contributions to optimization methods and practical implications for investment strategies.

Table 3. ACO on Compromise Programming with Different Numbers of Ants and Iterations

Total Ant	Maximum Iteration	KAEF	TLKM	GGRM	LPPF	GGIA	BBCA	Fitness
10	25	0	0.3561	0	0.3253	0	0.3186	0.00801
	50	0	0	0.8529	0.0155	0.1317	0	0.00591
	100	0.2365	0	0	0	0.7635	0	0.00178
20	25	0.2413	0	0	0	0.7587	0	0.00406
	50	0	0	0.3546	0	0.3182	0.3273	0.00165
	100	0	0	0.8461	0	0.1539	0	0.00147
30	25	0	0	0.8480	0	0.1520	0	0.00173
	50	0	0	0.1969	0	0.3775	0.4256	0.00039
	100	0.0371	0	0	0	0.4954	0.4676	0.00030

3.2 Simulation Result of Nadir Compromise Programming

The computation process for the hybrid Ant Colony Optimization (ACO) and Nadir Compromise Programming (NCP) is similar to that of Compromise Programming, with the primary difference being the deviation variables used. After computing each stock's expected return and risk, the ACO algorithm is constructed based on the defined parameters. In each iteration, ants randomly select candidate solutions. The best solution from all ants is then selected, and the pheromone levels for that solution are updated to increase its likelihood of being chosen in the subsequent iterations. The

simulation results for the hybrid ACO and Nadir Compromise Programming are shown in Figure 2.

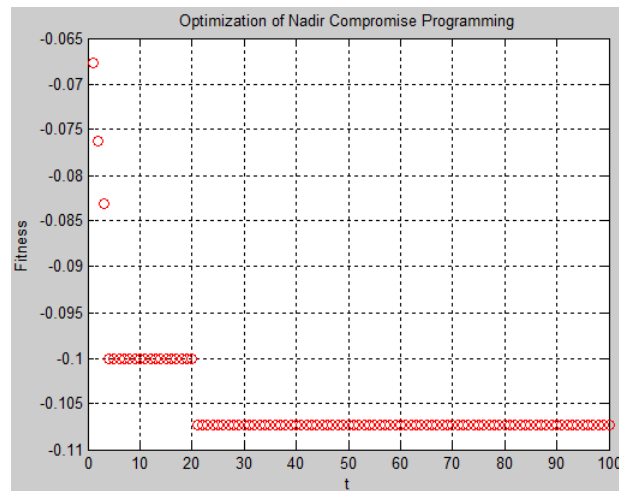


Figure 2. Simulation result of hybrid Ant Colony Optimization and Nadir Compromise Programming

In the initial iteration, ants select candidate solutions randomly due to the uniform pheromone distribution across all options. Once the best solution is identified, the probability of selecting that solution increases in subsequent iterations due to the pheromone update mechanism. The optimal investment proportions for each stock are determined upon reaching the maximum iteration. Table 4 shows that KAEF and TLKM stocks are selected for investment, with proportions of 10.88% and 89.12%, respectively.

Table 4. Investment proportion for each stock

KAEF	TLKM	GGRM	LPPF	GGIA	BBCA
0.1088	0.8912	0	0	0	0

Based on the sum of deviation variables, the resulting fitness value is -0.10835. Furthermore, we extended the experiment by varying the number of ants and iterations. Table 5 summarizes the results for different configurations. From the table, it can be observed that stocks KAEF and TLKM are frequently selected across different ant and iteration settings. This is because, as shown in Table 1, KAEF has the highest expected return, while TLKM exhibits the lowest risk.

Table 5. ACO on Nadir Compromise Programming with Different Numbers of Ants and Iterations

Total Ant	Maximum Iteration	KAEF	TLKM	GGRM	LPPF	GGIA	BBCA	Fitness
10	25	0.1203	0.8797	0	0	0	0	-0.10118
	50	0.1160	0.8840	0	0	0	0	-0.10344
	100	0.1088	0.8912	0	0	0	0	-0.10835
20	25	0	0.6827	0.0029	0	0	0.3145	-0.08453
	50	0.1429	0.8571	0	0	0	0	-0.08927
	100	0.1161	0.8839	0	0	0	0	-0.10341
30	25	0.1360	0.8640	0	0	0	0	-0.09290
	50	0.1269	0.8731	0	0	0	0	-0.09773
	100	0.1127	0.8873	0	0	0	0	-0.10519

The results of the Nadir Compromise Programming simulations indicate that the stocks KAEF and TLKM consistently emerge as the preferred investment options across various iterations and ant configurations. It is because KAEF has the highest expected return, making it an attractive option for maximizing profit. At the same time, TLKM exhibits the lowest risk, making it a stable choice for risk-averse investors. These findings demonstrate the effectiveness of the hybrid ACO and Nadir Compromise Programming approach in portfolio optimization, providing a robust method for balancing risk and return. The flexibility of this method allows for efficient exploration of multi-objective optimization problems, making it a valuable tool for investors seeking to construct well-balanced portfolios. Moreover, fine-tuning the number of ants and iterations offers additional control over the optimization process, ensuring that the results can be adapted to different investment scenarios.

4. Conclusion

This study demonstrates that the optimization models of Compromise Programming and Nadir Compromise Programming can effectively assist investors in determining the optimal portfolio composition, considering constraints such as investment amount, expected return, and risk. The critical contribution of this research is integrating the Ant Colony Optimization (ACO) algorithm with both Compromise Programming and Nadir Compromise Programming. Inspired by ants' behavior in searching for food and building nests through pheromone-based communication, ACO was used to explore feasible solutions for portfolio selection. As the iterations progress, ants refine their choices based on pheromone levels, leading to an optimal solution. The results indicate that Nadir Compromise Programming outperforms Compromise Programming by consistently selecting stocks with the highest expected return and the lowest risk. This makes it a more robust method for portfolio optimization. The approach minimizes deviation variables, resulting in a highly efficient fitness function that converges to the best portfolio configuration.

However, the study has some limitations. The model relies on historical stock data and does not account for potential future market changes or external factors that might influence stock performance. Additionally, the fixed weight assignment for portfolio components may limit the model's flexibility in handling more dynamic market conditions. For future research, exploring a fuzzy approach to determine the weight of each portfolio component in both Compromise Programming and Nadir Compromise Programming is recommended. This would allow for a more adaptable model to accommodate uncertainty and variability in market conditions better.

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