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A NUMERICAL STUDY OF EULER, RUNGE-KUTTA FOURTH-ORDER, AND RUNGE-KUTTA-FEHLBERG METHODS FOR SOLVING AN RLC CIRCUIT MODEL

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ABSTRACT

This study investigates the numerical solution of differential equation systems arising from resistor-inductor-capacitor (RLC) circuits using the Euler, fourth-order Runge-Kutta (RK4), and Runge-Kutta-Fehlberg (RKF) methods. The numerical solutions are evaluated against the analytical solution using the Root Mean Square Error (RMSE) under several step-size values. The results show that decreasing the step size consistently improves the numerical accuracy of all three methods. Among the methods considered, the Euler method produces the largest RMSE, while the RK4 method provides substantially higher accuracy. The RKF method achieves the smallest RMSE for all tested step sizes, demonstrating the highest numerical accuracy for solving the RLC circuit model. These findings provide practical guidance for selecting an appropriate numerical method and step size in RLC circuit analysis.

Keywords : Numerical Methods, RLC, Runge-Kutta, RMSE

I. INTRODUCTION

RLC circuits are widely utilized in electrical and electronic systems, particularly in signal filters, resonance circuits, frequency control devices, and communication equipment [1]. The behavior of these circuits can be modeled by a second-order differential equation involving current, voltage, resistance, inductance, and capacitance. Solving this equation is essential for analyzing system characteristics, including oscillation, damping, and the stability of voltage and current over time.

Hassan et al. [2] demonstrated that a two-loop RLC circuit can be modeled using differential equations and solved through both Laplace transform techniques and deep learning-based approaches. Furthermore, Idowu et al. [3] highlighted that second-order differential equation models can effectively describe the transient and steady-state behavior of RLC circuits under underdamped, critically damped, and overdamped conditions. Kafle et al. [4] further demonstrated that damping characteristics play a significant role in determining the transient response of RLC circuits.

The differential equation model of an RLC circuit can still be solved analytically. However, as the model becomes more complex or when system parameters vary with time, obtaining analytical solutions becomes increasingly challenging. Consequently, numerical methods provide an effective alternative for approximating solutions through iterative procedures. The fourth-order Runge-Kutta (RK4) method is widely recognized for producing highly accurate numerical solutions for ordinary differential equations [5]. This finding is consistent with the results reported by Malarvizhi and Karunanithi [6], who found that the RK4 method is effective in analyzing voltage and current behavior in electrical circuits.

Previous studies have demonstrated the effectiveness of higher-order numerical methods for solving differential equations. Abdul-Hassan et al. [7] proposed a third-order numerical scheme that combines the Runge-Kutta method with Taylor series expansion. Comparative studies by Banu et al. [8] showed that the Euler, RK2, RK4, and Runge-Kutta-Fehlberg (RKF) methods can provide accurate numerical solutions to initial value problems, although their performance depends on the choice of step size. Igwe et al. [9] reported comparable accuracy between RK4 and RKF when solving second-order differential equations, while highlighting the computational efficiency of RKF. Furthermore, Azarani and Sudin [10] concluded that RK4 performs well for problems with smooth solutions, whereas RKF offers greater flexibility through adaptive step-size control. These studies indicate that the performance of numerical methods depends not only on their order but also on the characteristics of the problem and the numerical settings employed, including the step size.

The effectiveness of numerical methods has also been demonstrated in several mathematical modeling applications. Alisa et al. [11] employed the RKF method to investigate the dynamics of disease transmission. Andiraja [12] utilized the RK4 method in the numerical simulation of a malaria transmission model. In addition, the Euler method has been widely used for the discretization of dynamical systems, including stability and bifurcation analyses [13], [14]. These findings demonstrate the broad applicability of numerical methods for solving systems of differential equations, including those arising from RLC circuit models. The accuracy of these methods can be assessed by comparing numerical and exact solutions using error measures such as the Root Mean Square Error (RMSE) [15].

The present study investigates the numerical solution of an RLC circuit model using the Euler, RK4, and RKF methods. The accuracy of each method is evaluated by comparing the numerical solutions with the analytical solution using RMSE under several step-size values. This analysis provides additional insight into the influence of step size on the numerical accuracy of the three methods for solving RLC circuit equations and offers practical guidance for selecting an appropriate numerical approach according to the required level of accuracy. The remainder of this paper is organized as follows. Section II presents the theoretical background and research methodology. Section III discusses the numerical results and their interpretation. Finally, Section IV summarizes the main findings and outlines directions for future research.

II. THEORETICAL BACKGROUND

2.1. Differential Equation Model of an RLC Circuit

An ordinary differential equation consists of a function and its derivatives with respect to a single independent variable. A general form of a first-order differential equation is presented in Equation (1).

$$\begin{aligned} \frac{dy}{dt} &= f(t,y), \\ y(t_0) &= y_0, \end{aligned} \tag{1}$$

where y denotes the dependent variable, t the independent variable, and y 0 the initial condition [16]. If the equation involves a second-order or higher-order derivative, it is classified as a higher-order differential equation. One important application of differential equations is the modeling of RLC circuit behavior.

A series RLC circuit consists of a resistor, an inductor, and a capacitor connected in a single electrical network. The resistor dissipates energy, whereas the inductor and capacitor store energy in magnetic and electric fields, respectively. The dynamic behavior of the circuit can be analyzed using Kirchhoff's Voltage Law, resulting in a second-order differential equation that describes the circuit response. The corresponding mathematical model is presented in Equation (2).

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t), \tag{2}$$

where L, R, C, q, and E denote the inductance, resistance, capacitance, electric charge, and the external voltage source, respectively [1].

2.2. Euler Method

The Euler method estimates the solution at the next time step using the slope evaluated at the current point. The numerical approximation is given by Equation (3).

$$y_{n+1} = y_n + hf(t_n, y_n), \tag{3}$$

where h denotes the step size and t_n represents the n-th time point. The method is simple to implement and computationally efficient. However, it has a global truncation error of order $O(h)$, which limits its accuracy compared with higher-order numerical methods [17].

2.3. Fourth-Order Runge-Kutta Method (RK4)

The RK4 method computes four intermediate slope approximations, namely κ_1, κ_2, κ_3, and κ_4, as given in Equation (4).

$$\begin{aligned}
\kappa_1 &= f(t_n, y_n),\\
\kappa_2 &= f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}\kappa_1\right),\\
\kappa_3 &= f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}\kappa_2\right),\\
\kappa_4 &= f(t_n + h, y_n + h\kappa_3).
\end{aligned} \tag{4}$$

The numerical solution at the next time step is obtained from the weighted average of these slope approximations, as expressed in Equation (5) [17].

$$y_{n+1} = y_n + \frac{h}{6}(\kappa_1 + 2\kappa_2 + 2\kappa_3 + \kappa_4). \tag{5}$$

2.4. Runge-Kutta-Fehlberg Method (RKF)

The RKF method extends the classical Runge-Kutta approach by generating both fourth-order and fifth-order approximations within a single computational step. The method is known for its high accuracy and is widely employed in the numerical solution of differential equations requiring precise approximations. The intermediate coefficients κ_1, κ_2, … κ_6 are evaluated as given in Equation (6).

$$\begin{aligned}
\kappa_1 &= hf(t_n, y_n),\\
\kappa_2 &= hf\left(t_n + \frac{1}{4}h, y_n + \frac{1}{4}\kappa_1\right),\\
\kappa_3 &= hf\left(t_n + \frac{3}{8}h, y_n + \frac{3}{32}\kappa_1 + \frac{9}{32}\kappa_2\right),\\
\kappa_4 &= hf\left(t_n + \frac{12}{13}h, y_n + \frac{1932}{2197}\kappa_1 - \frac{7200}{2197}\kappa_2 + \frac{7296}{2197}\kappa_3\right),\\
\kappa_5 &= hf\left(t_n + h, y_n + \frac{439}{216}\kappa_1 - 8\kappa_2 + \frac{3680}{513}\kappa_3 - \frac{845}{4104}\kappa_4\right),\\
\kappa_6 &= hf\left(t_n + \frac{1}{2}h, y_n - \frac{8}{27}\kappa_1 + 2\kappa_2 - \frac{3544}{2565}\kappa_3 + \frac{1859}{4104}\kappa_4 - \frac{11}{40}\kappa_5\right).
\end{aligned} \tag{6}$$

The fifth-order numerical approximation is then obtained through a weighted combination of these coefficients, as expressed in Equation (7) [8].

$$y_{n+1} = y_n + \frac{16}{135}\kappa_1 + \frac{6656}{12825}\kappa_3 + \frac{28561}{56430}\kappa_4 - \frac{9}{50}\kappa_5 + \frac{2}{55}\kappa_6. \tag{7}$$

2.5. Root Mean Square Error (RMSE)

The Root Mean Square Error (RMSE) is used to quantify the difference between numerical and exact solutions. The RMSE is computed using Equation (8), where a smaller RMSE value indicates higher numerical accuracy.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}, \tag{8}$$

where y_i, $\hat{y}_i$, and N represent the exact solution, numerical solution, and total number of data points, respectively. In this study, RMSE is employed as the primary criterion for evaluating and comparing the performance of the numerical methods [15].

III. RESULTS AND DISCUSSION

Numerical simulations were performed to solve the RLC circuit differential equation system using the Euler, RK4, and RKF methods over the interval $t \in [0,1]$ with a step size of $h = 0.01$. The governing equations for the current and voltage are given by Equations (9) and (10), respectively [18].

$$2I'' + 4I' + 20I = 10e^{-t}, \tag{9}$$

$$2V'' + \frac{1}{4}V' + 20V = 10e^{-t}. \tag{10}$$

To obtain the analytical solution of Equation (9), both sides are divided by 2, resulting in Equation (11),

$$I'' + 2I' + 10I = 5e^{-t}. \tag{11}$$

The corresponding homogeneous equation is obtained from Equation (11) by setting the right-hand side equal to zero, giving Equation (12),

$$I'' + 2I' + 10I = 0. \tag{12}$$

Substituting $I = e^{rt}$ into Equation (12) yields the characteristic equation shown in Equation (13),

$$r^2 + 2r + 10 = 0. \tag{13}$$

Solving Equation (13) gives the characteristic roots presented in Equation (14),

$$r = -1 \pm 3i. \tag{14}$$

Using the roots in Equation (14), the homogeneous solution is obtained as Equation (15),

$$I_h(t) = e^{-t}(C_1 \cos 3t + C_2 \sin 3t). \tag{15}$$

Substituting particular solution $I_p = Ae^{-t}$ into Equation (11) gives $A = \frac{5}{9}$. Therefore, the particular solution shown in Equation (16),

$$I_p(t) = \frac{5}{9}e^{-t}. \tag{16}$$

The general solution of Equation (9) is obtained by combining Equations (15) and (16), as shown in Equation (17),

$$I(t) = e^{-t}\left(C_1 \cos 3t + C_2 \sin 3t + \frac{5}{9}\right). \tag{17}$$

Applying the initial conditions $I(0) = -1$ and $I'(0) = 0$ on Equation (17) gives the constants $C_1 = -\frac{14}{9}$ and $C_2 = -\frac{1}{3}$. So that, the exact solution of $I(t)$ is

$$I(t) = e^{-t}\left(-\frac{14}{9}\cos 3t - \frac{1}{3}\sin 3t + \frac{5}{9}\right). \tag{18}$$

Following the same procedure, Equation (10) is rewritten as

$$V'' + \frac{1}{8}V' + 10V = 5e^{-t}. \tag{19}$$

The homogeneous solution of Equation (19) can be written as

$$V_h(t) = e^{-\frac{t}{16}}\left(K_1 \cos\frac{\sqrt{2559}}{16}t + K_2 \sin\frac{\sqrt{2559}}{16}t\right). \tag{20}$$

The particular solution of Equation (19) is

$$V_p(t) = \frac{40}{87}e^{-t}. \tag{21}$$

Adding Equations (20) and (21) produces the general solution,

$$V(t) = e^{-\frac{t}{16}}\left(K_1 \cos\frac{\sqrt{2559}}{16}t + K_2 \sin\frac{\sqrt{2559}}{16}t\right) + \frac{40}{87}e^{-t}. \tag{22}$$

The constants are evaluated by applying the initial conditions $V(0) = -1$ and $V'(0) = 0$ to Equation (22), giving

$$K_1 = -\frac{127}{87}, K_2 = \frac{513}{87\sqrt{2559}}. \tag{23}$$

Finally, inserting the constants from Equation (23) into Equation (22) yields

$$V(t) = e^{-\frac{t}{16}}\left(-\frac{127}{87}\cos\frac{\sqrt{2559}}{16}t + \frac{513}{87\sqrt{2559}}\sin\frac{\sqrt{2559}}{16}t\right) + \frac{40}{87}e^{-t}. \tag{24}$$

Equations (18) and (24) are employed as the exact solutions to validate the numerical results obtained using the Euler, RK4, and RKF methods.

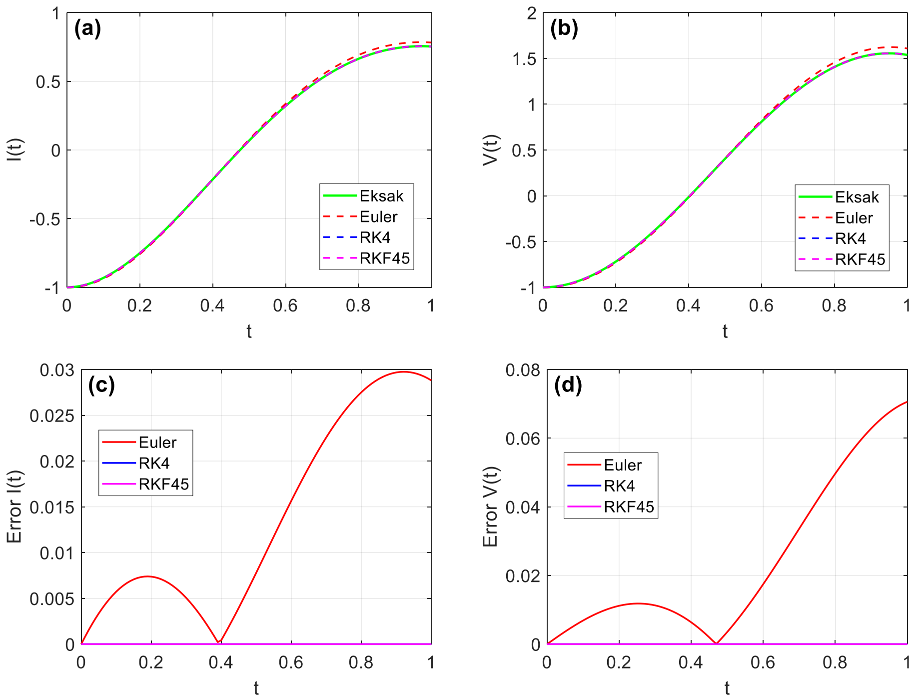

Figure 1: Plots of (a) $I(t)$, (b) $V(t)$, (c) Error of $I(t)$, and (d) Error of $V(t)$ Using the Euler, RK4, and RKF Methods

Figures 1(a) and 1(b) compare the numerical solutions of $I(t)$ and $V(t)$ with the corresponding exact solutions. The RK4 and RKF solutions are nearly indistinguishable from the exact solution, whereas the Euler solution exhibits noticeable deviations as t increases. Figures 1(c) and 1(d) show the corresponding error profiles of the numerical methods. The Euler method exhibits a substantial increase in error over the interval, whereas the RK4 and RKF methods maintain errors that remain close to zero. These results indicate that the RK4 and RKF methods provide significantly higher accuracy than the Euler method.

Table 1: Numerical Values of $I(t)$ Obtained Using the Euler, RK4, and RKF Methods

t	Solution of $I(t)$			
	Exact	Euler	RK4	RKF
$\mathbf{0}$	-1	-1	-1	-1
$\mathbf{0,1}$	-0.9311049958	-0.9368422989	-0.9311049998	-0.9311049958
$\mathbf{0,2}$	-0.7503782001	-0.7577550898	-0.7503782067	-0.7503782000
$\mathbf{0,3}$	-0.4982019231	-0.5032489536	-0.4982019306	-0.4982019231
$\mathbf{0,4}$	-0.2136925058	-0.2132577406	-0.2136925123	-0.2136925058
$\mathbf{0,5}$	0.0685510533	0.0763631563	0.0685510494	0.0685510533
$\mathbf{0,6}$	0.3207063991	0.3363591337	0.3207063990	0.3207063992
$\mathbf{0,7}$	0.5229714944	0.5455601435	0.5229714987	0.5229714944
$\mathbf{0,8}$	0.6638646964	0.6913717002	0.6638647050	0.6638646965
$\mathbf{0,9}$	0.7397250517	0.7694014171	0.7397250638	0.7397250517
$\mathbf{1,0}$	0.7536024629	0.7824057509	0.7536024773	0.7536024629

Table 1 presents the numerical values of $I(t)$ obtained using the Euler, RK4, and RKF methods together with the corresponding exact solution. All three methods are able to capture the overall behavior of the solution. However, differences in accuracy become more apparent as the simulation time increases. The Euler method exhibits progressively larger deviations from the exact solution, particularly near the end of the interval. For example, at $t = 1$, the Euler solution differs noticeably from the exact value. The RK4 and RKF methods produce results that are nearly identical to the exact solution throughout the interval. This behavior reflects the higher-order accuracy of RK4 and RKF, whereas the first-order Euler method accumulates larger numerical errors due to its reliance on a single slope evaluation at each time step.

Table 2 presents the absolute errors of the numerical solutions for $I(t)$ relative to the exact solution. It can be observed that at $t = 1$, the Euler method produces the largest errors, with values increasing over the interval and reaching 2.88033×10^{-2}. This indicates that the Euler method provides less accurate approximations of the current solution $I(t)$. The RK4 method yields substantially smaller errors 1.43569×10^{-8}. The RKF method achieves the highest level of accuracy, with errors 1.78657×10^{-12}. These results demonstrate that the RK4 and RKF methods provide significantly more accurate approximations of $I(t)$ than the Euler method, with RKF producing the smallest numerical errors.

Table 2: Numerical Error Values of the $I(t)$ Solution Using the Euler, RK4, and RKF Methods

t	Error of the $I(t)$ Solution		
	Euler	RK4	RKF
$\mathbf{0}$	0	0	0
$\mathbf{0,1}$	5.73730×10^{-3}	3.94000×10^{-9}	5.39169×10^{-12}
$\mathbf{0,2}$	7.37689×10^{-3}	6.63189×10^{-9}	1.58326×10^{-11}
$\mathbf{0,3}$	5.04703×10^{-3}	7.54132×10^{-9}	2.78125×10^{-11}
$\mathbf{0,4}$	4.34765×10^{-4}	6.54985×10^{-9}	3.81901×10^{-11}
$\mathbf{0,5}$	7.81210×10^{-3}	3.89359×10^{-9}	4.45919×10^{-11}
$\mathbf{0,6}$	1.56527×10^{-2}	6.58512×10^{-11}	4.56305×10^{-11}
$\mathbf{0,7}$	2.25886×10^{-2}	4.29697×10^{-9}	4.09528×10^{-11}
$\mathbf{0,8}$	2.75070×10^{-2}	8.52893×10^{-9}	3.11388×10^{-11}
$\mathbf{0,9}$	2.96764×10^{-2}	1.20333×10^{-8}	1.74962×10^{-11}
$\mathbf{1,0}$	2.88033×10^{-2}	1.43569×10^{-8}	1.78657×10^{-12}

Table 3 presents the numerical solutions of $V(t)$ obtained using the Euler, RK4, and RKF methods together with the corresponding exact solution. The results show that the RK4 and RKF methods produce values that are nearly identical to the exact solution throughout the interval, indicating a high level of numerical accuracy. In contrast, the Euler method exhibits increasingly larger deviations as t increases. For instance, at $t = 1$, the Euler method yields a value of 1.6085876639, whereas the exact solution is 1.5379937312. Meanwhile, the RK4 and RKF solutions remain extremely close to the exact value. These results demonstrate that the RK4 and RKF methods provide substantially more accurate approximations of $V(t)$ than the Euler method.

Table 3: Numerical Solutions for $V(t)$ Using the Euler, RK4, and RKF Methods

t	Solution of $V(t)$			
	Exact Solution	Euler	RK4	RKF
$\mathbf{0}$	-1	-1	-1	-1
$\mathbf{0,1}$	-0.9267376340	-0.9336224595	-0.9267376358	-0.9267376340
$\mathbf{0,2}$	-0.7184337622	-0.7296140243	-0.7184337677	-0.7184337622
$\mathbf{0,3}$	-0.4015408147	-0.4128401633	-0.4015408251	-0.4015408147
$\mathbf{0,4}$	-0.0124802464	-0.0188651838	-0.0124802617	-0.0124802464
$\mathbf{0,5}$	0.4060322626	0.4095586551	0.4060322436	0.4060322626
$\mathbf{0,6}$	0.8092399263	0.8267090612	0.8092399059	0.8092399263
$\mathbf{0,7}$	1.1547496060	1.1883986844	1.1547495872	1.1547496061
$\mathbf{0,8}$	1.4066518993	1.4563198246	1.4066518857	1.4066518994
$\mathbf{0,9}$	1.5389537737	1.6017985691	1.5389537684	1.5389537738
$\mathbf{1,0}$	1.5379937312	1.6085876639	1.5379937369	1.5379937314

Table 4: Numerical Error Values of the $V(t)$ Solution Using the Euler, RK4, and RKF Methods

t	Error of the $V(t)$ Solution		
	Euler	RK4	RKF
$\mathbf{0}$	0	0	0
$\mathbf{0,1}$	6.88483×10^{-3}	1.78780×10^{-9}	1.19416×10^{-11}
$\mathbf{0,2}$	1.11803×10^{-2}	5.53312×10^{-9}	1.86550×10^{-11}
$\mathbf{0,3}$	1.12993×10^{-2}	1.03906×10^{-8}	1.75344×10^{-11}
$\mathbf{0,4}$	6.38494×10^{-3}	1.52574×10^{-8}	7.38117×10^{-12}
$\mathbf{0,5}$	3.52639×10^{-3}	1.89466×10^{-8}	1.13671×10^{-11}
$\mathbf{0,6}$	1.74691×10^{-2}	2.03762×10^{-8}	3.66371×10^{-11}
$\mathbf{0,7}$	3.36491×10^{-2}	1.87463×10^{-8}	6.49696×10^{-11}
$\mathbf{0,8}$	4.96679×10^{-2}	1.36815×10^{-8}	9.19720×10^{-11}
$\mathbf{0,9}$	6.28448×10^{-2}	5.31755×10^{-9}	1.12927×10^{-10}
$\mathbf{1,0}$	7.05939×10^{-2}	5.68074×10^{-9}	1.23469×10^{-10}

The accuracy of the Euler, RK4, and RKF methods in approximating $V(t)$ can be further evaluated through the absolute error values presented in Table 4. The Euler method produces the largest errors, with values increasing throughout the interval and reaching 7.05939×10^{-2} at $t = 1$. This indicates that the Euler method provides less accurate approximations of the voltage solution. The RK4 method yields substantially smaller errors, ranging from 1.78780×10^{-9} to 2.03762×10^{-8}. The RKF method achieves the highest level of accuracy, with errors ranging from 7.38117×10^{-12} to 1.23469×10^{-10}. These results demonstrate that the RK4 and RKF methods provide significantly more accurate approximations of $V(t)$ than the Euler method, with RKF producing the smallest numerical errors. The observed differences in accuracy are closely related to the order of the numerical methods. As a first-order method, Euler accumulates larger numerical errors, whereas RK4, a fourth-order method, provides substantially more accurate approximations. The RKF method further improves accuracy through its higher-order formulation and built-in error estimation mechanism.

Table 5: RMSE of the Current $I(t)$ for Different Step Sizes

Step Size	Euler	RK4	RKF45
$\mathbf{0.1}$	2.132253×10^{-1}	7.648293×10^{-5}	3.364477×10^{-6}
$\mathbf{0.05}$	9.486987×10^{-2}	4.603697×10^{-6}	1.001240×10^{-7}
$\mathbf{0.01}$	1.733345×10^{-2}	7.174943×10^{-9}	3.073615×10^{-11}
$\mathbf{0.005}$	8.571767×10^{-3}	4.470778×10^{-10}	9.554736×10^{-13}
$\mathbf{0.001}$	1.699397×10^{-3}	7.134669×10^{-13}	2.234964×10^{-16}

Table 5: RMSE of the Current $I(t)$ for Different Step Sizes presents the RMSE values of the current $I(t)$ obtained using the Euler, RK4, and RKF45 methods for different step sizes, while Figure 2: RMSE of the Current $I(t)$ for Different Step Sizes illustrates the relationship between the step size and the RMSE on a logarithmic scale. As shown in Table 5: RMSE of the Current $I(t)$ for Different Step Sizes and Figure 2: RMSE of the Current $I(t)$ for Different Step Sizes, decreasing the step size consistently reduces the RMSE for all numerical methods. This indicates that a smaller step size reduces the discretization error, thereby improving the accuracy of the numerical solution.

The Euler method produces the largest RMSE for all tested step sizes. According to Table 5: RMSE of the Current $I(t)$ for Different Step Sizes, the RMSE decreases from 2.132253×10^{-1}at $h = 0.1$ to 1.699397×10^{-3} at $h = 0.001$. Although the error decreases as the step size becomes smaller, Figure 2: RMSE of the Current $I(t)$ for Different Step Sizes shows that the Euler method remains significantly less accurate than RK4 and RKF45 because it is a first-order numerical method. In contrast, the RK4 method exhibits a much faster reduction in RMSE. As reported in Table 5: RMSE of the Current $I(t)$ for Different Step Sizes, the RMSE decreases from 7.648293×10^{-5}at $h = 0.1$ to 7.134669×10^{-13}at $h = 0.001$. The same trend is clearly illustrated in Figure 2: RMSE of the Current $I(t)$ for Different Step Sizes, indicating that RK4 converges much faster than the Euler method.

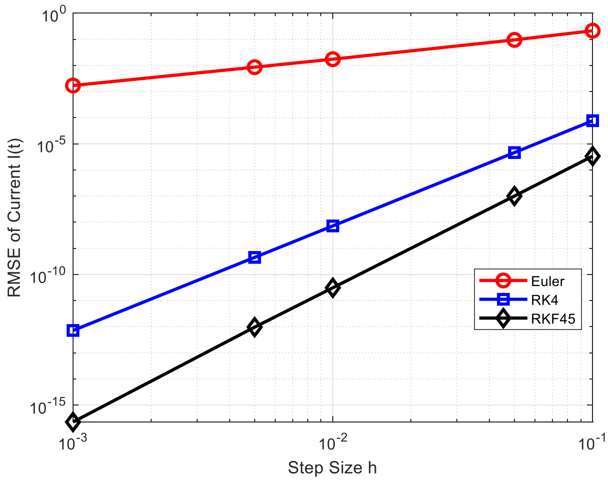

Figure 2: RMSE of the Current $I(t)$ for Different Step Sizes

Among the three methods, RKF45 achieves the highest numerical accuracy. As presented in Table 5: RMSE of the Current $I(t)$ for Different Step Sizes, the RMSE decreases from 3.364477×10^{-6}at $h = 0.1$ to 2.234964×10^{-16}at $h = 0.001$, which is close to machine precision. As shown in Figure 2: RMSE of the Current $I(t)$ for Different Step Sizes, the RKF45 curve consistently lies below those of Euler and RK4 for all tested step sizes, indicating that RKF45 produces the smallest numerical error throughout the simulations. These results are consistent with the theoretical properties of numerical methods, where higher-order methods have smaller truncation errors and therefore provide more accurate numerical solutions.

Table 6: RMSE of the Voltage $V(t)$ for Different Step Sizes

Step Size	Euler	RK4	RKF45
$\mathbf{0.1}$	3.854777×10^{-1}	1.427219×10^{-4}	5.917064×10^{-6}
$\mathbf{0.05}$	1.777487×10^{-1}	8.501220×10^{-6}	1.859855×10^{-7}
$\mathbf{0.01}$	3.284771×10^{-2}	1.305835×10^{-8}	5.974886×10^{-11}
$\mathbf{0.005}$	1.625240×10^{-2}	8.118622×10^{-10}	1.867646×10^{-12}
$\mathbf{0.001}$	3.223060×10^{-3}	1.293023×10^{-12}	4.883283×10^{-16}

Table 6: RMSE of the Voltage $V(t)$ for Different Step Sizes presents the RMSE values of the voltage $V(t)$ obtained using the Euler, RK4, and RKF45 methods for different step sizes, while Figure 3: Effect of Step Size on the RMSE of the Voltage $V(t)$ illustrates the effect of the step size on the RMSE. As shown in Table 6: RMSE of the Voltage $V(t)$ for Different Step Sizes and Figure 3: Effect of Step Size on the RMSE of the Voltage $V(t)$, reducing the step size consistently decreases the RMSE for all numerical methods. This indicates that a smaller step size improves the accuracy of the numerical solution by reducing the discretization error.

The Euler method produces the largest numerical error for all tested step sizes. According to Table 6: RMSE of the Voltage $V(t)$ for Different Step Sizes, the RMSE decreases from 3.854777×10^{-1} at $h = 0.1$ to 3.223060×10^{-3} at $h = 0.001$. Although the error decreases with a smaller step size, the

Euler method remains considerably less accurate than RK4 and RKF45. The RK4 method demonstrates a much faster reduction in numerical error. The RMSE decreases from 1.427219×10^{-4} at $h = 0.1$ to 1.293023×10^{-12} at $h = 0.001$, indicating that RK4 produces numerical solutions that are extremely close to the analytical solution even with relatively larger step sizes than the Euler method.

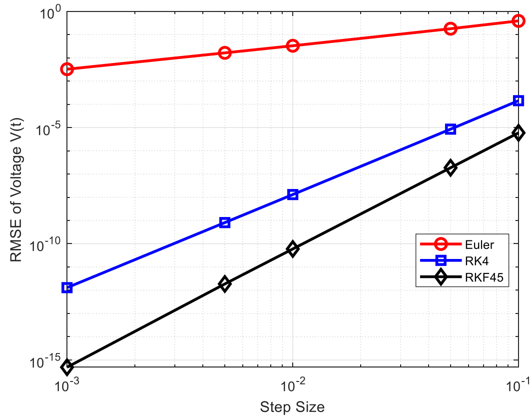

Figure 3: Effect of Step Size on the RMSE of the Voltage $V(t)$

Among the three methods, RKF45 again achieves the highest numerical accuracy. As presented in Table 6: RMSE of the Voltage $V(t)$ for Different Step Sizes, the RMSE decreases from 5.917064×10^{-6} at $h = 0.1$ to 4.883283×10^{-16} at $h = 0.001$, which is close to machine precision. Furthermore, Figure 3: Effect of Step Size on the RMSE of the Voltage $V(t)$ shows that the RKF45 curve consistently lies below the RK4 and Euler curves for all tested step sizes, indicating that RKF45 produces the smallest numerical error throughout the simulations. These findings are consistent with those obtained for the current $I(t)$, confirming that higher-order numerical methods provide superior accuracy compared with lower-order methods.

IV. CONCLUSION

This study investigated the numerical solution of an RLC circuit model using the Euler, RK4, and RKF45 methods. The results demonstrate that the choice of numerical method has a significant influence on the accuracy of the computed solutions. The Euler method produces the largest numerical errors and is therefore less suitable for applications requiring high accuracy. In contrast, the RK4 method provides numerical solutions that closely agree with the analytical solution while maintaining consistently low RMSE values. Among the three methods, RKF45 achieves the highest numerical accuracy, yielding the smallest RMSE for both current and voltage.

The analysis of different step sizes further shows that reducing the step size consistently decreases the RMSE for all three numerical methods. These results confirm that both the numerical method and the selected step size play important roles in determining the accuracy of the computed solutions. Although RKF45 provides the highest level of accuracy, RK4 also produces highly accurate results and can serve as an effective alternative for solving RLC circuit models. Future research may extend this work to more complex electrical circuit models, such as nonlinear systems, systems with time-varying parameters, or systems affected by noise. In addition, other numerical methods may be investigated and compared to further evaluate their performance in solving electrical circuit models.

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